# A New Method for Eliminating blur Caused by the Rotational Motion of the Images

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*Abstract*— **The aim of this investigation is to evaluate a new method for blur elimination caused by rotational motion of any object, assuming at the center and the angle of the rotation are known. In this new method, which is called Enhanced Rotational De-convolution Conjugate Gradient (ERD-CG), first we enhanced the RD method for the blurred images, and then we combined the ERD method with the CG method (which is the reconstruction of iterative method). The experimental results are promising and show that the proposed method outperforms both RD and CG, significantly.**

*Index Terms*— **Radial blurring, Image restoration, Image deblurring, Motion blur**

## I. INTRODUCTION

When we take a picture from an object with a rotational movement, this image will be blurred. De-blurring of the image is a complex procedure, because of its spatial variant. Processing of the pictures, include pictures of a real-world, three-dimensional scene on a two-dimensional plane. Depending on the application, different cameras to capture images of real scenes and punctuation, they can be used. In the ideal case, the camera records the image such that the intensity and the density of pixels in images are recorded exactly proportional to the intensity of the scene to be photographed.

However, it is very frequent that the recorded intensity of a pixel is coordinated with the intensity of larger neighborhood of the corresponding section of the scene. This effect is generally known as a blur. In order to make the first step of forming a blurry of conceptual interpretation of mathematical models, some samples may be considered. The description of the image formation process, which may be susceptible to camera movement, may produce spatially invariant or spatially variant blurs.

For a real imaging system, the captured image of an object that is rotating loses its sharpness. Complex movements such as rotation can cause spatially variant blurs [1].

Although the de-blurred image in digital photography is a known issue, most studies have focused on spatially invariant blurs and few investigations have been done on the compound and complex movements or spatially variant blurs. Removing rotational motion blur could be solved by using different techniques which could be divided in two classes, i.e., hardware and software techniques. An example of a hardware technique used by astronomical photographers is solving the problem by using extremely high quality tracking telescope mounts. Software techniques usually include post-processing of the image using various numerical algorithms to estimate the original, de-blurred image.

Although hardware techniques have better results, in most cases they are difficult and expensive, so software techniques are usually used to solve this problem. Although the de-blurred image in digital photography is a known issue, most studies have focused on linear blurring.

In some studies, Brenham's circle algorithm is used. These studies, first divide the image into a series of concentric blurring paths. The blur is then removed path by path [2].

Some other methods have re-covered the rotationally blurred images by transforming the rectangular coordinate of the image into polar coordinates or lattices, in which the transformed blur is spatially invariant. After that they applied some restoration techniques applicable in rectangular coordinates, and the de-blurred images were finally returned by the inverse transformation [3, 4].

Shan, et al. [5] designed an iterative optimization in three steps, to estimate all the unknowns:
1) Rotational motion of the object.
2) The de-blurred image.
3) The transparency map, which required a few of user's interactions.

The method proposed in present study, is different from all of the methods mentioned above. Our approach is somehow similar to the rotational de-convolution algorithm and we intend to improve this algorithm. In this method, it makes no difference if the blurriness of the image is caused by the movement of the object or the movement of the camera. We presume that the image is disrupted and try to fix and reconstruct the image anyways. We proposed a hybrid algorithm that uses two methods of de-blurring images in sequence.

The rest of the paper is organized as follows. Fundamental concepts of rotational blur are presented in Section 2. Section 3 is devoted to introduction of the proposed method. Experimental results are given in Section 4. Finally, Section5 concludes the paper.

## II. ROTATIONAL BLUR

Consider having a 3D point that is projected from the real world onto the image plane, where it is represented in 2D coordinates, so the exposure time T is taken into account and the acquired image can be written as the following integral:

$$I(x) = \int_0^T \Delta I(x, t) dt \qquad (1)$$

I.e. I(x); $X = (x_1, x_2) \epsilon X$, where X represents the image domain and $(x_1; x_2)$ the image coordinates[1, 6, 7, 8].

If the camera has a simple movement without acceleration and parallel to the screen shots during detection, each of the sub-images $\Delta I(x, t_i)$ proportional to each other are shifted. This is a spatially invariant blur that average neighboring pixels according to the direction and the length of this motion and the blurred image would become:

$$B(y) = \frac{1}{T} \int_0^T I_0(x - lt) dt \qquad (2)$$

Where $y \epsilon R^2$ are the blurred images spatial coordinate.



Fig.1 Rotational motion blur[15].

