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Component-oriented acausal modeling of the dynamical systems in Python language on the example of the model of the sucker rod string

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As a rule, the limitations of specialized modeling languages for acausal modeling of the complex dynamical systems are: limited applicability, poor interoperability with the third party software packages, the high cost of learning, the complexity of the implementation of hybrid modeling and modeling systems with the variable structure, the complexity of the modifications and improvements. In order to solve these problems, it is proposed to develop the easy-to-understand and to modify component-oriented acausal hybrid modeling system that is based on: (1) the general-purpose programming language Python, (2) the description of components by Python classes, (3) the description of components behavior by difference equations using declarative tools SymPy, (4) the event generation using Python imperative constructs, (5) composing and solving the system of algebraic equations in each discrete time point of the simulation. The classes that allow creating the models in Python without the need to study and apply specialized modeling languages are developed. These classes can also be used to automate the construction of the system of difference equations, describing the behavior of the model in a symbolic form. The basic set of mechanical components is developed — 1D translational components "mass", "spring-damper", "force". Using these components, the models of sucker rods string are developed and simulated. These simulation results are compared with the simulation results in Modelica language. The replacement of differential equations by difference equations allow simplifying the implementation of the hybrid modeling and the requirements for the modules for symbolic mathematics and for solving equations.

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2 # Component-oriented acausal modeling of the
3 dynamical systems in Python language on the
4 example of the model of the sucker rod string

5
6

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16

17 ## Abstract

18 As a rule, the limitations of specialized modeling languages for acausal modeling of the complex
19 dynamical systems are: limited applicability, poor interoperability with the third party software
20 packages, the high cost of learning, the complexity of the implementation of hybrid modeling
21 and modeling systems with the variable structure, the complexity of the modifications and
22 improvements. In order to solve these problems, it is proposed to develop the easy-to-understand
23 and to modify component-oriented acausal hybrid modeling system that is based on: (1) the
24 general-purpose programming language Python, (2) the description of components by Python
25 classes, (3) the description of components behavior by difference equations using declarative
26 tools SymPy, (4) the event generation using Python imperative constructs, (5) composing and
27 solving the system of algebraic equations in each discrete time point of the simulation. The
28 classes that allow creating the models in Python without the need to study and apply specialized
29 modeling languages are developed. These classes can also be used to automate the construction
30 of the system of difference equations, describing the behavior of the model in a symbolic form.
31 The basic set of mechanical components is developed — 1D translational components "mass",
32 "spring-damper", "force". Using these components, the models of sucker rods string are
33 developed and simulated. These simulation results are compared with the simulation results in
34 Modelica language. The replacement of differential equations by difference equations allow
35 simplifying the implementation of the hybrid modeling and the requirements for the modules for
36 symbolic mathematics and for solving equations.

37

38 ## Introduction

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39 As known, component-oriented simulation modeling is based on the separation of a complex
40 system model into simple components. The component describes the mathematical model of the
41 corresponding physical object (mass, spring, electrical resistance, hydraulic resistance, hydraulic
42 motor, etc.), which is formulated as an algebraic, differential or difference equation. Components
43 are connected with one another through ports (pins, flanges), which define a set of variables for
44 the interaction between components (Elmqvist, 1978; Fritzson, 2015). Components and ports are
45 stored in software libraries. Usually, it is possible to develop new components. The multi-domain
46 modeling allows to use together of components which differ in the physical nature (mechanical,
47 hydraulic, electric, etc.). The component-oriented modeling can be based on causal modeling or
48 acausal modeling (Fritzson, 2015). In the first case, the component receives the signal x at the
49 input, performs a certain mathematical operation $f(x)$ on it and returns the result y to the output.
50 In this case, the modeling is realized by imperative programming by assigning the value of the
51 expression $f(x)$ to the variable y. In the second case, the signal of the connected components can
52 be transmitted in two directions. Such modeling is realized by declarative programming by
53 solving the equation $y=f(x)$, where the unknown can be x or y. Here, the variables x and y are
54 some physical quantities, and the equation $y=f(x)$ is the physical law that describes their
55 relationship. It allows us to simplify the creation of the model, to focus on the physical
56 formulation of the problem, but not on the algorithm for solving it. It is also possible to avoid
57 errors that are typical for imperative programming.
58 Most often, the behavior of these models is described by the system of differential equations,
59 which are solved by the finite difference method — numerical method based on the replacement
60 of differential operators by difference schemes. As a result, the system of differential equations is
61 replaced by the system of algebraic equations.
62 The solution of non-stationary problems by the finite difference method is the iterative process
63 — at each iteration find the solution of the stationary problem for the given time point. Explicit
64 and implicit difference schemes are used for this purpose. Explicit schemes immediately find
65 unknown values, using information from previous iterations. Using of the implicit scheme
66 requires the solution of a difference equation because unknown values can be in the right and left
67 sides of the equation. The explicit Euler difference scheme is simple to implement, but it often
68 has numerical instability and low accuracy. To improve accuracy and stability it is desirable to
69 apply modified Euler methods, such as the Runge-Kutta method (Runge, 1895).
70

