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Modelling the effect of curves on distance running performance

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Background On a curve, the average axial leg force (F_a) of a runner is increased due to the need to exert centripetal force. The increased F_a presumably requires a greater rate of metabolic energy expenditure than straight running at the same velocity. We propose a model that explains the velocity reduction on curves, compared to straight running, assuming that runners maintain a constant metabolic rate.

Methods We combined published equations to estimate the change in the rate of gross metabolic energy expenditure as a function of F_a , where F_a depends on curve radius and velocity, with an equation for the gross rate of oxygen uptake as a function of velocity. We compared performances between straight courses and courses with different curve radii and geometries.

Results The differences between our model predictions and the actual indoor world records, are between 0.45 % in 3000 m and 1.78 % in the 1500 m for males, and 0.59 % in the 5000 m and 1.76 % in the 3000 m for females. We estimate that a 2:01:39 marathon on a 400 m track, corresponds to 2:01:32 on a straight path and to 2:02:00 on a 200 m track.

Conclusion Our model predicts that compared to straight racecourses, the increased time due to curves, is notable for smaller curve radii and for faster velocities. But, for larger radii and slower speeds, the time increase is negligible and the general perception of the magnitude of the effects of curves on road racing performance is not supported by our calculations.

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Abstract

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Introduction

The maximum sprint running velocities of humans are slower along curves compared to straight-ahead paths. Coherent explanations for this phenomenon, based on physics and biomechanics, are supported by substantial empirical evidence. The requirement to exert centripetal force (Greene 1985), and more specifically the force generated by the inside leg (Chang and Kram 2007; Churchill et al. 2016; Usherwood and Wilson 2005), adequate friction/traction (Alexander 2002; Luo and Stefanyshyn 2011; Luo and Stefanyshyn 2012), and ankle inversion/eversion torques (Greene 1987; Luo and Stefanyshyn 2012) have all been implicated as explanations for reduced sprint velocity along curves.

Here, we focus not on sprint running but on how curves affect the middle- and long-distance running performance of human athletes. Performances on standard indoor tracks (200 m/lap with curve radii of 17.2m (IAAF 2008a)) are generally slower than on tracks with larger radii. For example, the National Collegiate Athletics Association (NCAA) equates a 4:03.07 mile on a standard 200 m indoor track to a 4:00.00 mile on an “oversized” track (i.e. > 300 m/lap) (Pederson et al. 2012) (NCAA, 2012). What are the physiological/biomechanical mechanisms responsible for this effect?

A fundamental physiological limit to distance running performance is the ability to generate adequate energy (i.e. ATP) from aerobic metabolism. Three physiological factors determine distance running performance: maximal aerobic capacity ($\text{VO}_2 \text{ max}$), the submaximal rate of oxygen consumption required to run at a specified velocity (aka “running economy”) and the % or fraction of the maximal aerobic capacity that can be sustained (Bassett and Howley 2000; Coyle 1995; Joyner and Coyle 2008; Ferretti et al. 2011). For example, an elite male marathon runner might have a $\text{VO}_2 \text{ max}$ of 86 $\text{mlO}_2/\text{kg}/\text{min}$ and the physiology to sustain 85% of that $\text{VO}_2 \text{ max}$ (71 $\text{ml O}_2/\text{kg}/\text{min}$) for more than 2 hours at a velocity of 5.55 m/sec in a straight line (Joyner 1991).

The rates of oxygen uptake or metabolic energy required to run straight-ahead at a specified velocity are proportional to the force applied to the ground (Arellano and Kram 2014; Kipp et al. 2018b; Kram and Taylor 1990). During distance running, the vertical ground reaction force (GRF) vs. time pattern resembles a half-sine wave with a peak magnitude of 2.5 to 4 x body weight depending on velocity. During straight-line running, the vertical GRF averaged over a complete stride is equal to 1.0 x body weight (BW). But, when a person runs along a curved path on a flat surface, they lean in towards the center of the curve and the required average force axial to the leg is greater than 1.0 BW due to the need to exert a centripetal force. The greater total axial leg force presumably requires a greater rate of metabolic energy expenditure than straight running. Centripetal force is equal to mv^2/r , where m is body mass, v is tangential velocity and r is the curve radius. Accordingly, an athlete must run slower on a curved path to maintain the same metabolic energy expenditure.

Overall, our objective was to combine physics and physiology to model the energetics of running on curved paths. From these energetic cost estimates, we then calculated race time differentials for various race distances and velocities. A few distance running races occur along straight paths (e.g. the Fifth Avenue Mile in New York City) but most involve at least one turn. We begin our analysis by modelling an out-and-back racecourse with a single 180° turn. We then move to the much more

common track races which are contested on standard 200 m indoor oval tracks (inner edge radius = 17.2 m) and standard 400 m outdoor oval tracks (inner edge radius = 36.5 m). According to the rules of the governing body for athletics, the IAAF (IAAF 2008a; IAAF 2008b), both indoor and outdoor tracks must be measured 0.3 m from the raised curb positioned at the inner edge of lane 1. The added 0.3 m takes in account the theoretical line of running (IAAF 2008b) of athletes who will run on curves with actual radii of 17.5 m (IAAF 2008a) and 36.8 m (IAAF 2008b) on indoor and outdoor tracks respectively. We then consider races longer than 10000 m which are predominately conducted on road surfaces. Road-racing courses typically involve multiple turns of different radii and we demonstrate how we can apply our model to any course configuration. Finally, urban races often involve negotiating city blocks comprising 90 degree angles. According to the IAAF, road racing courses must be measured 0.3 m from the curb (IAAF) which equates to 0.3 m radius. In the appendix, we consider the special case of running races on rectangular city blocks.

Methods

Gross metabolic energy expenditure as a function of body weight

Using a spring and harness system, Teunissen et al. (2007) quantified how simulated reduced gravity decreased the gross rate of metabolic energy expenditure during treadmill running. We utilized their data to calculate the fractional change in the rate of gross metabolic energy expenditure f as a function of the average axial leg force (\overline{F}_a)

$$f = 0.6234\overline{F}_a + 0.3766 \quad (\text{Eq. 1})$$

where \overline{F}_a is expressed as multiples of body weight (BW) and is calculated over an entire stride cycle (from touch down of one leg to the next touch down of the same leg). While Teunissen et al. (2007) only measured metabolic energy expenditure in normal and simulated reduced gravity ($\overline{F}_a \leq 1 \text{ BW}$), we assume that the slope in Eq. 1 extends to $\overline{F}_a > 1 \text{ BW}$. According to equation 1, when a person is running in a straight line ($\overline{F}_a = 1 \text{ BW}$), $f = 1$ (i.e. no change). If $\overline{F}_a = 1.25$ (a 25% increase in average axial force), $f = 1.16$ (a 16% increase in gross metabolic energy expenditure). Note that we calculated Eq. 1 based on the data of Teunissen et al. (2007) but to calculate the gross rate of metabolic energy expenditure, we added Teunissen et al.'s standing metabolic rate value of 1.87 W/kg to the net values reported in their tables. Further, we forced the regression to have an exact value of $f = 1$ when $\overline{F}_a = 1$.

Body weight as a function of velocity and curve radius

A person with a body mass m (kg), running with a tangential velocity v (m/s) on a curve of radius r (m) is subject to two forces (Greene 1985): \overline{F}_v the average force in the vertical direction due to gravity,

$$\overline{F}_v = mg \quad (\text{Eq. 2})$$

where g is gravitational acceleration, and \overline{F}_c the average centripetal force,

$$\overline{F}_c = mv^2/r \quad (\text{Eq. 3})$$

The vector sum of F_v and F_c is the average axial leg force:

$$\overline{F}_a = \sqrt{\overline{F}_v^2 + \overline{F}_c^2} \quad (\text{Eq. 4})$$

where \overline{F}_a is measured in newtons (N). Dividing Eq. 4 by the body weight of the runner ($1 BW = mg$) and combining it with Eq. 2 and 3, the axial force \overline{F}_a can be calculated in multiples of body weight:

$$\overline{F}_a = \sqrt{1 + \frac{v^4}{(gr)^2}} \quad (\text{Eq. 5})$$

Gross rate of metabolic energy expenditure during curve running

By inserting Eq. 5 into Eq. 1, it is possible to calculate the fractional increase, f , in the gross rate of metabolic energy expenditure for a runner with a tangential velocity v , on a curve of radius r compared to running straight-ahead at the same velocity:

$$f = 0.6234 \sqrt{1 + \frac{v^4}{(gr)^2}} + 0.3766 \quad (\text{Eq. 6})$$

As $r \rightarrow \infty$ (straight running), $\overline{F}_a \rightarrow 1$ in Eq. 5 (Fig. 1a) and therefore Eq. 6 reduces to $f \rightarrow 1$ (Fig. 1c) irrespective of running velocity v . At slower velocities, as $v \rightarrow 0$, $\overline{F}_a \rightarrow 1$ in Eq. 5 (Fig. 1b) and therefore Eq. 6 reduces to $f \rightarrow 1$ (Fig. 1d) irrespective of curve radius r .