Now, each of the aforementioned sub-images would be rotated with respect to each other, i.e. if a more complex motion such as a rotation is considered now, $\Delta I(x, t_i) \approx \Delta I(Rx, t_{i-l}) \approx I_0(R_i x)$ where R is a

rotation matrix and $R_i = \prod_{j=l}^{i} R_j$. The blur is now space variant and the arc of the blur increases proportionally to the image radius, with neighboring pixels along that arc being averaged. If is imaginable an image in polar coordinates ($\rho$,$\theta$), the blur then becomes spatially invariant along the $\theta$ axis and can be modeled exactly like the linear motion example. The blurred image will be:

$$B(y) = \frac{1}{T} \int_0^T I_0(R_t x) dt \qquad (3)$$

Here $R_t$ is the rotation matrix at t (time) [1, 6, 7, 8].

In the uniform circular motion, if a pixel impacted the adjacent pixels, the result will be circular blurring. Therefore, image blurring caused by the uniform circular motion, can be modeled using a rotational convolution. If the content of all pixels over the circumference of a supposed circle (the center of this supposed circle, coincides with the center of the blurred image) Fig. 2, were recollected in a linear array and to repeat this same operation for each radius, by using the rotational convolution of each linear arrays with blur vector (BV), we can construct the circular blurring. BV is also a linear vector and its length is equal to the number of pixels affected. As we get far away from the center of the circulation, larger the length of the blurring vector (Obviously, whatever the greater the radius of the arc, the BV becomes larger and affecting on each other).
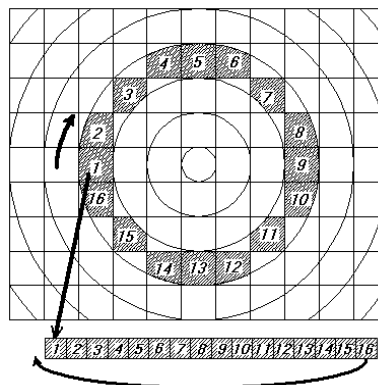


Fig 2. Pixels placed on the supposed circle[9].

In a linear vector, the amount of each element of BV is equal to the inverse of vector's length and by increasing the length of this vector; one pixel affects the following pixels. This affection will decrease gradually.
Rotational convolution can be defined as the following:

$$y[n] = \sum_{m=0}^{N-1} x_2[m] \, x_1 \left[ \left( (n-m) \right)_N \right] \qquad (4)$$

If the elements of an image, are placed on an arc, they will be organized respectively, x1[m] will be created and $x_2[m]$ is the same BV. Also the symbol of $\left( (n-m) \right)_N$ indicates the residue modulo N.

In blurring image process, each pixel has an effect on several following pixels. In the blurring circle, the first and the last element are jointed between each other, therefor we have to presume that the image is intermittent and this intermittence in the rotational convolution is considered as relation equation .4 [9].

## III. The Proposed Method: ERD-CG

As explained before, circular de-blurring can be done via several methods. In this work, the basic method is the circular de-convolution. The first goal of the present study was to improve this method. Circular de-convolution is a fast and simple method in which the image is firstly mapped to a greater matrix, considering the coincidence of the center of blurring with the center of image.

There is a function with the following inputs:

    A) Coordinates of the rotating center.

    B) The angle of rotation.

Using this function, we take samples from the elements of each circle, and we store their addresses. Referring to obtained addresses, for different radiuses, stored the light of the main image's points, are stored in a linear array.

First of all, the discrete Fourier transform is extracted from BV[9]:

$$Y[k] = DFT\{y[n]\} = \sum_{n=0}^{N-1} y[n]e^{-j\frac{2\pi}{N}kn} \qquad (5)$$

$$X_2[k] = DFT\{x_2[n]\} = \sum_{n=0}^{N-1} x_2[n]e^{-j\frac{2\pi}{N}kn} \qquad (6)$$

According to equation (6), each of the linear vectors of the image's array, will be divided into the corresponded BV, and then the inverse discrete Fourier transform is taken:

$$\hat{X}_1[k] = \frac{Y[k]}{X_2[k]} \qquad (7)$$

$$\hat{x}_1[k] = IDFT\{\hat{X}_1[k]\} = \sum_{n=0}^{N-1} \hat{X}_1[k]e^{j\frac{2\pi}{N}kn} \qquad (8)$$

Since the discrete Fourier transform of BV, becomes similar to the sync function (in its real quantities, is equal to zero), placing it in the denominator, is invalid. To solve this problem by using the interpolation function, the vector X is calculated from the vector K, thus the obtained vector will not be zero[9]:

$$\hat{X}_1[k] = \frac{Y[k]}{\hat{X}_2[k]} = \frac{Y[k]}{X_2[k+\delta]} \qquad (9)$$



Fig 3. Deblurring image by rotational de-convolution[9].

As shown in fig. 3, the rotational de-convolution's method has three major problems:

- The corners of the image cannot restored and are usually dark.
- The rotating center is not perfectly restored, and even in some cases, no restoration is seen.
- The restored image includes high amount of noise.