71 Statement of the problem

72 For the simulation of complex dynamic multi-domain systems such specialized modeling
73 languages are developed: Dymola (Elmqvist, 1978), APMonitor (Hedengren et al., 2014),
74 ASCEND (Piela, McKelvey & Westerberg, 1993), gPROMS (Barton & Pantelides, 1994),
75 Modelica (Fritzson & Engelson, 1998) , MKL, Modelyze (Broman, 2010). Among them,
76 Modelica is the most popular free language for component-oriented modeling of such systems.
77 Its main features: free, object-oriented, declarative, focused on hybrid (continuous and discrete)
78 component-oriented modeling of complex multi-domain physical systems, it supports the

79 construction of hierarchical models, adapted for visual programming, widely used for research in
80 various fields (Fritzson, 2015). Free Modelica Standard Library has about 1280 components.
81 There are free and commercial simulation environments in Modelica language —
82 OpenModelica, JModelica.org, Wolfram SystemModeler, SimulationX, MapleSim, Dymola,
83 LMS Imagine.Lab AMESim.
84 As a rule, the limitations of such modeling languages are: limited applicability, poor
85 interoperability with the third party software packages, the high cost of learning, the complexity
86 of the modifications and improvements, the complexity of the implementation of hybrid
87 modeling and modeling variable structure systems where the structure and number of equations
88 can change at run-time (Fritzson, Broman & Cellier, 2008; Nikolić, 2016). Some problems can
89 be solved by using interfaces to general-purpose languages (Akesson et al., 2010; Hedengren et
90 al., 2014). But it is usually more difficult to learn a new language than to learn a component or
91 library of a familiar programming language.
92 These problems are less common in modeling systems that are based on general-purpose
93 programming languages: GEKKO (Beal et al., 2018), Ariadne (Benvenuti et al., 2014), SimuPy
94 (Margolis, 2017), Sims.jl (Short, 2017), Modia.jl (Elmqvist, Henningsson & Otter, 2016),
95 PyDSTool (Clewley et al., 2007), DAE Tools (Nikolić, 2016), Assimulo (Andersson, Führer &
96 Åkesson, 2015). The implementation of such systems can be simplified if the difference
97 equations are used to describe the model instead of differential equations. Many high-level
98 general-purpose languages are suitable for implementing component-based modeling because
99 they have convenient imperative and object-oriented constructions and allow declarative
100 programming. The advantages of modeling systems based on general-purpose programming
101 languages are described in detail in paper (Nikolić, 2016). Python language (Van Rossum &
102 Drake, 1995) is a good choice mainly due to its features: multi-paradigm, object-oriented,
103 intuitive with code readability and improved programmer's productivity, highly extensible,
104 portable, open source, large community and extensive libraries as mathematical libraries SymPy
105 and SciPy. SymPy is a Python library for symbolic mathematics (Meurer et al., 2017). SciPy is a
106 fundamental library for scientific computing (Jones et al., 2001).
107 The purpose of this work is to develop of the easy-to-understand and to modify component-
108 oriented acausal hybrid modeling system that is based on: (1) the use of general-purpose
109 programming language Python, (2) the description of components by Python classes, (3) the
110 description of components behavior by difference equations using declarative tools SymPy, (4)
111 the event generation using Python imperative constructs, (5) composing and solving a system of
112 algebraic equations in each discrete time point of the simulation. The principles of the system are
113 described using the example of the model of the sucker rod string that is used in the oil industry
114 to join together the surface and downhole components of a rod pumping system. Let's take a
115 look the steel rod string, in which the length is 1500 m and sucker rod diameter is 19 mm. This
116 column will have a mass of 3402 kg, a weight in the liquid of 29204 N, a spring constant of
117 39694 N/m, a damping constant of 1856 N·s/m. Liquid weight above the pump with a diameter
118 of 38 mm will be 16688 N.