Running velocity on straight and on curves

The following equation, derived by Kipp et al. (2019), expresses the relationship between gross rate of oxygen uptake ($\dot{V}O_{2s}$) and overground running velocity (v_s) on a straight path:

$$\dot{V}O_{2s} = 0.02724v_s^3 + 1.7321v_s^2 - 0.4538v_s + 18.91 \quad (\text{Eq. 7})$$

where $\dot{V}O_{2s}$ is measured in $\text{mlO}_2/\text{min}/\text{kg}$. The cubic term in Eq. 7 takes into account air resistance (Pugh 1970).

To calculate the gross rate of oxygen uptake of a person running on a curve ($\dot{V}O_{2c}$) with a tangential velocity along the curve v_c , we can combine Eq. 6 and 7:

$$\dot{V}O_{2c} = (0.6234 \sqrt{1 + \frac{v_c^4}{(gr)^2}} + 0.3766)(0.02724v_c^3 + 1.7321v_c^2 - 0.4538v_c + 18.911) \text{ (Eq. 8)}$$

where $\dot{V}O_{2c}$ is measured in $\text{mlO}_2/\text{min}/\text{kg}$. Note: we and others prefer to express running economy in units of energy or power (e.g. W/kg or $\text{kcal}/\text{min}/\text{kg}$) (Beck et al. 2018; Fletcher et al. 2009; Kipp et al. 2018a; Shaw et al. 2014) to account for differences in substrate utilization and therefore, in the amount of energy liberated per liter of oxygen uptake. However, Pugh (1970) used oxygen uptake rates. For our purpose here, assuming equivalence between rates of metabolic energy utilization and oxygen uptake incurs an insignificant error.

A runner maintaining a constant velocity on both straight and curved portions ($v_s = v_c$), would therefore alternate their gross rates of metabolic energy expenditure according to Eq. 7 and 8 respectively, where $\dot{V}O_{2c} > \dot{V}O_{2s}$. A runner performing at the maximal sustainable percentage of their aerobic capacity on the straight portion of a race cannot sustain an equal tangential velocity on the curve, since would increase their rate of metabolic energy expenditure. Rather, in order to maintain the same metabolic energy expenditure throughout the race, running velocity on the curve must be reduced ($v_c < v_s$) so that $\dot{V}O_{2c} = \dot{V}O_{2s}$.

To calculate the running velocity on the curve (v_c) for a given velocity on the straight (v_s), we used numerical approximation methods (see Appendix for algorithm 1). To calculate the increased time during a single 180° turn in an out-and-back race, we used Eq. 7 to calculate the required $\dot{V}O_{2s}$ for a straight racecourse, we then calculated the running velocity v_c on the curved portion according to Eq. 8 given the same metabolic energy expenditure ($\dot{V}O_{2c} = \dot{V}O_{2s}$) for a range of radii from 0.3 m (minimum radius according to IAAF rules (IAAF)) up to 36.8 m (outdoor track (IAAF 2008b)). The increased time during the curved portion is calculated as:

$$\Delta t_{180^\circ} = \frac{d_c}{v_c} - \frac{d_c}{v_s} \text{ (Eq. 9)}$$

where d_c is the distance run in the 180° turn, corresponding to πr .

In order to calculate the time difference between a straight race course (t_{straight}) and the same racing distance on indoor or outdoor tracks, we used the same approach described above, with the curve radii set at 17.5 m for indoor track (IAAF 2008a) and 36.8 m for the outdoor track (IAAF 2008b). The total time (t_{track}) on the track is then calculated as:

$$t_{\text{track}} = \frac{d_s}{v_s} + \frac{d_c}{v_c} \text{ (Eq. 10)}$$

where d_s and d_c are the total distances run on the straight and curved portions of the track, respectively, and the total racing distance is $d_{\text{tot}} = d_s + d_c$.

Vice-versa, when a certain time t_{track} on the track is known, it is possible to calculate the respective velocities on the straight and curved paths, v_s and v_c , that satisfy Eq. 10 (see Appendix for algorithm 2). The respective time on a straight racecourse would then be:

$$t_{straight} = \frac{d_{tot}}{v_s} \quad (\text{Eq. 11})$$

The same procedure can be used to convert times between tracks with different curve radii and/or sizes: for example between indoor ($r_{indoor} = 17.5$ m, distance of one lap $d_{lap,indoor}=200$ m) vs. outdoor tracks ($r_{outdoor} = 36.8$ m, $d_{lap,outdoor}=400$ m).

For a given racing distance, it is possible to calculate the time difference Δt as follows:

$$\Delta t = t_{indoor\ track} - t_{outdoor\ track} \quad (\text{Eq. 12})$$

to compare indoor vs. outdoor tracks,

and:

$$\Delta t = t_{straight} - t_{outdoor\ track} \quad (\text{Eq. 13})$$

to compare straight racecourses vs. outdoor tracks.

Given that $r_{indoor} < r_{outdoor}$, $\Delta t > 0$ in Eq. 12 represents the increased amount of time for running on an indoor track while keeping the same rate of oxygen uptake maintained on the outdoor track. On the other hand, given that $r_{straight} \rightarrow \infty$, $\Delta t < 0$ in Eq. 13 represents the increased amount of time for running on a straight racecourse while keeping the same rate of oxygen uptake maintained on the outdoor track. We used the outdoor 400 m track as a reference because the majority of racing distances (1500 m, 3000 m, 5000 m and 10000 m) are commonly run on outdoor tracks, compared to indoor tracks (1500 m, 3000 m and 5000 m) and very few races are contested on straight racecourses.

We also determined the ideal geometry of an outdoor track, where we kept the track lap distance constant ($d_{lap,outdoor}=400$ m) and changed curve radii from $r=6$ m, corresponding to an oval track with a total straight portion of 362.3 m and a total curved portion of 37.7 m per lap, to $r=63.66$ m, corresponding to a perfectly circular track with 400 m run on the curved portion.

We selected the world record times t_{WR} on a standard outdoor track for 1500 m, 3000 m, 5000 m and 10000 m as a reference, we then calculated the total racing time $t(r)$ as a function of the different curve radii r according to Eq. 10. The time difference Δt :

$$\Delta t = t(r) - t_{WR} \quad (\text{Eq. 14})$$

is the increased time ($\Delta t > 0$) or decreased time ($\Delta t < 0$) as a function of radius r compared to the respective world record. The ideal track geometry corresponds to the curve radius that allows the biggest time savings.

More generally, these algorithms can be used to convert times between straight racecourses and the same distance run on a path with a series of curves with different radii:

$$t_{path} = \frac{d_s}{v_s} + \sum \frac{d_{c,i}}{v_{c,i}} \quad (\text{Eq. 15})$$

where $d_{c,i}$ is the distance ran on the i -th curve, with a given radius r_i , and $v_{c,i}$ is the velocity on the i -th curve.

Breaking 2 on a straight path

Using Equation 15, we analyzed the racetrack in Monza, Italy used for the “Breaking 2” marathon exhibition (<https://en.wikipedia.org/wiki/Breaking2>). We divided the total lap distance, $d_{lap, Monza}$ = 2424.4 m (17.4 laps to run a full marathon: 42195 m) into a straight portion d_s = 1907 m and 6 different curves (see Fig. 2, racetrack blueprints: personal communication, Brett Kirby, Ph.D.). We divided the “Curva parabolica” into three different portions in order to account for the non-constant radius of this specific section. All other curves were assumed to have a fixed radius throughout each section. We then applied the same algorithm described in the previous paragraph: we calculated the running velocities on the straight and on each of the curved portions of the track assuming that Eliud Kipchoge maintained a constant $\dot{V}O_{2s} = \dot{V}O_{2c}$. We then converted the total time $t_{Monza} = 7225$ s (2 hours and 25 seconds) to the time $t_{straight}$ that Kipchoge might have run on a straight path with a length d_{tot} = 42195 m, while maintaining all the other factors (drafting, shoes, hydration etc.) adopted during the Breaking 2 attempt.

Results

Increased time for a single 180° turn

We report the increased time Δt_{180° as a function of radius r according to Eq. 9 for three different representative velocities ($v_1 = 7.3$ m/s, corresponding to Hicham el Guerrouj's 1500 m world record; $v_2 = 6$ m/s, corresponding to the men's half marathon world record; and a recreational running velocity, $v_3 = 4$ m/s) in Fig. 3. The radius r influences both the distance run on the curve d_c and the velocity on the curve v_c in Eq. 9. As $r \rightarrow \infty$, $d_c \rightarrow \infty$, but given that $v_c \rightarrow v_s$ (Eq. 8), the increased time $\Delta t_{180^\circ} \rightarrow 0$. As r decreases, $v_c < v_s$ and Δt_{180° starts to increase up to a specific radius \hat{r} , different for each velocity. In particular $\hat{r}_1 = 2.7$ m and $\Delta t_{180^\circ} = 0.261$ s for v_1 ; $\hat{r}_2 = 1.9$ m and $\Delta t_{180^\circ} = 0.232$ s for v_2 ; $\hat{r}_3 = 0.8$ m and $\Delta t_{180^\circ} = 0.193$ s for v_3 . \hat{r} , therefore, represents the worst radius in terms of velocity reduction ($v_c < v_s$) and non-trivial distance run on the curve ($d_c > 0$). As r further decreases ($r < \hat{r}$), $d_c \rightarrow 0$, leading to an overall decrease in Δt_{180° .

