Noise and metabolic free energy in high-order biocognition

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We examine the possible role of ‘noise’ as a control signal for large-scale cognitive biological phenomena that recruit simpler cognitive modules into temporary, dynamic working coalitions. Noise color, as well as magnitude, may convey essential control information, a possibly important extension of the Data Rate Theorem. An adaptation of the Black-Scholes model suggests the availability of metabolic free energy can determine rates of coalition biocognition in the presence of noise. Evolutionary process may have exapted colored noise as a subtle tool for the regulation of biological phenomena, supplementing direct molecular signals. Experimental verification of this conjecture may be similarly subtle.

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The life course of a higher organism is dominated by a spectrum of sophisticated systems that recruit lower level cognitive modules into dynamic coalitions to meet changing patterns of threat and affordance [1-3]. These systems include gene expression, wound healing, immune response, animal consciousness, and so on, and operate at vastly different rates. For a large animal, these particular examples typically have time constants of years, months, hours, and fractions of a second.

Most notably, neural phenomena are strongly dominated by signals that are difficult to interpret as anything but ‘noise’, making determination of any ‘neural code’ a challenging enterprise indeed. As Koch [4] puts the matter, it has recently been realized that cortical networks are primarily driven by stochastic internal fluctuations of inhibition rather than by excitatory feed-forward input – spikes are preceded by a momentary reduction in inhibition rather than by an increase in excitation. For microscopic models of individual neurons, the phenomenon has been characterized as ‘inverse stochastic resonance’ [5,6]. Here, we will provide a model of macroscopic stochastic logic gates similarly triggered by a reduction in ‘noise’.

That is, we suggest ‘noise’, tinted in various colors, can be a powerful control signal for such large-scale recruitment processes. We use methods similar to those applied to the study of noise-driven phase transitions [7]. A further noise argument, via a Black-Scholes model, illuminates the influence of metabolic free energy supply rate on the rate of coalition cognition.

The Baars model of animal consciousness [3,8,9] posits the phenomenon as the assembly of unconscious cognitive modules into shifting working arrangements that address sensory or internal signals exceeding a dynamic threshold. Wallace [3] argues that the underlying mechanism is general, a consequence and evolutionary exaptation of the inevitable information crosstalk between cognitive processes. In contrast to the simple on-off threshold operation of neurons that may involve noise through stochastic resonance (e.g., [10]), the assembly of such working coalitions is usually a large-scale explosive ‘reentrant’ autocatalytic phenomenon in which signal boosts signal until a regulated maximum is reached [11]. We model that dynamic to include noise, first taken as ‘white’, i.e., the mathematically tractable fiction of having a uniform spectrum across frequencies.

A simple picture of explosively reentrant phenotype emergence in the presence of noise can be represented by the Ito stochastic differential equation

$$dX_t = \alpha X_t(1 - \frac{X_t}{K})dt + \sigma X_t dW_t \tag{1}$$

where $\alpha > 0$ is a rate constant, $K > 0$ a ‘carrying capacity’, and $\sigma > 0$ the amplitude of the white noise dW_t, its only characteristic.

Applying the Ito chain rule to $\log[X_t]$ in equation (1) [12,13] gives, as a consequence of the Ito correction factor, the long-time limits

$$\begin{aligned} X_t &\to 0,\ \alpha < \frac{\sigma^2}{2} \\ X_t &\to K(1 - \frac{\sigma^2}{2\alpha}),\ \alpha \geq \frac{\sigma^2}{2} \end{aligned} \tag{2}$$

In this model, the ratio between the reentrant autocatalytic amplification parameter α and the squared magnitude of the white noise, σ^2, determines whether, and to what extent, the large-scale logic switch defined by equation (1) operates. α and the ‘carrying capacity’ K are presumed to be genetically determined or to have been learned, while σ represents an internal control signal that can proportionally activate the biological logic gate, making it an analog system.

There is a strikingly similar control theory result, the Data Rate Theorem (DRT), a generalization of the classic Bode Integral Theorem for linear control systems. The DRT bridges a longstanding gap between information theory and control theory, describing the stability of feedback control under data rate constraints [14]. Given a noise-free data link between a discrete linear plant and

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its controller, unstable modes can be stabilized only if the feedback data rate $\mathcal{H}$ is greater than the rate of 'topological information' generated by the unstable system. For the simplest incarnation, if the linear matrix equation of a 'plant' has the form $x_{t+1} = \mathbf{A}x_t + ...$, where x_t is the n-dimensional state vector at time t, then the necessary condition for stabilizability is that

$$\mathcal{H} > \log[|det\mathbf{A}^u|] \tag{3}$$

where det is the determinant and $\mathbf{A}^u$ is the decoupled unstable component of $\mathbf{A}$, the part with eigenvalues ≥ 1. The determinant represents a generalized volume. Thus there is a critical positive data rate below which there does not exist any quantization and control scheme able to stabilize an unstable system.