119

Model in Modelica language

First, we will simulate the free vibrations of the string using the Modelica language. We will develop the model of the simple mechanical translational oscillator, which consists of such components as `Mass`, `SpringDamper` and `Fixed` (Fig. 1). Component `SpringDamper` is designed to simulate the elastic-damper properties of the string. Component `Mass` simulates the inertial properties of the string. Component `Fixed` simulates the fixed point at the top of the string. The module code which describes this model is shown below (Listing S1). In order to simplify the model, these classes differ slightly from the corresponding classes of the standard Modelica library (Fritzson, 2015).

```
connector Flange // class-connector
  Real s; // variable (positions at the flange are equal)
  flow Real f; // variable (sum of forces at the flange is zero)
end Flange;

model Fixed // class-model
  parameter Real s0=0; // parameter (constant in time)
  Flange flange; // object of class Flange
equation // model equations
  flange.s = s0;
end Fixed;

partial model Transl // class-model
  Flange flange_a; // object of class Flange
  Flange flange_b; // object of class Flange
end Transl;

model Mass // class-model
  extends Transl; // inheritance of class Transl
  parameter Real m(min=0, start=1); // parameter
  Real s; // variable
  Real v(start=0); // variable with initial condition
  Real a(start=0); // variable with initial condition
equation // model equations
  v = der(s);
  a = der(v);
  m*a = flange_a.f + flange_b.f;
  flange_a.s = s;
  flange_b.s = s;
end Mass;

model SpringDamper // class-model
```

```
162     extends Transl; // inheritance of class Transl
163     parameter Real c(final min=0, start=1); // parameter
164     parameter Real d(final min=0, start=1); // parameter
165     Real s_rel(start=0); // variable
166     Real v_rel(start=0); // variable
167     Real f; // variable
168   equation // model equations
169     f = c*s_rel+d*v_rel;
170     s_rel = flange_b.s - flange_a.s;
171     v_rel = der(s_rel);
172     flange_b.f = f;
173     flange_a.f = -f;
174   end SpringDamper;
175
176   model Oscillator // class-model
177     Mass mass1(s(start=-1), v(start=0), m=3402.0); // object with
178   initial conditions
179     SpringDamper spring1(c=39694.0, d=1856.0); // object
180     Fixed fixed1(s0=0); // object
181   equation // additional equations
182     // creates a system of equations (see Flange class)
183     connect(fixed1.flange, spring1.flange_a);
184     connect(spring1.flange_b, mass1.flange_a);
185   end Oscillator;
186
```

187 The Modelica language class describes the set of similar objects (components). The `Flange`
188 class describes the concept of a mechanical flange. Its real-type variable `s` corresponds to the
189 absolute position of the flange. Its value should be equal to the value of the variables `s` of the
190 other flanges connected to this flange. The real-type variable `f` corresponds to the force on the
191 flange. It is marked by the `flow` keyword, which means that the sum of all forces at the
192 connection point is equal to zero. The `Fixed` class describes the concept of a fixed component
193 with one flange, for example `fixed1` (Fig. 1). It has the real-type variable `s0`, which
194 corresponds to the absolute position of the flange, and the object `flange` of the `Flange` class,
195 designed to connect this component to others. The variable `s0` is marked by the `parameter`
196 keyword, which means that it can be changed only at the start of the simulation. After the
197 `equation` keyword, an equation describing the behavior of this component is declared — the
198 `flange` object position must be equal to the `s0` value. The `Transl` class describes an abstract
199 component that has two flanges — `flange_a` and `flange_b`. It is the base class for
200 mechanical translational components with two flanges. The `Mass` class inherits the class
201 `Transl` and describes the sliding mass with inertia. The example of such component is `mass1`
202 (Fig. 1). The command `extends Transl` means inheriting members of the `Transl` class in
203 such a way that they become members of the `Mass` class. That is, the `Mass` components will