Hence, we tried to enhance the method of rotational de-convolution. As the first step, we enhanced the RD method of the blurring images, and called it ERD (Enhanced RD). The failure observed in reconstruction of the image's corners, was due to the following action done by the RD method: the largest radius of circular vectors was determined as half of the image size. Thus, the algorithm is not able to reconstruct the image for larger radiuses. By using zero padding in the corners of the blurred image and converting it into a new image, we become able to reconstruct the corners of the image. In the next step, using Wiener filter, the noise of the resulting image was reduced was reduced to an acceptable level.

After the above process was carried out, the first problem mentioned for RD was highly solved, however, the failure in reconstruction of the rotating center as well as the high level of noise were still present. Therefore, we decided to combine the ERD method with CG [10, 11].

Conjugate Gradient (CG) is a very useful and famous method in solving the large scale problem in numerical analysis, which is actually a sub system of Krylov solutions. The (CG) method is for the right positive rotating. Thus, it is possible to use the (CG) method also in normal systems of equations as well. A pseudo code for this method is given below [12, 13]:

$$x^{(0)} = \text{starting vector } (\text{e.g}, \text{zero})$$
$$r^{(0)} = b - Ax^{(0)}$$
$$d^{(0)} = A^T r^{(0)}$$
$$\text{for } k = 1, 2, \dots$$
$$\bar{a}_k = \frac{\left\| A^T r^{(k-1)} \right\|_2^2}{\left\| A^T d^{(k-1)} \right\|_2^2}$$
$$x^{(k)} = x^{(k-1)} + \bar{a}_k d^{(k-1)}$$
$$r^{(k)} = r^{(k-1)} + \bar{a}_k A d^{(k-1)}$$
$$\bar{\beta}_k = \frac{\left\| A^T r^{(k)} \right\|_2^2}{\left\| A^T r^{(k-1)} \right\|_2^2}$$
$$d^{(k)} = A^T r^{(k)} + \bar{\beta}_k d^{(k-1)}$$

In present study, we tried two different algorithms in turn, in order to reconstruct images. First, we used the ERD method for blur elimination of a blurred image and took the output as the initial guess. Then we gave this initial guess to the CG algorithm, so this algorithm will process the de-blurring action for the second time. The new output obtained, is our final de-blurred image. We denoted this hybrid algorithm as ERD-CG.
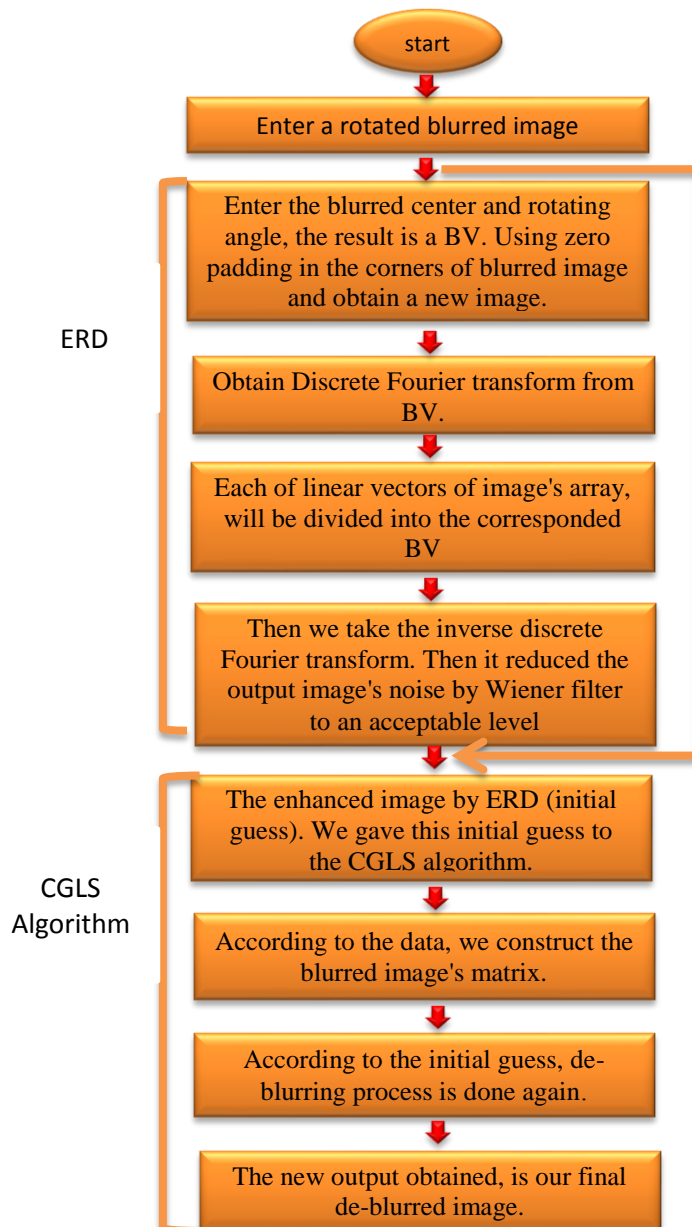
```
                        ┌──────────┐
                        │  start   │
                        └────┬─────┘
                             ▼
              ┌──────────────────────────────────┐
              │    Enter a rotated blurred image   │
              └──────────────┬───────────────────┘
                             ▼
              ┌──────────────────────────────────┐
              │ Enter the blurred center and rotating │
              │ angle, the result is a BV. Using zero │
              │ padding in the corners of blurred image │
    ERD       │ and obtain a new image.          │
              └──────────────┬───────────────────┘
                             ▼
              ┌──────────────────────────────────┐
              │ Obtain Discrete Fourier transform from │
              │ BV.                              │
              └──────────────┬───────────────────┘
                             ▼
              ┌──────────────────────────────────┐
              │ Each of linear vectors of image's array, │
              │ will be divided into the corresponded │
              │ BV                              │
              └──────────────┬───────────────────┘
                             ▼
              ┌──────────────────────────────────┐
              │ Then we take the inverse discrete │
              │ Fourier transform. Then it reduced the │
              │ output image's noise by Wiener filter │
              │ to an acceptable level           │
              └──────────────┬───────────────────┘
                             ▼
              ┌──────────────────────────────────┐
              │ The enhanced image by ERD (initial │
              │ guess). We gave this initial guess to │
              │ the CGLS algorithm.              │
              └──────────────┬───────────────────┘
   CGLS                      ▼
  Algorithm   ┌──────────────────────────────────┐
              │ According to the data, we construct the │
              │ blurred image's matrix.          │
              └──────────────┬───────────────────┘
                             ▼
              ┌──────────────────────────────────┐
              │ According to the initial guess, de- │
              │ blurring process is done again.  │
              └──────────────┬───────────────────┘
                             ▼
              ┌──────────────────────────────────┐
              │ The new output obtained, is our final │
              │ de-blurred image.                │
              └──────────────────────────────────┘
```