Here, in contrast, 'noise', measured by σ, controls an 'explosive' instability transition to a fraction of the carrying capacity K determined by the ratio between the amplification parameter α and σ^2.

Other forms of noise, for example fractional Brownian motion, produce more complicated results, as do geographic extensions of the model [15,16]. The most complete treatment currently available appears to be via the Doleans-Dade exponential ([12] Theorem 36).

Equation (1) is expressed in terms of classic white noise having the simple quadratic variation [12,13] $[W_t, W_t]_t = \sigma^2 t$, where, again, σ is the single available parameter, the noise magnitude. These and similar arguments can be extended to other kinds of noise, having arbitrary quadratic variation and discontinuous (Levy-like) stochastic jumps, provided they have the characteristic form

$$dX_t = X_{t-}dY_t \tag{4}$$

where Y_t is a stochastic process, and $t-$ indicates left-continuous. Letting $\Delta Y_t = Y_t - Y_{t-}$, representing the jump process, the solution is given by the Doleans-Dade exponential as [12]

$$X_t = \exp(Y_t - \frac{1}{2}[Y_t, Y_t]_t^C)\Pi_{s \leq t}(1 + \Delta Y_s)\exp(-\Delta Y_s) \tag{5}$$

where $[Y_t, Y_t]_t^C$ is the path-by-path continuous part of the quadratic variation of Y_t. This is written as

$$[Y_t, Y_t]_t^C = [Y_t, Y_t] - \sum_{0 \leq s \leq t} (\Delta Y_s)^2 \tag{6}$$

The product term in equation (5), with jump processes having nonzero Δ, converges.

If the control signal 'noise' represented by the continuous part of the quadratic variation $[Y_t, Y_t]_t^C$ is monotonically increasing in time at a greater rate than Y_t, the Doleans-Dade expression collapses to zero, and the biological logic gate is not activated. The similarity with the Data Rate Theorem thus arises inversely, through a nonactivation condition as

$$\mathcal{I} = \frac{1}{2}d[Y_t, Y_t]^C/dt > dY_t/dt \tag{7}$$

While degrees of reentrant, autocatalytic activation – when $\mathcal{I} \leq dY_t/dt$ – depend on the details of system structure, that activation represents the coordination of independent underlying cognitive modules according to the (inverse) control signal $\mathcal{I}$.

Note that $[Y_t, Y_t]_t^C$ may itself be parameterized, as in the well-known case of fractional Brownian motion (fBM) (e.g., [17]). For fBM, having Hurst parameter $0 \leq H \leq 1$, the covariance between noise at times t and s is

$$cov[W^H(t), W^H(s)] = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H}) \tag{8}$$

If $H = 1/2$ the process is Brownian white noise. Following [18], the quadratic variation $[W_t^H, W_t^H]$ is $+\infty$ for $H < 1/2$, proportional to t for $H = 1/2$, and 0 for $H > 1/2$.

Although equation (5) provides a general model of large-scale noise-driven analog logic gates, an unresolved question concerns their rates of operation. Gene expression, wound healing, the immune response, and animal consciousness involve the macroscopic assembly of sets of lower level cognitive modules into similar Barrs-like dynamic 'global workspaces' that operate at vastly different rates [3]. How is this possible? The key lies in the observation that neural tissues consume metabolic free energy at an order of magnitude greater rate than other kinds of tissue [19], although the argument requires some work.

Biological molecular energetics are striking. At 300 K, molecular energies represent about 2.5 KJ/mol in available free energy. By comparison, the basic biological energy reaction, the hydrolysis of adenosine triphosphate (ATP) to adenosine diphosphate, under proper conditions at 300 K, produces some 50 KJ/mol. in reaction energy, equivalent to a 'reaction temperature' of 6000 K. A high rate of ATP delivery thus provides sufficient energy for very rapid biocogniton, taking a simple chemical reaction perspective from the Arrhenius relation of physical chemistry (e.g., [20]).

The argument, however, must be adapted to highly nonequilibrium cognitive physiological processes, as opposed to simple equilibrium chemical reactions. Interest focuses on the Rate Distortion Function (RDF) associated with the channel connecting the organism with an embedding and embodying environment [21]. Recall that $R(D)$ defines the minimum rate of information transmission needed to ensure that the mean distortion between what is sent and what is received is less than or equal to $D \geq 0$, according to an appropriate distortion measure.