Outdoor tracks vs. indoor tracks vs. straight races

We report the time difference Δt a function of running velocity v in Fig. 4a (1500 m, 5000 m and 10000 m) and Fig. 4b (half marathon and marathon) respectively, the maximum velocity v for each distance corresponds to the respective current men's world record. In both figures, $\Delta t > 0$ represents the increased amount of time for running on an indoor track ($r = 17.5$ m) compared to the outdoor track ($r = 36.8$ m), while $\Delta t < 0$ represents the decreased amount of time for running on a straight racecourse compared to an outdoor track. The increased or decreased amount of time compared to an outdoor track increases non-linearly with velocity v and is inversely proportional to the curve radius r (see Appendix for step-by-step algorithms). In addition, we selected four racing distances commonly contested on outdoor oval tracks (1500 m, 3000 m, 5000 m and 10000 m). Based on the actual outdoor 400 m track world records for both males and females, we calculated the respective time the same athlete would have run on an indoor track and on a straight racecourse, while keeping the same rate of oxygen uptake maintained on the outdoor track (Table 1). For comparison, we report the actual world record on the indoor track for 1500 m, 3000 m and 5000 m distances. 10000 m is not officially run on an indoor track (IAAF 2018).

According to our model, an athlete running a marathon in 2:01:39 (corresponding to the actual world record, $v = 5.78$ m/s) on an outdoor track, would run 2:01:32 on a straight path and 2:02:00 on an indoor track. According to our model, an athlete running a half marathon in 58:18 (corresponding to the actual world record, $v = 6.03$ m/s) on an outdoor track, could run 58:14 on a straight path and 58:30 on an indoor track (Fig. 4b).

Ideal geometry of 400 m track

For a track constrained to comprise a 400 m lap, we report the time difference Δt a function of curve radius r in Fig. 5 for 1500 m, 3000 m, 5000 m, and 10000 m respectively. The plots intersect at ($r=36.8$ m, $\Delta t=0$ s), where $t(r) = t_{WR}$. For $r < 36.8$ m, $\Delta t > 0$ s, indicating that a reduction in curve radius, compared to standard outdoor tracks, is detrimental for performance. For example, when $r=6$ m, Δt values range between +1.18 s for 1500 m and +7.14 s for 10000 m. On the other hand, for $r > 36.8$ m, $\Delta t < 0$ s for all distances, indicating that an increase in curve radius, compared to standard outdoor tracks, favors performance; in particular, at the maximum radius ($r=63.66$ m) Δt equals -0.15 s for 1500 m, -0.31 s for 3000 m, -0.48 s for 5000 m and -0.86 s for 10000 m

Breaking 2 on a straight path

We report the velocities v_c on each of the curve portions and the velocity v_s on the straight portions, calculated assuming that Kipchoge maintained a constant oxygen uptake ($\dot{V}O_{2c} = \dot{V}O_{2s}$) in Table 2. Note that combining each velocity with the respective distance, the time for a full lap (2424.4 m) is $t_{lap, Monza} = 415.1$ s, and the total time for a full marathon (17 full laps plus the remaining 0.4 laps, i.e. 980.2 m on the last straight portion) coincides with $t_{Monza} = 7225$ s.

To calculate the time $t_{straight}$ that Kipchoge could have run on a straight marathon course, it is sufficient to divide the total distance by the velocity on the straight:

$$t_{straight} = \frac{d}{v_s} = \frac{42195}{5.8414} = 7223.48 \text{ s (Eq. 16)}$$

leading to an overall time difference of only $\Delta t=1.52$ s.

Discussion

According to our model, the increased time Δt_{180° for an out-and-back race course (i.e.: with a single 180° turn) is less than 0.27 s even in the worst-case scenario (high velocity, 7.3 m/s, and small curve radius, 2.7 m). Nevertheless, race organizers trying to keep Δt_{180° to a minimum, should aim for the largest curve radius allowed by road widths, the presence of buildings/sidewalks, median strip etc.. IAAF rules (2018) require that, in order for a race course to be record-eligible, the start and finish points of any road race shall not be further apart than 50% of the total race distance (Rule 260.21), making the presence of at least one curve mandatory for all record-eligible courses. While having the largest possible radius is still a valid recommendation, on races measuring 5000 m and above, athletes can maintain a lower velocity compared to the scenario described above (6.6 m/s being the average velocity for Bekele's 5000 m world record), and would therefore experience even lower values for Δt_{180° . In addition, Δt_{180° becomes trivial in terms of percentage of the total race time especially in races like the half marathon or the marathon, while other factors, like change in elevation (Giovanelli et al. 2016; Hoogkamer et al. 2014), surface type (Kerdok et al. 2002), drafting (Hoogkamer et al. 2019) etc., have a much greater effect on running energetics and therefore on the overall time (Hoogkamer et al. 2016).

Lacking empirical data from a controlled study, we can only evaluate the validity of our model by comparing our predictions to actual race performances. Starting with the outdoor 400 m track records, our model predicts faster indoor world records compared to the actual record times in all distances for both males and females (Table 1).

The differences between our model predictions and the actual indoor world records, based on the times run on 400 m outdoor tracks (Table 1), are 3.75 s (1.78 %) in the 1500, 2.2 s (0.45 %) in 3000 m, and 8.82 s (1.14 %) 5000 m for males, and 4.1 s (1.74%) in the 1500, to 8.73 s (1.76%) in the 3000 m and 5.09 (0.59%) in the 5000 m for females. It must be noted that multiple factors can contribute to these differences between predicted and actual times. Indoor races are typically run in winter, while outdoor races are run in spring/summer and athletes tend to reach peak fitness for outdoor races when major international competitions (Olympics, World Championship etc.) are held. Only the men's 5000 m indoor and outdoor records were run by the same athlete (Kenenisa Bekele) in the same year. All other indoor and outdoor records were run by the same athlete, but in different years, or by different athletes. Pacing and drafting play important roles when trying to run a world record time (Hoogkamer et al. 2019): it is likely more difficult for athletes on indoor tracks to negotiate the smaller curve radii while following or overtaking other competitors, compared to outdoor tracks.

Our model, and Table 1, can be used to identify which, among the actual indoor world records, is the hardest or easiest to beat, assuming the outdoor world record is a "benchmark performance" corresponding to the current "physiological limits" of males and female athletes respectively. Our model, in fact, calculates what time an athlete with the exact same fitness level and all the conditions (drafting, motivation etc.) found during the outdoor world record could run on tracks with different curve radii or on the straight. For males, it is evident that Daniel Komen's 3000 m indoor world record is only 0.45 % slower compared to the "physiological limit" he himself

reached on the outdoor track two years earlier. In order for an athlete to break the indoor world record, he must be close to being able to run under the current outdoor world record. On the other hand, the current 1500 m indoor world record is 1.78% slower compared to the “physiological limit” set by Hicham el Guerrouj on an outdoor track and seems therefore relatively easier to break. In order to break the current indoor world record by 0.01 s, an athlete must be able to run 3:29.80 on a 400 m outdoor track. For females, the 5000 m indoor world record is only 0.59% slower compared to the “physiological limit” set by Tirunesh Dibaba on an outdoor track. The women’s outdoor 5000 m record is relatively harder to break compared to the 3000 m indoor world record, 1.76% slower compared to the “physiological limit” set by Junxia Wang on a 400 m outdoor track.

Our prediction that a perfectly circular track is optimal for distance running performance concurs with Greene’s model for sprint running (Greene and Monheit 1990). This is true for all racing distances. A 1500 m runner is more affected by the velocity reduction on the curve ($v_c < v_s$) because of their faster average velocity compared to longer distances. However, runners competing in longer events have to perform more laps around the track (up to 25 laps for the 10000 m). The number of laps seems therefore the dominant factor on the overall increased/decreased time as a function of curve radius. It would also be interesting to see if and how a circular track, compared to the standard track, could influence race tactics.

Some insight into the validity of our model can also be gained by comparing the best performances of two world-class athletes (Jenny Simpson and Sydney Maree) when racing one mile (1609 m) indoors, outdoors and on a straight racecourse (5th Avenue Mile, NY). Considering their outdoor personal best as their “benchmark performance” (4:17.30 for Simpson, 3:48.83 for Maree), our model predicts times on a 200 m indoor track of 4:18.27, 2.87% faster than the time of 4:25.91 run by Simpson, and 3:50.10, 0.99% faster compared to the time of 3:52.40 run by Maree. For a straight race, our model predicts times of 4:16.98, only 0.14% slower compared to the actual time of 4:16.6 run by Simpson and 3:48.40, only 0.39% slower compared to 3:47.52 run by Maree. While the same considerations highlighted above must be taken in account when comparing different races (different years or racing seasons, different fitness levels), we must also take in account that the 5th Avenue Mile is slightly net downhill which may explain why both our predictions seems slower than the actual race time.

The NCAA indoor track time conversion system provides another validity test. The NCAA conversion factors were developed using thousands of race performances, comparing times of the same athlete in different indoor facilities (Pederson et al. 2012). However, these conversions do not specifically take in account the exact curve radius of each indoor track. Rather, the NCAA categorizes them as “undersized” (<200 m per lap, like the Madison Square Garden track, which is 146.3 m per lap (Attwood 2012)), “standard” (200 m per lap) and “oversized” (>200 m per lap, typically 300 m (Pederson et al. 2012)). In addition, racing velocity is accounted for only in terms of male vs. female athletes and in terms of racing distances. For example, as the racing distance increases from 200 m to 5000 m, the NCAA conversion factor from “oversized” to “standard” indoor tracks decreases from 1.0179 to 1.0107 for males, and from 1.0155 to 1.0077 for females. Despite these limitations, we can compare the NCAA conversions with our model predictions for 3000 m and 5000 m for male and female athletes (see Table 3).