Fig 4. The flowchart of Suggested algorithm[15]


## IV. EXPERIMENTAL RESULTS

Our obtained algorithm was implemented by the MATLAB software, and applied on different images with different angles of rotation. The results are shown here.

In researches those have similarity to our investigation; we could find that in most of them, the rotating center was exactly coinciding with the image's center. But in our investigation, we try to de-blur images with a center of rotation in anywhere placed on the image (coinciding the rotating center with the center of image or not).

Quality of image is one of the most practical and important topic among the image processing threads. Determining the loss of image quality is one of the key points in the evaluation of most of relevant methods, specially the noise elimination and de-blurring of images. Some methods like the Mean Square Error, and Peak Signal to Noise Ratio (PSNR), they don't consider the object's structure in images, they

do not also consider the role of pixel's position in images, even the different impacts of pixels with respect to their position to the human eye.

There are two major categories of image quality diagnosis:

1- Quality diagnosis by having the fully reference image (FR).
2- Quality diagnosis without a reference image (NR).

Obviously the NR model is more complex, so that most of quality diagnosis models are FR (the NR model are in experimental phase yet). In traditional method the quality diagnosis is based on parametrical signals like SNR, PSNR, MES and etc. In these methods, image quality, is based on the mean change of different pixels gray scale, to the original value obtained.

There is also another new method to evaluate the quality of a reconstructed image which is Structural Similarity Index (SSIM)[14].
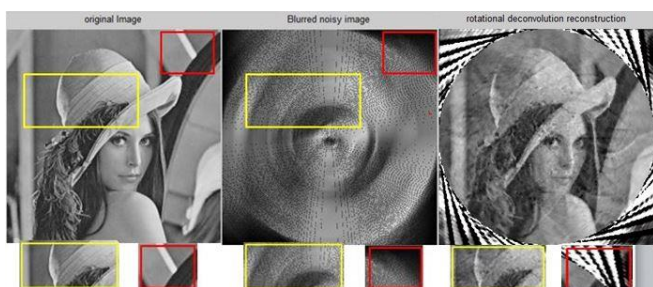