Let R_t be the RDF of the cognitive channel at time t. A general relation can, under conditions of both white noise and volatility, be written as

$$dR_t = f(t, R_t)dt + bR_t dW_t \tag{9}$$

where the magnitude of the noise is now expressed as b to avoid confusion with the earlier development.

Let $M(R_t, t)$ represent the rate of incoming metabolic free energy that is needed to achieve R_t at time t, and

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expand using the Ito chain rule,

$$dM_t = [\partial M/\partial t + f(R_t,t)\partial M/\partial R + \frac{1}{2}b^2 R_t^2 \partial^2 M/\partial R^2]dt + [bR_t \partial M/\partial R]dW_t \quad (10)$$

Define $\mathcal{L}$ as the Legendre transform of M

$$\mathcal{L} = -M + R\partial M/\partial R \quad (11)$$

Using the heuristic of replacing dX with ΔX in these expressions, and applying the results of equation (10), gives the relation

$$\Delta\mathcal{L} = (-\partial M/\partial t - \frac{1}{2}b^2 R^2 \partial^2 M/\partial R^2)\Delta t \quad (12)$$

Analogous to the classic Black-Scholes calculation of financial engineering [22], the terms in f and dW_t cancel out, so that the effects of noise are subsumed in the Ito correction involving b. This invokes powerful regularity assumptions that may be violated. Matters then revolve about model robustness in the face of such violation.

$\mathcal{L}$, as the Legendre transform of M, can be expected to reach a constant rate of production at nonequilibrium steady state (nss). Then $\Delta\mathcal{L}/\Delta t = C \geq 0, \partial M/\partial t = 0$, so that

$$-\frac{1}{2}b^2 R^2 \partial^2 M/\partial R^2 = C \quad (13)$$

The most interesting solution is

$$M_{nss} = \frac{2C}{b^2}\log[R_{nss}] + \kappa \quad (14)$$

Thus

$$R_{nss} = R_0 \exp[\frac{b^2 M_{nss}}{2C}]$$
$$R_0 = \exp[\frac{-\kappa b^2}{2C}] \quad (15)$$

at nonequilibrium steady state, so that the rate of cognition, as measured by R, increases exponentially with available metabolic free energy, in this approximation.

Taking $C = 0$ – so that $\mathcal{L}$ is a maximum at nss – R is linear in M with a threshold, since $R \geq 0$, and may be expected to be sharply rising in M thereafter.

A complete solution can be found to equation (13) involving the Lambert W-function, but it is quite obscure.

Extension of the Black-Scholes approach to colored or Levy noise is not entirely trivial, and is a central theme in much financial engineering literature (e.g., [23]).

It is interesting to compare these results with a simpler Arrhenius reaction rate calculation [20], taking the probability of the Rate Distortion Function above a Barrs-like consciousness threshold in a Boltzmann expression as the rate index, and M as a temperature analog. This produces a slightly different exponential expression that increases rapidly at first, and then tops out with M as

$$Pr[R \geq R_0] = \frac{\int_{R_0}^{\infty}\exp[-R/\omega M]dR}{\int_0^{\infty}\exp[-R/\omega M]dR} = \exp[-R_0/\omega M] \quad (16)$$

ω is an appropriate constant and R_0 the Baars threshold value of the Rate Distortion Function.

The general inference – whichever model is chosen – is that animal consciousness, which must operate with a time constant of about 100ms, appears to do so by providing metabolic free energy to neural tissues at a rate an order of magnitude greater than other tissues.

Stochastic resonance is a local phenomenon in which applied noise raises an information-carrying signal above the triggering threshold of a nonlinear on-off mechanism, in a large sense. By some contrast, here we examine the possible role of noise color as well as amplitude as a control signal for large-scale cognitive biological phenomena particularly involving dynamic recruitment of simpler cognitive modules into temporary working coalitions. A central inference, then, is that, for some processes, the color of the noise may constitute an – or, indeed, the – essential matter, in addition to magnitude measures. Further mathematical development would involve Fourier expansions of our results to more explicitly tease out the influence of 'color' per se.

Extension of the argument leads to consideration of the cognitive rate of such coalitions, finding that plausible models imply sharply increasing rates of cognition with increase in energy supply.

In sum, evolutionary process may have exapted mechanisms incorporating colored noise as a subtle tool in the regulation of large-scale cognitive biological phenomena, in addition to the usual well understood role of individual molecular signals. Experimental test of this conjecture may be similarly subtle.

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