Comparing standard 200 m vs. undersized indoor tracks, the difference between the current model and NCAA conversions range from 0.01 s (<0.01 %) for males in the 3000 m, to 1.54 s (0.2%) for females in the 5000 m. Comparing standard vs. oversized indoor tracks, the differences between the current model and NCAA conversions range from 3.39 s (0.63 %) for females in the 3000 m, to 6.52 s (0.78 %) for males in the 5000 m. While both NCAA conversion tables and our models agree on the overall effect of smaller vs larger radii on performance (i.e. the larger the radius, the better the overall time), our model predicts a slightly greater time when going from standard to undersized tracks, while it predicts smaller time reductions when going from standard to oversized tracks compared to NCAA conversion tables. These differences could be explained by the fact that NCAA tables provide an average conversion factor for a given race, independent of the actual performance of the athlete in that race, while in our model, velocity has a non-linear effect on the decreased or increased time on tracks of different radii (see Fig. 4).

When Eliud Kipchoge participated in the Breaking 2 attempt, he completed 17.4 laps around the Monza racetrack, totaling 105 curves (note that we divided the “Curva parabolica” into three sections, but even considering it as one single curve the total number of curves would still be 71). Our model predicts a trivial 1.52 s time difference between the Breaking 2 attempt and a marathon run on a straight racecourse. This is due to the fact that the smallest radius on the Monza racetrack is still 23 m (curve # 1), a value 31% bigger than the radius of indoor tracks (17.5 m), therefore the velocity reduction on curves is hardly noticeable. A similar number of curves can be counted for two of the most famous marathon racecourses: ~50 curves for the Berlin marathon (<https://www.bmw-berlin-marathon.com/en/your-race/start-course-finish/course/>) and ~70 for the London marathon (<https://www.virginmoneylondonmarathon.com/en-gb/event-info/runner-info/>). Even though we could not measure the radii of these curves, our model predicts that the increased time due to curve negotiation, compared to a straight racecourse, is negligible and the general perception of the magnitude of the effects of curves on road racing performance is not supported by our calculations.

Limitations and future studies

Running economy is affected by a multitude of biomechanical factors. In combination with the axial leg force that drives our model, contact time of the foot with the ground and the rate of force production (Roberts et al. 1998), antero-posterior ground reaction forces (Chang and Kram 1999), stride length (Cavanagh and Kram 1989) and stride frequency (Snyder and Farley 2011) all affect the energetic cost of running. When running at maximum speed on curves with small radii ($r \leq 6$ m), runners increase their contact time, decrease antero-posterior ground reaction forces and stride length compared to straight running (Chang and Kram 2007). It is unclear if these biomechanical differences are maintained at sub-maximal speeds and at the larger radii. We have no knowledge of studies that measured biomechanics and/or, more crucially, energetics of curve running that could validate our model. It is important to empirically verify the key assumption of our model - that athletes run slower on curves compared to straight portions of a track when performing at sub-maximal (aerobic) velocities.

For longer (5000 m and above) races, the aerobic system comprises nearly 100% of the energy demands, but the relative contribution of the non-oxidative systems in shorter races like the 1500 m is not trivial (Hill 1999). Our model assumes that all of the energy is derived through aerobic

metabolism. Despite this limitation, the differences between actual and predicted indoor world records for 1500 m, based on outdoor performances, are indeed quite small (<1.8% error, Table 1). Future studies should investigate the relative contribution of aerobic and anaerobic systems to the energetics of curve running. That could extend our approach to even shorter races such as the 800 m.

The data collected by Teunissen et al. (2007) that allowed us to derive Eq. 1 were collected at one fairly slow velocity (3 m/s), but to our knowledge there are no equivalent data for faster running velocities. In addition, we extrapolated Eq. 1 beyond normal gravity, assuming the same slope is maintained when the average axial force acting on the runner is increased ($\overline{F}_a > 1 BW$). Additional experiments are needed to quantify the effects of different velocities and increased gravity on Eq. 1 and verify our assumption. In addition, our model does not distinguish between male and female athletes. While Eq. 1 can be applied to both male and female athletes, given that Teunissen et al. (2007) included both sexes in their study, Eq. 7 was derived for male runners only (Kipp et al. 2019). Generally, studies find that males are slightly more economical than females at matched absolute running velocities (Daniels and Daniels 1992) Eq. 7 should therefore be adapted for female athletes with a different set of parameters that take in account these differences.

In order to run 42195 m on the Monza racetrack in a total time $t_{Monza} = 7225$ s or 5.84 m/s, Equation 7 predicts that Eliud Kipchoge sustained a rate of oxygen uptake $\dot{V}O_{2s} = 80.87$ mlO₂/min/kg (see algorithm 2 in Appendix for details). This incredible value suggests that either that the Pugh (1970) factor for air resistance is too large or that Kipchoge is much more economical runner than the subjects tested by Kipp et al. (2019). Fortunately, the absolute value does not affect our calculations of the effects of curve running.

When we model an athlete transitioning from straight to curved running, such as when running on a track or on a non-straight road race, we assume that the change in velocity (from v_s to v_c and vice-versa) is instantaneous, i.e. there is no deceleration or acceleration phase between straight and curved portions. This assumption may be reasonable for larger radii, such as outdoor or indoor racing tracks. If $r=17.5$ m, when the velocity on the straight is $v_s = 7.00$ m/s, the velocity on the curve is reduced to $v_c = 6.91$ m/s, allowing an athlete to decelerate and re-accelerate in one single step. But, for much smaller radii (e.g. $r=1$ m) when the velocity on the straight is $v_s = 7.00$ m/s, the velocity on the curve is $v_c = 4.69$ m/s, an athlete would likely need more than one step to decelerate and then re-accelerate). Non-trivial decelerations and accelerations increase the metabolic cost of running (di Prampero et al. 2005) and should therefore be factored into our model, especially for smaller (<6 m) curve radii. This approach, while theoretically possible, can lead to accurate calculations only if the exact values of deceleration and accelerations are known. Future studies (from video and/or from lab-based measurements) could provide such information and fill this gap to create a more realistic model.

When an athlete is running on a curve with large radius, even for faster (>7 m/s) velocities, the increase in axial force \overline{F}_a is relatively small (see Fig. 1b) and it is reasonable to assume that, as modeled in this paper, the limiting factor on v_c is mainly the metabolic cost of running. However, at smaller radii the increase in \overline{F}_a is much more marked (for $r=1$ m, when $v_s=7.00$ m/s and $v_c=4.69$

m/s, $\overline{F_a}=2.45$ BW). $\overline{F_a}$ is the axial force calculated over a full step, assuming a duty factor of 45% (Chang and Kram 2007) the average force during contact reaches an even higher value of 2.72 BW. Chang and Kram (2007) measured velocities and ground reaction forces of recreational athletes sprinting on the straight and on curves of small (6 m or less) radii. While subjects were able to reach $v_s=7.70$ m/s on the straight, the maximum velocity on a curve when $r=1$ m was only 2.99 m/s, well below the velocity predicted by our model. In addition, the *peak* axial forces reached only 1.87 BW for the inside leg and 2.25 BW for the outside leg. In the Chang and Kram (2007) study, subjects were instructed to run as fast as possible but only for a very limited amount of time. Therefore, the maximum velocity they were able to attain on curves was not limited by metabolic cost, but by other constraints. Chang and Kram (2007) concluded that during small radius curve sprinting, the ability to generate force, in particular from the inside leg, limits maximum curve velocity. When athletes run on curves at sub-maximal speeds (i.e. for a prolonged period of time), it is likely that both mechanisms play roles. When transitioning from straight to curved running, at larger radii, the main driving factor in the velocity reduction is maintaining a constant metabolic rate. But, at progressively smaller radii, the increase of centripetal, and therefore axial forces, is amplified and velocity is further reduced as the athlete is limited by his/her ability to generate forces.

Conclusions

Our model assumes that runners reduce their velocity on curves, compared to straight running, to maintain a constant metabolic rate for the whole duration of the event. This reduction is marked for smaller curve radii, such as indoor tracks. At faster velocities, the effect becomes negligible, in terms of overall performance, for larger radii and slower speeds, such as those seen in city marathons.

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Conflict of interest

The authors declare that they have no conflicts of interest.