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204 also have two flanges `flange_a` and `flange_b`. In addition, this class has the parameter `m`
205 (mass) and variables `s` (position), `v` (speed), `a` (acceleration). Expression `start=0` is the
206 default initial condition. After `equation` keyword the system of the differential and algebraic
207 equations which describes behavior of this component is given. The keyword `der` means the
208 derivative with respect to time t (v=ds/dt, a=dv/dt).
209 The class `SpringDamper` inherits the class `Transl` and describes the linear 1D translational
210 spring and damper in parallel. The example of such component is `springDamper1` (Fig. 1).
211 Class has the parameters `c` (spring constant), `d` (damping constant) and the variables `s_rel`
212 (relative position), `v_rel` (relative speed), `f` (force at `flange_b`). After `equation` keyword
213 the system of differential-algebraic equations of this component is given.
214 The `Oscillator` class describes spring-mass system (Fig. 1). It contains three components
215 `mass1`, `spring1`, `fixed1`, which are described by the classes `Mass`, `SpringDamper` and
216 `Fixed`, respectively. The values of parameters and initial conditions of these components are
217 shown in round brackets. The additional equations which are obtained from component
218 connections are given after `equation` keyword. So, for example
219 `connect(fixed1.flange, spring1.flange_a)` command connects the flanges of
220 the `fixed1` and `spring1` components and creates the additional system of equations:
221

```
222 fixed1.flange.s = spring1.flange_a.s;
223 fixed1.flange.f = -spring1.flange_a.f
```

224
225 The model code can be prepared using any text editor or the Modelica Development Tooling
226 (MDT) module (Pop et al., 2006) of the Eclipse development environment. Simulation of model
227 requires the OpenModelica environment (Fritzson et al., 2005). To start calculations enter this in
228 MDT console:
229

```
230 simulate(Oscillator, stopTime=10)
```

231
232 To plot the curve that describes the position of `mass1` component with time enter the following into the
233 console:
234

```
235 plot(mass1.s)
```

236

237 Model in Python language

238 Description of components by Python-classes

239 Now we will develop the module `pycodyn` with similar components in Python (Listing S2). In
240 addition, we will develop the `Force` class for simulating the external forces acting on the string.
241 The behaviour of the components will be described by means of the difference equations. As a
242 result, the system of components connected by flanges will be described by the system of the
243 difference equations.