Fig 5. De-blured image by RD method with a rotation angle of 20 degree[15].
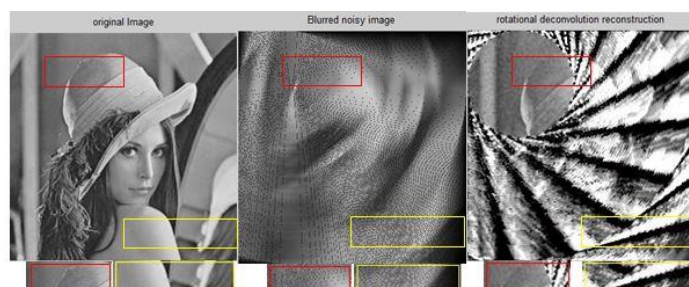


Fig 6. De-blurred image by RD method with rotation angle of 20 degree, If the of rotating center doesn't coincide with the center of image[15].
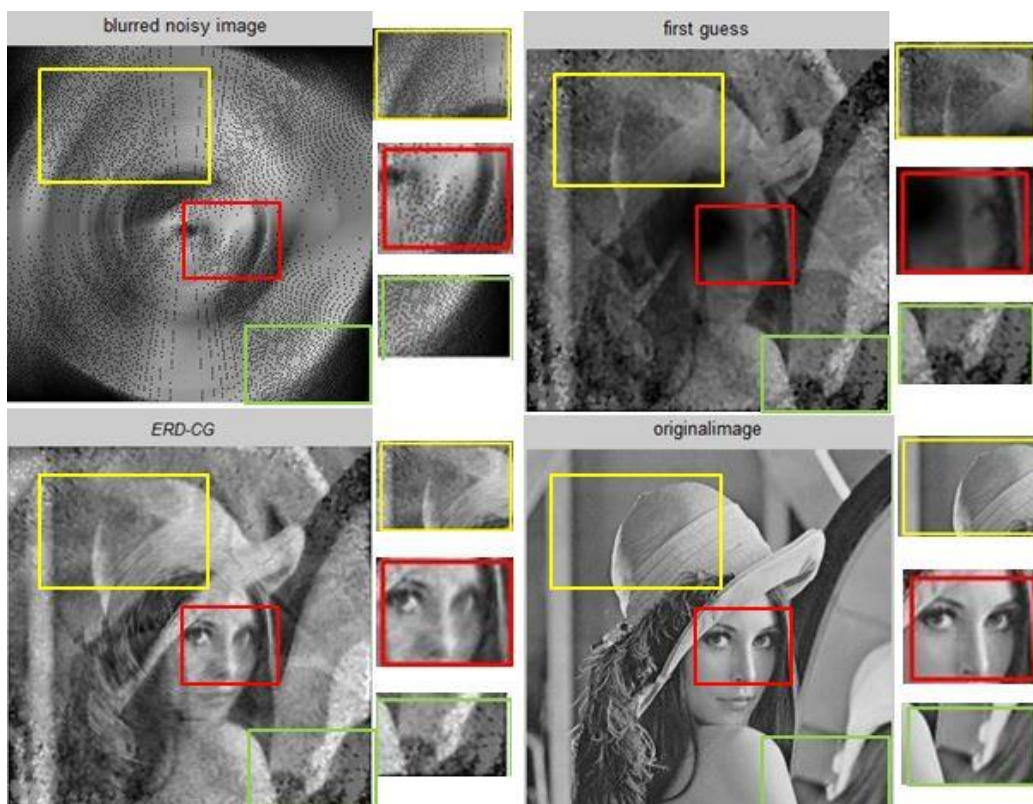