References

- Alexander RM (2002) Stability and manoeuvrability of terrestrial vertebrates. *Integr Comp Biol* 42:158-164 doi:10.1093/icb/42.1.158
- Arellano CJ, Kram R (2014) Partitioning the metabolic cost of human running: a task-by-task approach. *Integr Biol* 54:1084-1098 doi:10.1093/icb/icu033
- Attwood E (2012) NCAA institutes indexing system for 2012-13 indoor track and field. <https://www.athleticbusiness.com/college/ncaa-institutes-indexing-system-for-2012-13-indoor-track-and-field.html>. Accessed 3/1/2019
- Bassett DR, Jr., Howley ET (2000) Limiting factors for maximum oxygen uptake and determinants of endurance performance. *Med Sci Sports Exerc* 32:70-84
- Beck ON, Kipp S, Byrnes WC, Kram R (2018) Use aerobic energy expenditure instead of oxygen uptake to quantify exercise intensity and predict endurance performance. *J Appl Physiol* 125:672-674 doi:10.1152/jappphysiol.00940.2017
- Cavanagh PR, Kram R (1989) Stride length in distance running: velocity, body dimensions, and added mass effects. *Med Sci Sports Exerc* 21:467-479
- Chang YH, Kram R (1999) Metabolic cost of generating horizontal forces during human running. *J Appl Physiol* 86:1657-1662
- Chang YH, Kram R (2007) Limitations to maximum running speed on flat curves. *J Exp Biol* 210:971-982 doi:10.1242/jeb.02728
- Churchill SM, Trewartha G, Bezodis IN, Salo AI (2016) Force production during maximal effort bend sprinting: Theory vs reality. *Scand J Med Sci Sports* 26:1171-1179 doi:10.1111/sms.12559
- Coyle EF (1995) Integration of the physiological factors determining endurance performance ability. *Exerc Sport Sci Rev* 23:25-63
- Daniels J, Daniels N (1992) Running economy of elite male and elite female runners. *Med Sci Sports Exerc* 24:483-489
- di Prampero PE, Fusi S, Sepulcri L, Morin JB, Belli A, Antonutto G (2005) Sprint running: a new energetic approach. *J Exp Biol* 208:2809-2816 doi:10.1242/jeb.01700
- Ferretti G, Bringard A, Perini R (2011) An analysis of performance in human locomotion. *Eur J Appl Physiol* 111:391-401 doi:10.1007/s00421-010-1482-y
- Fletcher JR, Esau SP, Macintosh BR (2009) Economy of running: beyond the measurement of oxygen uptake. *J Appl Physiol* 107:1918-1922 doi:10.1152/jappphysiol.00307.2009
- Giovanelli N, Ortiz AL, Henninger K, Kram R (2016) Energetics of vertical kilometer foot races; is steeper cheaper? *J Appl Physiol* 120:370-375 doi:10.1152/jappphysiol.00546.2015
- Greene PR (1985) Running on flat turns: experiments, theory, and applications. *J Biomech Eng* 107:96-103
- Greene PR (1987) Sprinting with banked turns. *J Biomech* 20:667-680
- Greene PR, Monheit MA (1990) Optimal geometry for oval sprint tracks *J Biomech* 23:447-452
- Hill DW (1999) Energy system contributions in middle-distance running events. *J Sports Sci* 17:477-483 doi:10.1080/026404199365786
- Hoogkamer W, Kipp S, Spiering BA, Kram R (2016) Altered running economy directly translates to altered distance-running performance. *Med Sci Sports Exerc* 48:2175-2180 doi:10.1249/MSS.0000000000001012

- 601 Hoogkamer W, Snyder KL, Arellano CJ (2019) Reflecting on Eliud Kipchoge's marathon world
602 record: an update to our model of cooperative drafting and its potential for a sub-2-hour
603 performance. *Sports Med* 49:167-170 doi:10.1007/s40279-019-01056-2
- 604 Hoogkamer W, Taboga P, Kram R (2014) Applying the cost of generating force hypothesis to
605 uphill running *PeerJ* 2:e482 doi:10.7717/peerj.482
- 606 IAAF The Measurement of Road Race Courses
607 [https://www.iaaf.org/download/download?filename=1d445793-24b4-4821-98e4-](https://www.iaaf.org/download/download?filename=1d445793-24b4-4821-98e4-38fc55b9f8ef.pdf&urlslug=IAAF%20Road%20Running%20Manual)
608 [38fc55b9f8ef.pdf&urlslug=IAAF%20Road%20Running%20Manual](https://www.iaaf.org/download/download?filename=1d445793-24b4-4821-98e4-38fc55b9f8ef.pdf&urlslug=IAAF%20Road%20Running%20Manual).
- 609 IAAF (2008a) IAAF Track and Field Facilities Manual 2008 - chapters 4-8.
- 610 IAAF (2008b) Track and field facilities manual - chapters 1-3
- 611 IAAF (2018) IAAF Competition Rules 2018-2019
- 612 Joyner MJ (1991) Modeling: optimal marathon performance on the basis of physiological
613 factors. *J Appl Physiol* 70:683-687 doi:10.1152/jappl.1991.70.2.683
- 614 Joyner MJ, Coyle EF (2008) Endurance exercise performance: the physiology of champions. *J*
615 *Physiol* 586:35-44 doi:10.1113/jphysiol.2007.143834
- 616 Kerdok AE, Biewener AA, McMahon TA, Weyand PG, Herr HM (2002) Energetics and
617 mechanics of human running on surfaces of different stiffnesses. *J Appl Physiol* 92:469-
618 478 doi:10.1152/japplphysiol.01164.2000
- 619 Kipp S, Byrnes WC, Kram R (2018a) Calculating metabolic energy expenditure across a wide
620 range of exercise intensities: the equation matters. *Appl Physiol Nutr Metab* 43:639-642
621 doi:10.1139/apnm-2017-0781
- 622 Kipp S, Grabowski AM, Kram R (2018b) What determines the metabolic cost of human running
623 across a wide range of velocities? *J Exp Biol* 221 doi:10.1242/jeb.184218
- 624 Kipp S, Kram R, Hoogkamer W (2019) Extrapolating metabolic savings in running: implications
625 for performance predictions. *Front Physiol* 10:79 doi:10.3389/fphys.2019.00079
- 626 Kram R, Taylor CR (1990) Energetics of running: a new perspective. *Nature* 346:265-267
627 doi:10.1038/346265a0
- 628 Luo G, Stefanyshyn D (2011) Identification of critical traction values for maximum athletic
629 performance. *Footwear Sci* 3:127-138 doi:10.1080/19424280.2011.639807
- 630 Luo G, Stefanyshyn D (2012) Limb force and non-sagittal plane joint moments during
631 maximum-effort curve sprint running in humans. *J Exp Biol* 215:4314-4321
632 doi:10.1242/jeb.073833
- 633 Pederson K, Larson G, Jones S, Podkaminer B (2012) Indoor facility indexing for NCAA
634 running events performances
- 635 Pugh LG (1970) Oxygen intake in track and treadmill running with observations on the effect of
636 air resistance. *J Physiol* 207:823-835
- 637 Roberts TJ, Kram R, Weyand PG, Taylor CR (1998) Energetics of bipedal running. I. Metabolic
638 cost of generating force. *J Exp Biol* 201:2745-2751
- 639 Shaw AJ, Ingham SA, Folland JP (2014) The valid measurement of running economy in runners.
640 *Med Sci Sports Exerc* 46:1968-1973 doi:10.1249/MSS.0000000000000311
- 641 Snyder KL, Farley CT (2011) Energetically optimal stride frequency in running: the effects of
642 incline and decline. *J Exp Biol* 214:2089-2095 doi:10.1242/jeb.053157
- 643 Teunissen LP, Grabowski A, Kram R (2007) Effects of independently altering body weight and
644 body mass on the metabolic cost of running. *J Exp Biol* 210:4418-4427
645 doi:10.1242/jeb.004481

646 Usherwood JR, Wilson AM (2005) Biomechanics: no force limit on greyhound sprint speed.
647 Nature 438:753-754 doi:10.1038/438753a

Table 1 (on next page)

Actual and predicted world records at various racing distances.

A 400 m outdoor track is used as the baseline reference for the record predictions on a 200 m indoor track and straight path races. The curve radii are the actual radii run by athletes for indoor ($r=17.5$ m, (IAAF 2008a)) and outdoor tracks ($r=36.8$ m, (IAAF 2008b)) respectively. 10000 m is not officially run on an indoor track (IAAF 2018).

Males	1500 m		3000 m		5000 m		10000 m	
Indoor track ($r=17.5$ m)	Actual record: 3:31.04	Predicted record: 3:27.29	Actual record: 7:24.90	Predicted record: 7:22.88	Actual record: 12:49.60	Predicted record: 12:40.78	Actual record: N.A.	Predicted record: 26:23.78
Outdoor Track ($r=36.8$ m)	3:26.00		7:20.67		12:37.35		26:17.53	
Straight	Predicted record: 3:25.56		Predicted record: 7:19.93		Predicted record: 12:36.20		Predicted record: 26:15.47	

Females	1500 m		3000 m		5000 m		10000 m	
Indoor track ($r=17.5$ m)	Actual record: 3:55.17	Predicted record: 3:51.07	Actual record: 8:16.60	Predicted record: 8:07.87	Actual record: 14:18.86	Predicted record: 14:13.77	Actual record: N.A.	Predicted record: 29:22.32
Outdoor Track ($r=36.8$ m)	3:50.07		8:06.11		14:11.15		29:17.45	
Straight	Predicted record: 3:49.74		Predicted record: 8:05.53		Predicted record: 14:10.30		Predicted record: 29:15.88	

Table 1. Actual and predicted world records at various racing distances. A 400 m outdoor track is used as the baseline reference for the record predictions on a 200 m indoor track and straight path races. The curve radii are the actual radii run by athletes for indoor ($r=17.5$ m, (IAAF 2008a)) and outdoor tracks ($r=36.8$ m, (IAAF 2008b)) respectively. 10000 m is not officially run on an indoor track (IAAF 2018).