244 First, we'll import the `sympy` module and the standard mathematical module `math`. It is
245 important to distinguish the functions of these modules.
246
```
247 from sympy import *
248 import math
```
249
250 Create the global variable `dt` (time step).
251
```
252 dt=0.1
```
253
254 If you only need to obtain the system of equations in a symbolic form, then this variable must be
255 an instance of the `Symbol` class of the `sympy` module:
256
```
257 dt=Symbol('dt')
```
258
259 `Translational1D` is the basic class of mechanical 1D components that have translational
260 motion. The constructor function `__init__` is called when an object of this class is created and
261 has two parameters — name of the component `name` and the dictionary of its attributes `args`.
262 For component attribute naming, we use the following notation. The symbols `x`, `v`, `a`, `f` at the
263 beginning of the name mean position, speed, acceleration and force, respectively. The symbol `p`
264 at the end of the name means the value at time `t-dt`. The numerical index at the end
265 corresponds to the flange number. To distinguish the variables of various components in the
266 system, each of them begins with the name of the component followed by the symbol "_". For
267 example, the name `s1_x2p` means the position of the second flange of the component `s1` at
268 time `t-dt`. The constructor for each name-value pair of the dictionary `args` (except `name` and
269 `self`) creates SymPy variables. The symbolic variable of the `Symbol` class is created if its
270 value is not known. The numeric variable of the `Number` class is created if its value is known.
271 The `self.eqs` list contains the component equations, and the `self.pins` list contains the
272 component flanges. Each equation is created using SymPy class `Eq`. Each flange is described by
273 a dictionary whose keys are `x`, `xp`, `f`, and the values are the corresponding attributes of the
274 component (see `Mass`, `SpringDamper`, `Force` classes). The `pinEqs` function returns a list
275 of equations for the component flange that is connected to the flanges of the other components. It
276 has the parameter `pindex` — the index of the flange (for example 0), and the parameter `pins`
277 — the list of flanges of the other components. Always the positions of the mechanical 1D
278 translational components on the flange are equal, and the sum of the forces on this flange is zero.
279 For example, if the flange 2 of the component `s1` is connected to the flange 1 of the component
280 `m1` then `pinEqs` function of the `s1` component returns the list of equations `[s1_x2==m1_x1,`
281 `s1_x2p==m1_x1p, s1_f2==-m1_f1]`.
282
```
283 class Translational1D(object):
```

```
284     def __init__(self, name, args):
285         self.name=name # component name
286         for k,v in args.iteritems(): # for each key-value pair
287             if k in ['name','self']: continue # except name and self
288             if v==None: # if value is None
289                 # create symbolic variable with name name+'_'+k
290                 self.__dict__[k]=Symbol(name+'_'+k)
291             elif type(v) in [float,Float]: # if value is float
292                 self.__dict__[k]=Number(v) # create constant
293         self.eqs=[] # equations list
294         self.pins=[] # pins list
295
296     def pinEqs(self,pindex,pins):
297         eqs=[] # equation list of the flange
298         f=Number(0) # sum of forces on flanges of other components
299         for pin in pins: # for each flange of the other components
300             # add equations describing the equality on the flange:
301             # positions
302             eqs.append(Eq(self.pins[pindex]['x'], pin['x']))
303             # positions at time t-dt
304             eqs.append(Eq(self.pins[pindex]['xp'], pin['xp']))
305             f+=pin['f'] # add to the sum of forces
306         # equality to zero the sum of forces on the flange
307         eqs.append(Eq(self.pins[pindex]['f'], -f))
308         return eqs
309
```

310 The class `Mass` describes the mass concentrated at a point, which has translational motion. It
311 inherits `Translational1D` class. The constructor `__init__` calls the constructor of the
312 base class `Translational1D` and send to it the parameters `name` and `locals()`. The latter
313 is a dictionary of local variables `self`, `name`, `m`, `x`, `xp`, `v`, `vp`, `a`, `f1`, `f2`. The behavior of this
314 component is described by a system of equations `self.eqs`. For example, for the component
315 `m1`:
316

```
317 [m1_m*m1_a == m1_f1+m1_f2, m1_a == (m1_v- m1_vp)/dt,
318 m1_v == (m1_x-m1_xp)/dt]
319
```

320 A list of additional equations can be generated for each component flange using the function
321 `pinEqs` described above. The first element of the `self.pins` list is the dictionary
322 `dict(x=self.x, xp=self.xp, f=self.f1)` which means that the positions `x`, `xp` on
323 the flange will be equal to the `self.x`, `self.xp` attributes of this component respectively,
324 and the force `f` on the flange will be equal to the `self.f1` attribute. The same applies to the
325 second element of the list.
326

```
class Mass(Translational1D):
    def __init__(self, name, m=1.0, x=None, xp=None, v=None, vp=None,
a=None, f1=None, f2=None):
        # base class constructor call
        Translational1D.__init__(self, name, locals())
        # system of equations
        self.eqs=[Eq(self.m*self.a, self.f1+self.f2),
                  Eq(self.a, (self.v-self.vp)/dt),
                  Eq(self.v, (self.x-self.xp)/dt)]
        # two flanges
        self.pins=[dict(x=self.x, xp=self.xp, f=self.f1),
dict(x=self.x, xp=self.xp, f=self.f2)]
```

The `SpringDamper` class describes the translational 1D spring and damper, which are connected in parallel. It inherits `Translational1D` class. In addition to the attributes described above, it has the following attributes: spring constant `c`, damping constant `d`, relative velocity between flanges `vrel`. The behavior of this component is described by a system of equations `self.eqs`. For example, for the component `s1`:

```
[s1_c*( s1_x2-s1_x1)+ s1_d*s1_vrel == s1_f2, -s1_f2 == s1_f1,
s1_vrel == (s1_x2-s1_x2p)/dt-(s1_x1-s1_x1p)/dt]
```