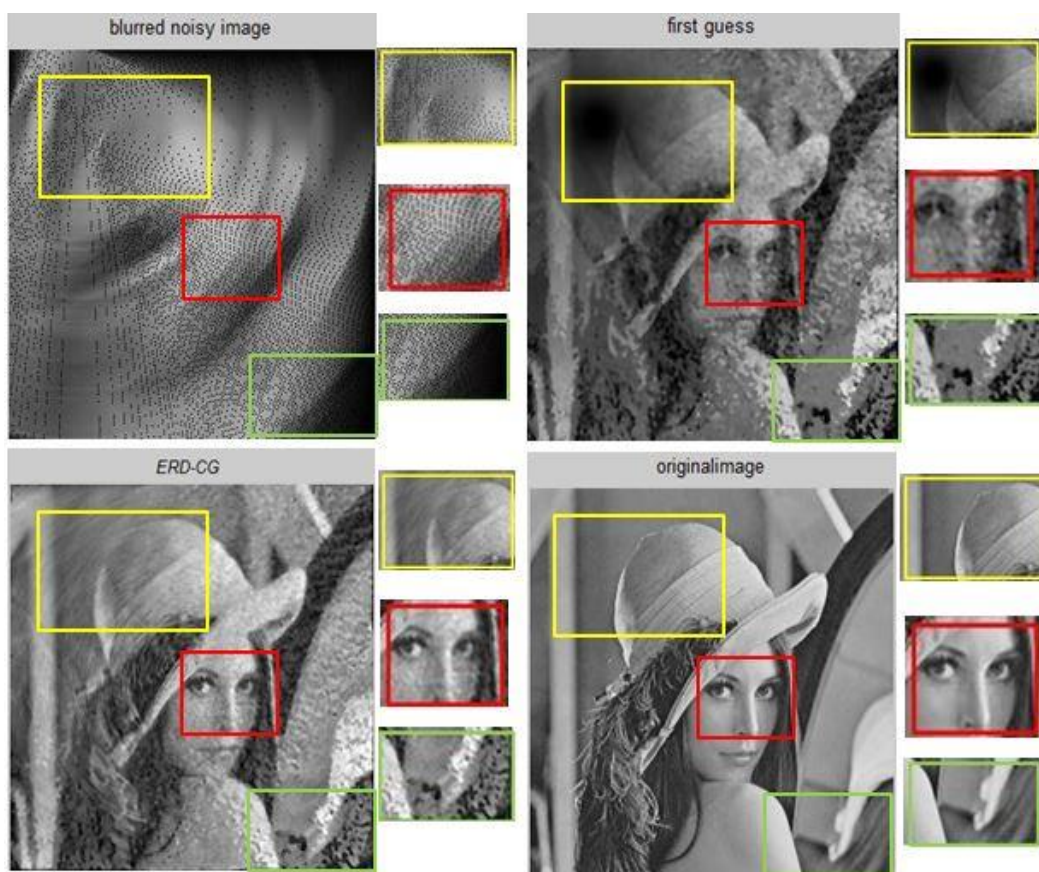Fig 7. De-blurred image by ERD and ERD-CG methods with a rotation angle of 20 degree and 2% of noise[15].

Fig 8. De-blurred image by ERD and ERD-CG methods with rotation of angle 20 degree and 2% of noise, the rotating center, doesn't coincide with the image's center[15].

Table I. The comparison of PSNR of each method (here the rotating center, coincides with the image's center)[15].

| rotation angle | Lucy-Rich | RD | ERD | Landweber | ERD-CG |
|---|---|---|---|---|---|
| 10 | 10.01 | 14.34 | 18.74 | 19.34 | 23.34 |
| 20 | 9.94 | 13.19 | 17.5 | 18.5 | 22.33 |
| 40 | 8.45 | 13.54 | 13.68 | 14 | 21.51 |
| 60 | 8.45 | 12.55 | 12.17 | 11.8 | 20.04 |
| 80 | 8.3 | 12.83 | 10.49 | 11.7 | 19.69 |

Table II. The comparison of PSNR of each method (here the rotating center, doesn't coincide with the image's center)[15].

| rotation angle | Lucy-Rich | RD | ERD | Landweber | ERD-CG |
|---|---|---|---|---|---|
| 10 | 5.5 | 10.58 | 17.81 | 11.34 | 23.83 |
| 20 | 6.03 | 9.61 | 16.08 | 11.18 | 22.85 |
| 40 | 3.05 | 8.79 | 15.74 | 10.32 | 22.39 |
| 60 | 2.68 | 6.98 | 10.41 | 9.01 | 18.77 |
| 80 | 2.4 | 7.85 | 8.63 | 7.43 | 18.76 |

Table III. The comparison of SSIM of each method (here the rotating center, coincides with the image's center)[15].

| rotation angle | RD | ERD | ERD-CG |
|---|---|---|---|
| 10 | 0.45 | 0.52 | 0.48 |
| 20 | 0.44 | 0.48 | 0.49 |
| 40 | 0.50 | 0.41 | 0.52 |
| 60 | 0.47 | 0.39 | 0.37 |
| 80 | 0.41 | 0.35 | 0.40 |

Table IV. The comparison of SSIM of each method (here the rotating center, doesn't coincide with the image's center)[15].