Table 2 (on next page)

Radii and distances of each of the six curves we identified for the Monza racetrack

The Straight row represents the sum of all the straight portions of the racetrack. For each portion, we calculated the running velocities on the straight and on each of the curved portions of the track according assuming that Eliud Kipchoge maintained a constant $VO_{2c}=VO_{2s}$ (see algorithm 2 in Appendix for details).

Curve #	Radius (m)	Distance (m)	Velocity (m/s)
1	23	32	5.8165
2	24	21	5.8185
3	25	62	5.8202
4	350	116.7	5.8413
5	164	151	5.8409
6	80	134.7	5.8393
Straight		1907	5.8414

Table 2. Radii and distances of each of the six curves we identified for the Monza racetrack, the Straight row represents the sum of all the straight portions of the racetrack. For each portion, we calculated the running velocities on the straight and on each of the curved portions of the track according assuming that Eliud Kipchoge maintained a constant $\dot{V}O_{2c} = \dot{V}O_{2s}$ (see algorithm 2 in Appendix for details).

Table 3(on next page)

Comparison of the NCAA conversion factors vs. the current proposed model for 3000 m and 5000 m racing distances.

For each distance and for each sex, we identified sample performances on a standard indoor track (200 m per lap, curve radius $r=17.5$ m) and converted them to an undersized track (corresponding to the Madison Square Garden indoor track, 146.3 m per lap and $r=11.7$ m (Attwood 2012) and to an oversized track (300 m per lap, $r=35$ m, <https://www.pl-linemarking.co.uk/300-metre-track-line-marking.html>) using NCAA conversion tables and our model. Performances are reported in minutes:seconds.hundredths.

Males	3000 m		5000 m	
Undersized indoor track (1 lap 146.3 m, $r = 11.7$ m)	NCAA: 8:02.27	Current model: 8:02.28	NCAA: 14:03.29	Current model: 14:03.45
Standard indoor track (1 lap = 200 m, $r = 17.5$ m)	8:00.00		14:00.00	
Oversized indoor track (1 lap = 300 m, $r = 35$ m)	NCAA: 7:54.50	Current model: 7:58.40	NCAA: 13:51.11	Current model: 13:57.63

Females	3000 m		5000 m	
Undersized indoor track (1 lap 146.3 m, $r = 11.7$ m)	NCAA: 9:01.03	Current model: 9:01.79	NCAA: 16:01.06	Current model: 16:02.60
Standard indoor track (1 lap = 200 m, $r = 17.5$ m)	9:00.00		16:00.00	
Oversized indoor track (1 lap = 300 m, $r = 35$ m)	NCAA: 8:55.40	Current model: 8:58.79	NCAA: 15:52.66	Current model: 15:58.25

Table 3. Comparison of the NCAA conversion factors vs. the current proposed model for 3000 m and 5000 m racing distances. For each distance and for each sex, we identified sample performances on a standard indoor track (200 m per lap, curve radius $r = 17.5$ m) and converted them to an undersized track (corresponding to the Madison Square Garden indoor track, 146.3 m per lap and $r = 11.7$ m (Attwood 2012) and to an oversized track (300 m per lap, $r = 35$ m, <https://www.pl-linemarking.co.uk/300-metre-track-line-marking.html>) using NCAA conversion tables and our model. Performances are reported in minutes:seconds.hundredths.

Figure 1

Axial leg force F_a , measured in BW, as a function of curve radius r for a person running at different velocities (panel A), and as a function of running velocity v on different curve radii (panel B). Fractional increase in gross rate

The standard radii for indoor and outdoor running tracks, are $r=17.5$ m and $r=36.8$ m respectively. Inset figure representing centripetal (F_c), vertical (F_v) and axial (F_a) forces, modified with permission from Chang and Kram (2007). Values of F_a and f when running at fast velocities (>8 m/s) on small radii (<6 m) are not physiological, we depict them only to illustrate the effects of velocity and radius in extreme conditions.

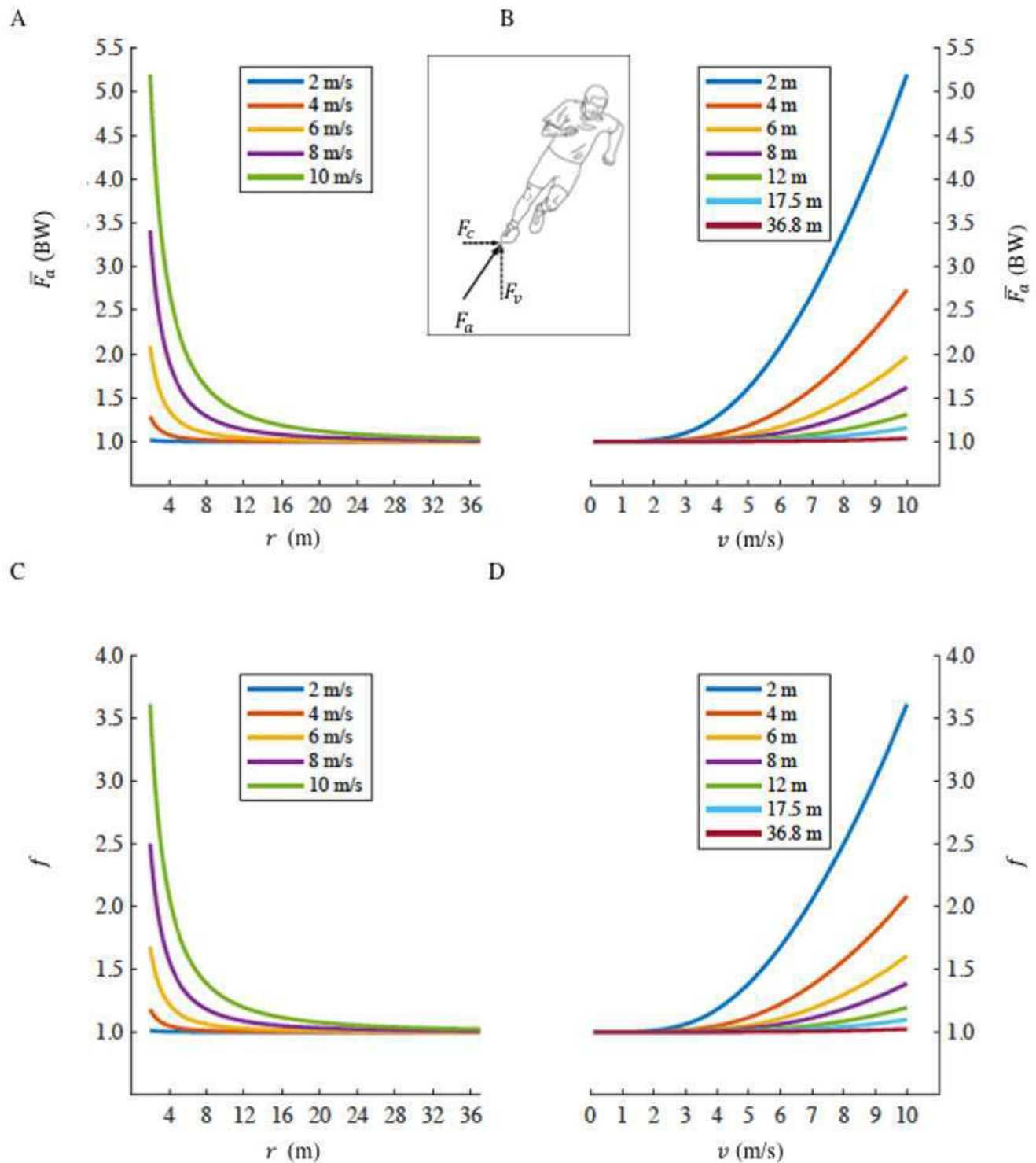


Figure 2

Outline of the Monza racetrack utilized during the “Breaking 2” project.

We divided the south curve (“Curva parabolica”) into three different portions (labeled 4, 5 and 6) in order to account for the non-constant radius of this specific section.

