The simplest mathematical model of consciousness

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Abstract

Adaptation of the Arrhenius relation to cognitive process explains how an order of magnitude difference in the rate of metabolic free energy metabolism for neural tissues translates into many orders of magnitude increase in the rate of assembling sets of cognitive submodules into the temporary, tunable, 'global workspace' coalitions of consciousness, much faster than such closely analogous processes as wound healing, immune response, and gene expression. The model strongly contradicts Tononi's 'panpsychic' integrated information treatment of consciousness.

Key Words: Arrhenius' law; crosstalk; evolutionary exaptation; information theory; metabolic free energy; mitochondria

1 Introduction

Currently popular 'panpsychic' models of consciousness (Tononi 2008) are computationally and mathematically intractable (e.g., Maguire et al. 2014). Here, by contrast, we take Baars' (1988) formulation as a starting point to produce what is perhaps the most direct possible mathematical model of the phenomenon. Following Wallace (2012), gene expression, wound healing, the immune response, tumor control, and animal consciousness all represent the evolutionary exaptation of information crosstalk into processes that recruit sets of simpler cognitive modules into temporary working coalitions to address patterns of threat and opportunity confronting an organism. Such tunable coalitions operate, however, at markedly different rates. Wound healing, depending on the extent of injury, may take 18 months to complete its work (e.g., Mindwood et al. 2004). Animal consciousness typically operates with a time constant of a few hundred milliseconds. How can phenomena acting on such different rates be subsumed under the same underlying mechanism? A heuristic answer is relatively straightforward: neural tissues in humans consume metabolic free energy at ten times the rate of other tissues (e.g., Clarke and Sokoloff 1999), and adaptation of Arrhenius' law (e.g., Laidler 1987), which predicts exponential differences in reaction rate with 'temperature', in a large sense, produces the result.

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2 The model

In more detail, given a simple chemical reaction of the form $aA + bB \rightarrow pP + qQ$, the rate of change in (for example) the concentration of chemical species P (written $[P]$) is typically determined by an equation of the form

$$d[P]/dt = k(T)[A]^n[B]^m \quad (1)$$

where n and m are constants depending on the reaction details. The reaction rate k is expressed by the Arrhenius relation as

$$k = \alpha \exp[-E_a/RT] \quad (2)$$

where α is another characteristic constant, E_a is the reaction activation energy, T is the Kelvin temperature and R a universal constant. $\exp[-E_a/RT]$ is, using the Boltzmann distribution, the fraction of molecular interactions having energy greater than E_a. Figure 1 shows the form of this expression for $k = \exp[-1/M]$, where M is taken as an index of the rate of available metabolic free energy.

3 Discussion

Since, in mammals, body temperature remains constant, the rate of available metabolic free energy – dependent on mitochondrial function – serves as the temperature-like index determining the characteristic rate of any chemically-generated generalized consciousness-analog, or of the individual lower-level cognitive modules that come together in a temporary assemblage to form such an analog. Neural tissues, in humans consuming metabolic energy at an order of magnitude greater rate than other tissues, can thus easily provide cognitive function many orders of magnitude faster than similar physiological phenomena, although the mechanism illustrated by figure 1 does reach a point of diminishing returns. Another interpretation, however, is that decline in the rate of available metabolic free energy can, below the shoulder of the figure, cause sudden catastrophic collapse of cognitive function both directly and by interference with basic physiological regulatory processes. This is, perhaps, the most parsimonious model possible for a spectrum of mitochondrial-related disorders. More complicated mathematical analyses treat such sudden change as a phase transition at a critical M, or as violation of the stability condition of the Data Rate Theorem (Nair et al. 2007; Wallace 2012, 2015).

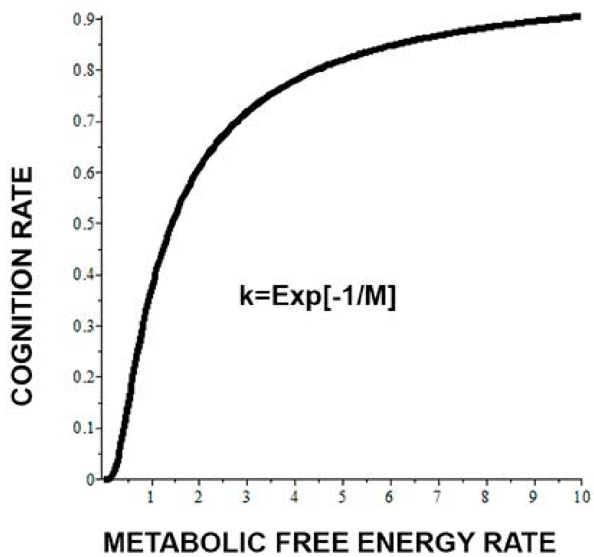

Figure 1: Arrhenius-like relation for rate of a generalized consciousness or other cognitive physiological process as a function of the rate of available metabolic free energy M. An order of magnitude increase in such free energy can enable several orders of magnitude increase in the rate of cognition. Decline in M below the shoulder of the curve, however, triggers catastrophic pathological collapse of cognition. More complicated approaches treat such failure as a phase transition at a critical value of M or as violation of the necessary condition of the Data Rate Theorem.

4 References

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