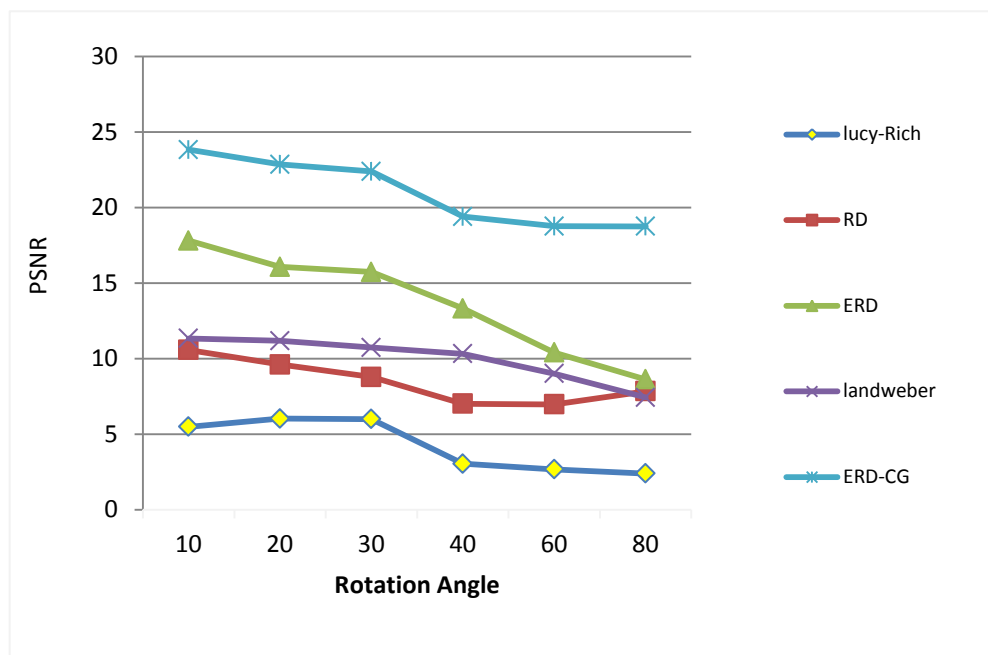| rotation angle | RD | ERD | ERD-CG |
|---|---|---|---|
| 10 | 0.10 | 0.49 | 0.49 |
| 20 | 0.23 | 0.45 | 0.51 |
| 40 | 0.21 | 0.43 | 0.53 |
| 60 | 0.24 | 0.33 | 0.35 |
| 80 | 0.23 | 0.27 | 0.29 |

Fig 9. The comparison of PSNR according to the rotating angle of each method
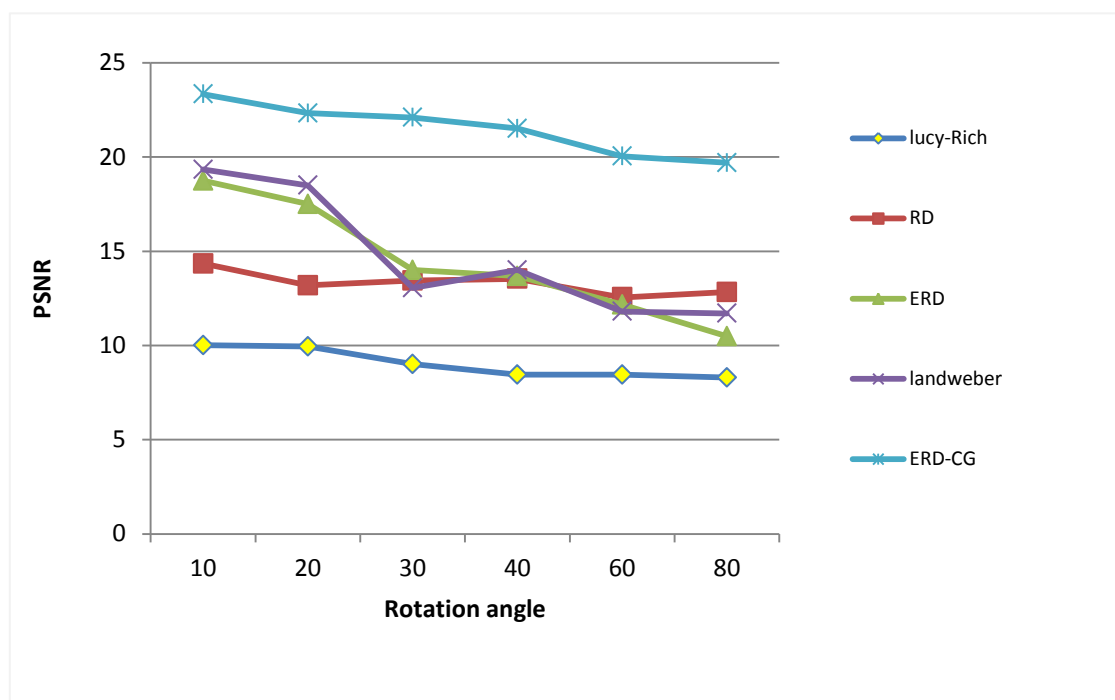(here the rotating center, coincides with the image's center)[15].



Fig 10. The comparison of PSNR according to the rotating angle of each
method (here the rotating center, doesn't coincide with the image's center)[15].