This component also has two flanges and it is possible to generate a list of additional equations using the `pinEqs` function.

```
class SpringDamper(Translational1D):
    def __init__(self, name, c=1.0, d=0.1, x1=None, x2=None, x1p=None,
x2p=None, vrel=None, f1=None, f2=None):
        Translational1D.__init__(self, name, locals())
        # system of equations
        self.eqs=[Eq(self.c*(self.x2-self.x1)+self.d*self.vrel,
self.f2), Eq(-self.f2, self.f1), Eq(self.vrel, (self.x2-self.x2p)/dt-
(self.x1-self.x1p)/dt)]
        # two flanges
        self.pins=[dict(x=self.x1, xp=self.x1p, f=self.f1),
dict(x=self.x2, xp=self.x2p, f=self.f2)]
```

The `Force` class describes a 1D force whose application point has translational motion. The value of the force `f` can be constant or variable. It inherits `Translational1D` class and has one flange.

```
class Force(Translational1D):
    def __init__(self,name,f=None,x=None,xp=None):
```

```
            Translational1D.__init__(self, name, locals())
            self.pins=[dict(x=self.x, xp=self.xp, f=-self.f)] # one flange
```

The class `System` describes the system of components connected by flanges. The constructor `__init__` gets two parameters — the list of components `els` and the list of additional equations `eqs`, which usually are created using `pinEqs` functions. The system components are stored in the `self.els` list and the `self.elsd` dictionary. The list `self.eqs` contains all system equations and is created by joining the equations of all components with additional equations `eqs`.

The function of this class `solve` solves a stationary problem. It returns the solution of a system of equations with conditions `ics` — a dictionary with known values of variables. To solve a system of equations, it can use the SymPy `solve` function, but its algorithm is very slow. It is possible to use fast algorithms for solving equations, for example, the function `scipy.optimize.root` from the SciPy library, which supports many effective methods for solving systems of equations. In this case, the call of the SymPy function `solve(eqs)` must be replaced with the call of the function `self.solveN(eqs)`, which adapts the system of equations for SciPy and solves it using `scipy.optimize.root`.

The function `solveDyn` solves a non-stationary problem. It receives three parameters — the dictionary with initial conditions `d`, the final time value `timeEnd` and the function `fnBC` that returns the dictionary to update the boundary conditions. First, the time variable `t` is assigned an initial value. In the `while` loop with the condition `t<timeEnd`, the following instructions are executed: the positions and velocities of the components in the previous steps `xp`, `x1p`, `x2p`, `vp` are assigned the values of the initial conditions `d`, the values of the boundary conditions are updated, the system of equations is solved by calling the `solve` function, solutions are assigned to the dictionary `d`, the results are saved, the time value increases by `dt`. After the loop is completed, the function returns the results as `T` and `Res` lists. These results can be represented in the form of plots using the `matplotlib` library.

```
class System(object):
    def __init__(self, els, eqs):
        self.els=els # components list
        self.elsd=dict([(e.name,e) for e in els]) # same, but dict.
        self.eqs=[] # list of system equations
        for e in self.els: # for each component
            self.eqs+=e.eqs # join with component equations
        self.eqs=self.eqs+eqs # join with additional equations

    def solveN(self, eqs): # solves the static problem
        # code is not shown here

    def solve(self, ics): # solves the dynamic problem
```

```
411         eqs=[e.subs(ics) for e in self.eqs] # substitution of ics
412         # discard all degenerate equations
413         eqs=[e for e in eqs if e not in (True,False)]
414         # solve the system of equations by:
415         #sol=solve(eqs) # SymPy (slow)
416         sol=self.solveN(eqs) # SciPy (faster)
417         sol.update(ics) # update dictionary by dictionary ics
418         return sol
419
420     def solveDyn(self, d, timeEnd, fnBC):
421         t=0.0 # time variable
422         T=[] # list of time values
423         Res=[] # list of results
424         ics={} # dictionary with values of variables
425         while t<timeEnd: # while t < final time value
426             for e in self.els: # for each component
427                 # save positions and velocities
428                 if 'x' in e.__dict__:
429                     ics.update({e.xp:d[e.x]})
430                 if 'x1' in e.__dict__:
431                     ics.update({e.x1p:d[e.x1]})
432                 if 'x2' in e.__dict__:
433                     ics.update({e.x2p:d[e.x2]})
434                 if 'v' in e.__dict__:
435                     ics.update({e.vp:d[e.v]})
436             ics.update(fnBC(self.elsd, d, t)) # update BC
437             d=self.solve(ics) # solve the problem
438             print t
439             T.append(t)
440             Res.append(d) # save results
441             t+=dt # increase time value
442             #if some_condition: # changing the system structure
443             #    self.__init__(new_els, new_eqs)
444         return T,Res
445
```