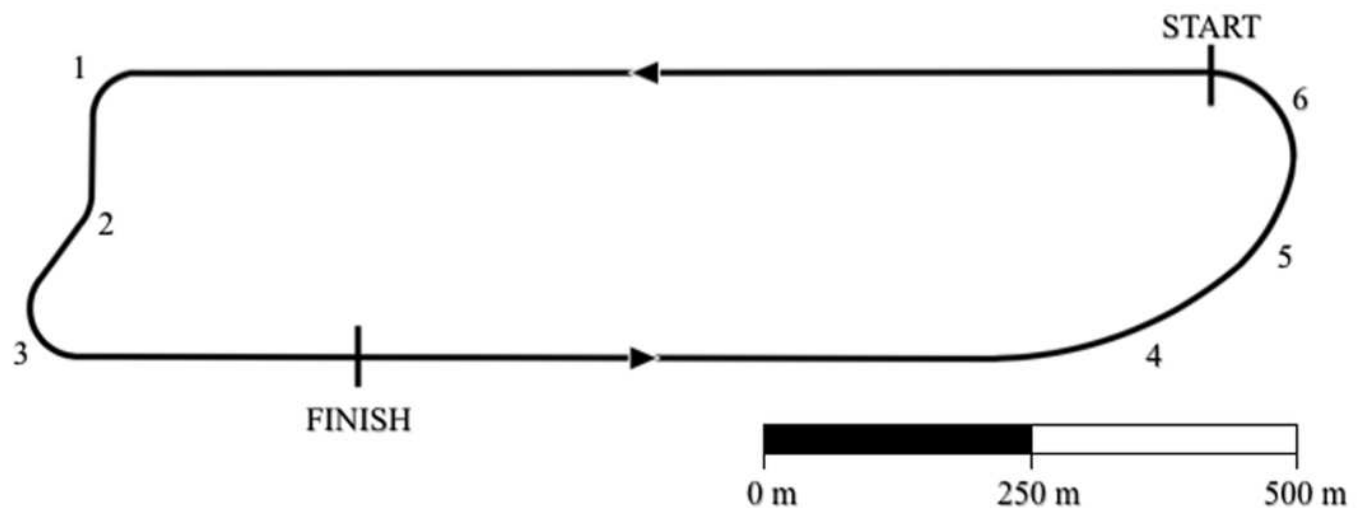


Figure 3

Increased time for a 180°turn as a function of radius

We selected three velocities: $v_1=7.3$ m/s (dashed line, corresponding to Hicham el Guerrouj's 1500 m world record), $v_2=6$ m/s (continuous line, corresponding to men's half marathon world record) and $v_3=4$ m/s (dash-dotted line). For each line, the maximum Delta t is located at $r_1=2.8$ m, $r_2=1.9$ m and $r_3=0.8$ m for v_1 , v_2 and v_3 respectively. Caution should be used when applying our model at very small radii (<6 m). Chang and Kram (2007) report a maximum *sprinting* velocity $v=5.66$ m/s for $r=6$ m, while our model predicts an unrealistic sustained velocity $v_c=6.71$ m/s.

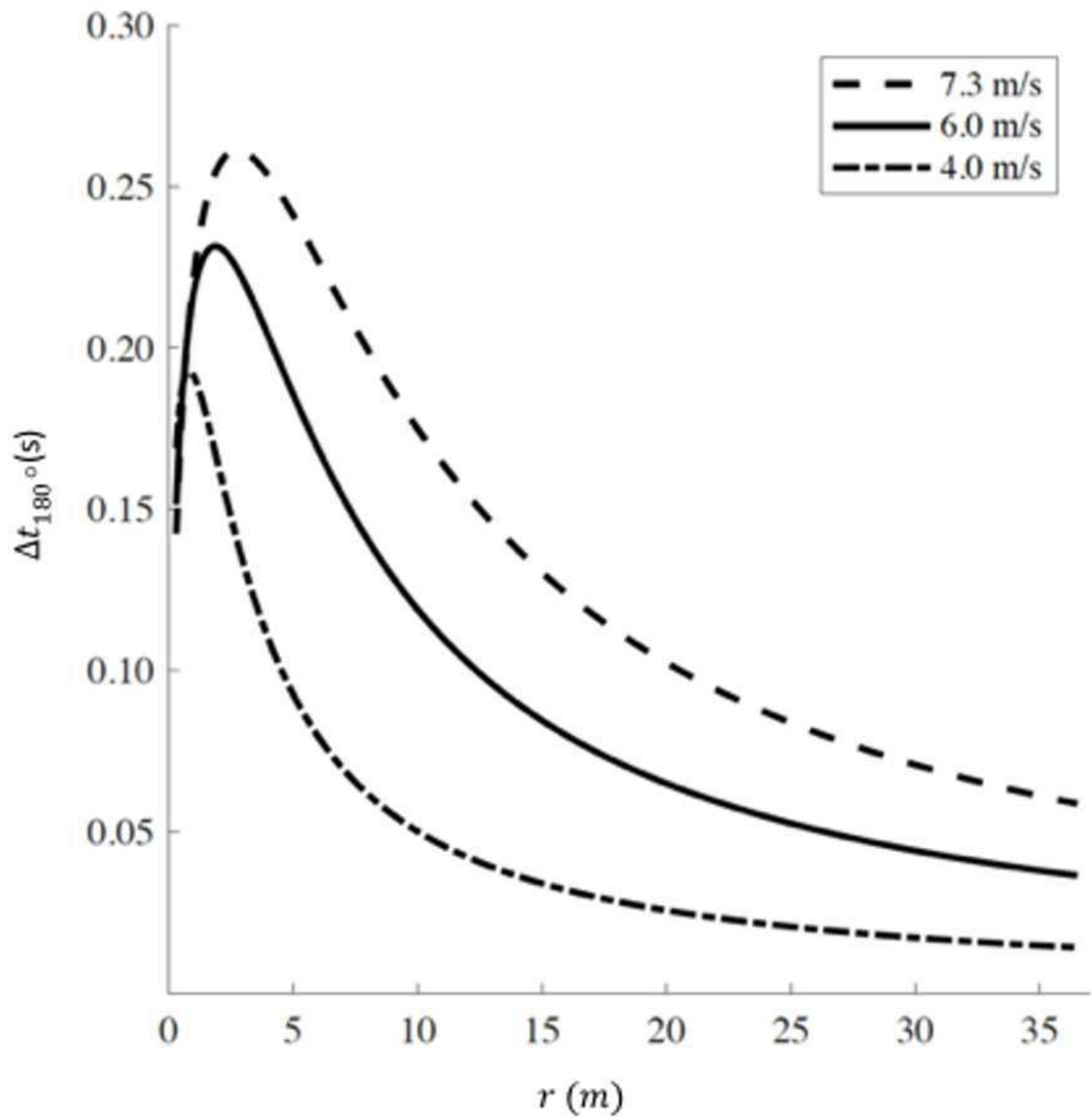


Figure 4

Time difference (Δt) for a given racing distance (1500 m, 5000 m and 10000 m in panel A, half marathon and marathon in panel B) as a function of velocity (v).

For a given racing distance, run on a 400 m outdoor track (curve radius $r_{\text{outdoor}}=36.8$ m), we calculated how much time would increase ($\Delta t > 0$) on a 200 m indoor track (curve radius $r_{\text{indoor}}=17.5$ m), or decrease ($\Delta t < 0$) on a straight path. For each racing distance, the maximum velocity corresponds to the respective current men's world record.

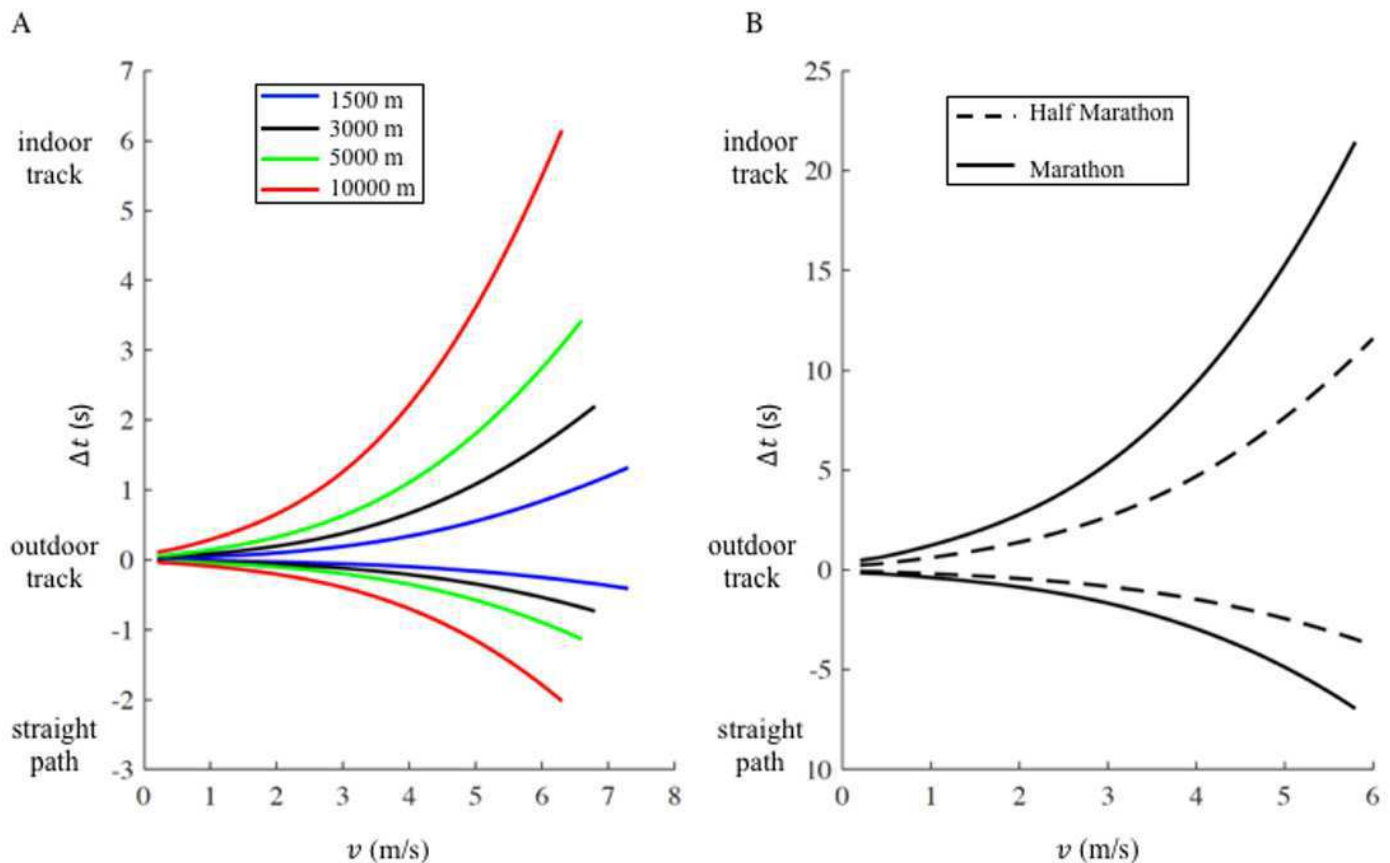


Figure 5

Time difference (Δt) for various racing distances (1500 m, 3000 m, 5000 m, and 10000 m) as a function of radius (r) for tracks constrained to be 400 m lap distance.