Fig 11. De-blurred image by Lucy-Richardson, RD, Landweber, ERD and ERD-CG methods with rotation of angle degree and 2% of noise[15].
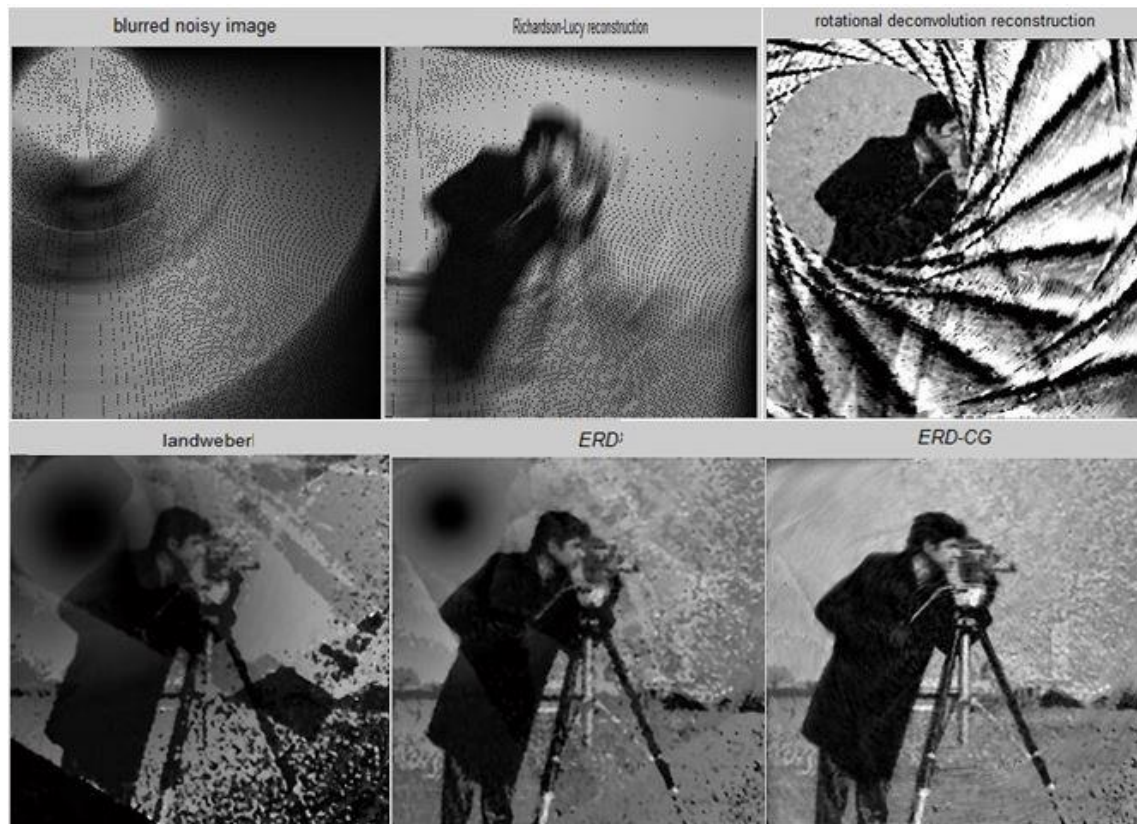


Fig 12. De-blurred image by Lucy-Richardson, RD, Landweber, ERD and ERD-CG methods with rotation of angle 80 degree and 2% of noise, the rotating center, doesn't coincide with the image's center[15].

As can be seen from the charts above, our suggested algorithm succeeded in to restore the image to the desired shape. For blurred images in which the rotating center doesn't coincide with the image's center, the result is about twice as better as the existing methods. Also, when the rotation center coincides with the image center, existing methods fail to act. Especially in the reconstruction of RD, and the reconstructed image is not acceptable. However, as shown through experimental results, the proposed algorithm (ERD-CG) is a very good and effective solution for both cases, i.e., when the rotating center coincide with the image's center and vice versa.

## V. CONCLUSION

The method introduced above is a non-blind method that needs user's information for image reconstruction. But two reasons made this justifiable:  first, in astronomical applications as astrophotography that is aimed for future work, we have access to this information (blurred center and rotating angle) precisely. Second according to two-step construction, it is observed that this method is robust against non-precise information. Although in De-convolution methods the rotational speed is very high, it is unable to reconstruct the image corners successfully, and the resulting image has noises. This occurs when the center of rotation coincides with the center of image, and the result is not clear in such cases. The method proposed in the present study (ERD-CG) is a hybrid of our enhanced version of rotational de-convolution method (ERD) and the iterative reconstruction method. As seen through the experiments, the proposed method solves the main challenges of the existing methods and significantly outperforms its counterparts. Using ERD-CG, restoration of the corners, the edges and the center of rotation are all performed effectively.

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