446 You can easily implement modeling of variable structure systems by overriding the `solveDyn`
447 method and calling in it the constructor of the `System` class with new values of arguments `els`,
448 `eqs`. Usually this call should occur after a certain condition.
449

450 **Simulation of free vibrations of the sucker rod string**

451 Let's perform the simulation of free vibrations of the sucker rod string (Fig. 1). In the separate
452 module (Listing S3) we will create the components: spring-damper `s1` and mass `m1`. In round
453 brackets there are the values of the attributes — the name and the known parameters values.
454

```
from pycodyn import *
s1=SpringDamper(name='s1', c=39694.0, d=1856.0)
m1=Mass(name='m1',m=3402.0)
```

Create the list of additional equations, formed by connecting the flanges of the components. Then create the object of the component system.

```
peqs=s1.pinEqs(1,[m1.pins[0]])
s=System(els=[s1,m1], eqs=peqs)
```

A list of the model equations can be printed using the command `print s.eqs`. To obtain equations only in the symbolic form, the numerical values of the constructors parameters `c`, `d`, `m` should be replaced by `None`:

```
[s1_c*(-s1_x1 + s1_x2) + s1_d*s1_vrel == s1_f2,
-s1_f2 == s1_f1,
s1_vrel == -(s1_x1 - s1_x1p)/dt + (s1_x2 - s1_x2p)/dt,
m1_a*m1_m == m1_f1 + m1_f2,
m1_a == (m1_v - m1_vp)/dt,
m1_v == (m1_x - m1_xp)/dt,
s1_x2 == m1_x, s1_x2p == m1_xp, s1_f2 == -m1_f1]
```

Let's solve the static problem — the column is stretched by 1 m.

```
ics={m1.x:-1.0,m1.v:0.0,m1.a:0.0,s1.x1:0.0,s1.x1p:0.0,m1.vp:0.0}
d=s.solve(ics)
```

The boundary conditions depend on the type of the problem. If this is the problem of free oscillations, then the position of the string top point `elsd['s1'].x1` and the force on the plunger `elsd['m1'].f2` are zero. Create the function to update the boundary conditions at time `t` for the `elsd` components. Then solve the dynamic problem — free vibrations of the string.

```
def fnBC(elsd, d, t):
    return {elsd['s1'].x1:0.0, elsd['s1'].x1p:0.0, elsd['m1'].f2:0.0}
T,R=s.solveDyn(d, timeEnd=10, fnBC=fnBC)
```

The comparison of oscillator simulation results for Python and Modelica is shown in Fig. 2. The differences are explained by the use of unequal difference schemes in the Python model and the Modelica solver. It is possible to improve the results in the Python model by using the more accurate but more complex difference schemes. For example, if the trapezoidal rule is used

(Listing S4, Listing S5), the second equation for the `Mass` should be `Eq((self.a+self.ap)/2, (self.v-self.vp)/dt)`.