When $r=36.8$ m, $\Delta t=0$ s corresponding to the respective world records on a standard 400 m outdoor track. $\Delta t>0$ indicates that any radius $r<36.8$ m is detrimental for performance, while $\Delta t<0$ indicates that any radius $r>36.8$ m favors performance. The ideal geometry for a 400 m track is a perfect circle with radius $r=63.66$ m. (Note: r is the actual radius that includes the 0.3 m offset from the inner edge of the curb, to take in account the theoretical line of running according to IAAF rules (IAAF 2008b)).

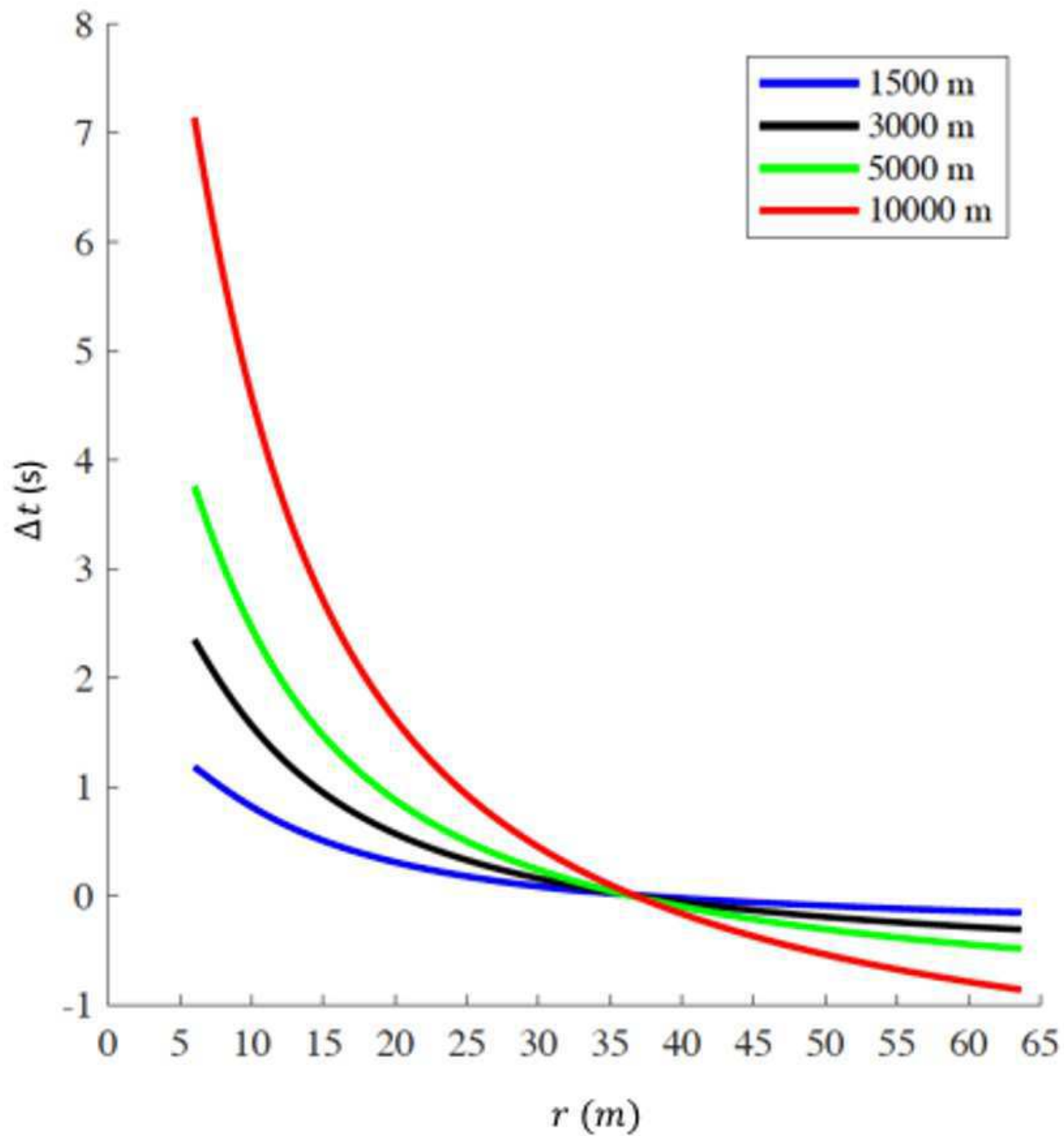


Figure 6

One of the four corners of a city block

The sides, measured along the curbs/walls, are A and B respectively. Dashed line: curbs/walls, red line: course measurement according to IAAF rules (IAAF 2018)

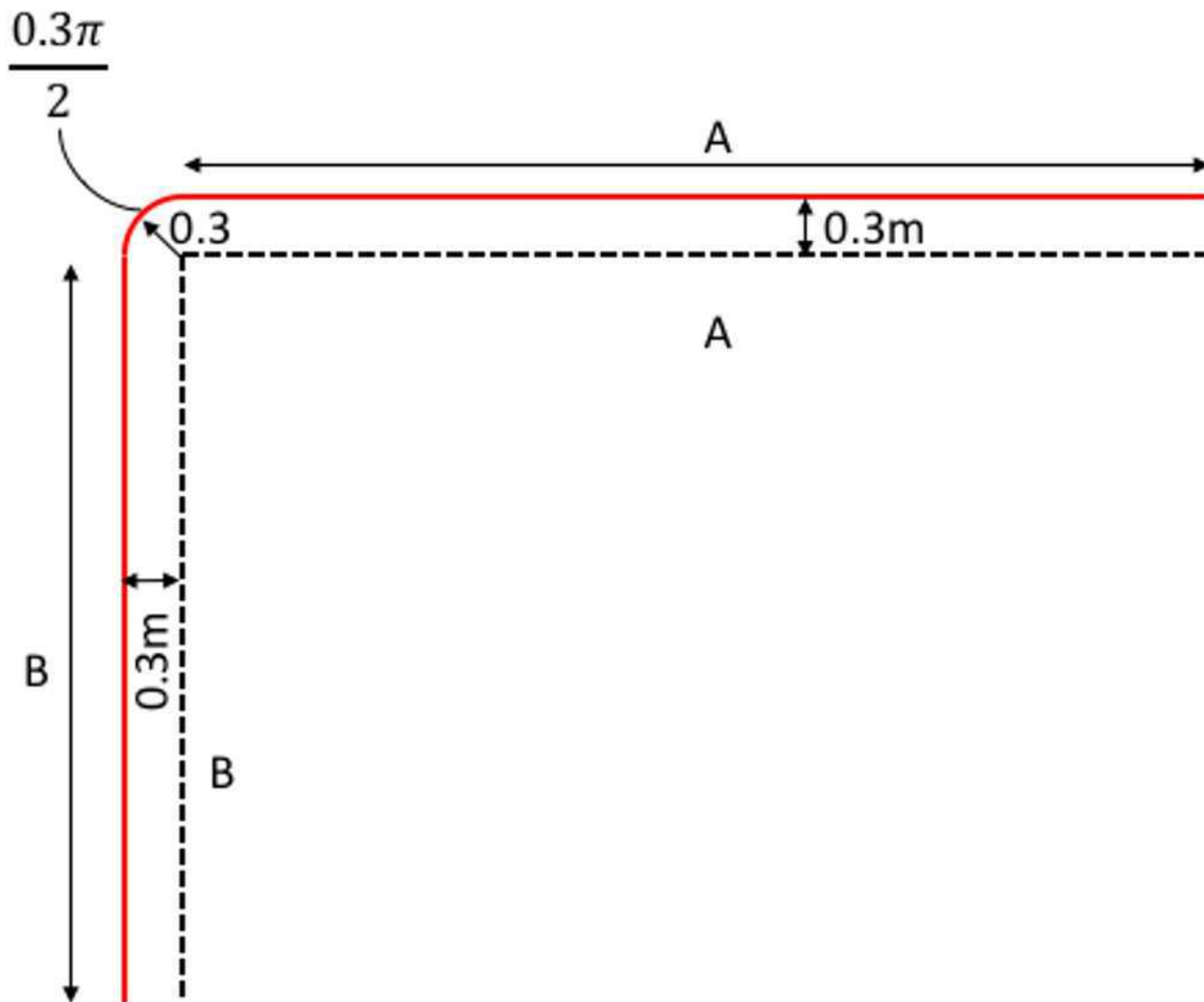


Figure 7

Running around a corner of a city block.

Dashed line indicates curb. Black line: the official path according to IAAF rules. Red line: the actual path ran by the runner.