Simulation of the pumping process by the two-section string

Now in the new module (Listing S6) we will create the model of the sucker rod string, which contains two sections. The model of each section consists of three 1D mechanical translational components: `SpringDamper`, `Mass` and `Force` (Fig. 3). The `SpringDamper` component is designed to simulate the elastic-damper properties of the string section, the `Mass` component simulates the inertial properties of the section, and the `Force` component simulates the section weight in the fluid and other external forces acting on the section.
Assign values to the variable of sections weights `fs` and the variable of liquid weight above the plunger `fr`.

```
from pycodyn import *
fs=(-14602.0, -14602.0)
fr=-16688.0
```

Let's create the components: the spring-damper of the first section `s1`, the mass of the first section `m1`, the weight of the first section `f1`, the spring-damper of the second section `s2`, the mass of the second section `m2`, the weight of the second section with the weight of the liquid `f2`.

```
s1=SpringDamper(name='s1', c=79388.0, d=3712.0)
m1=Mass(name='m1',m=1701.0)
f1=Force(name='f1', f=fs[0])
s2=SpringDamper(name='s2', c=79388.0, d=3712.0)
m2=Mass(name='m2', m=1701.0)
f2=Force(name='f2', f=fs[1]+fr)
```

Form the list of the additional equations of the string model, formed by connecting of the components flanges. And create the object of the component system (string model).

```
peqs=s1.pinEqs(1,[m1.pins[0]])
peqs+=m1.pinEqs(1,[s2.pins[0],f1.pins[0]])
peqs+=s2.pinEqs(1,[m2.pins[0]])
peqs+=m2.pinEqs(1,[f2.pins[0]])
s=System(els=[s1,m1,s2,m2,f1,f2], eqs=peqs)
```

The complete list of equations for this system `s.eqs` in the SymPy format:

```
[s1_c*(-s1_x1 + s1_x2) + s1_d*s1_vrel == s1_f2,
-s1_f2 == s1_f1,
s1_vrel == -(s1_x1 - s1_x1p)/dt + (s1_x2 - s1_x2p)/dt,
```

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```
m1_a*m1_m == m1_f1 + m1_f2,
m1_a == (m1_v - m1_vp)/dt,
m1_v == (m1_x - m1_xp)/dt,
s2_c*(-s2_x1 + s2_x2) + s2_d*s2_vrel == s2_f2,
-s2_f2 == s2_f1,
s2_vrel == -(s2_x1 - s2_x1p)/dt + (s2_x2 - s2_x2p)/dt,
m2_a*m2_m == m2_f1 + m2_f2,
m2_a == (m2_v - m2_vp)/dt,
m2_v == (m2_x - m2_xp)/dt,
s1_x2 == m1_x, s1_x2p == m1_xp,
s1_f2 == -m1_f1, m1_x == s2_x1,
m1_xp == s2_x1p, m1_x == f1_x,
m1_xp == f1_xp, m1_f2 == f1_f - s2_f1,
s2_x2 == m2_x, s2_x2p == m2_xp,
s2_f2 == -m2_f1, m2_x == f2_x,
m2_xp == f2_xp, m2_f2 == f2_f]
```

Let's solve the static problem — the string under the maximum static loads.

```
ics={m1.v:0.0, m1.a:0.0, m2.v:0.0, m2.a:0.0}
ics.update({s1.x1:0.0, s1.x1p:0.0})
d=s.solve(ics)
```

Dictionary `d` contains the results. To display the position value for the bottom point of the second section, enter the command `print d[m2.x]`. We get the result -0.972. This is the elongation value of the string under maximum load. Let's solve the dynamic problem — the upper point has a harmonic motion. The stroke length of the upper point is 3 m, the number of double strokes per minute is 6.5. The `motion` function describes the harmonic motion of the upper point and returns its position at time `t`.

```
def motion(t):
    A=3.0/2 # amplitude
    n=6.5/60 # frequency
    return A*math.sin(2*math.pi*n*t) # position
```

The `force` function returns the value of the force on the pump plunger `F`, depending on the value of its speed `v`. If the speed is less than zero (downstroke of the string), the function returns the weight value of the second section. Otherwise, the function returns the sum of the second section weight and the liquid weight above the plunger. This function should be smoothed when the sign of the velocity changes, for example, using the hyperbolic tangent function `math.tanh`.

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```
def force(v):
    F=fs[1] # weight of the second section
    if v>0: # if upperstroke
        F+=fr # increase the force by value of the fluid weight
    return F*math.tanh(abs(v)/0.01) # smoothing near the point v=0
```

Create the function to update the boundary conditions at time `t` for `elsd` components. Here `d` is the dictionary of the results calculated in the previous step. Then solve the problem.

```
def fnBC(elsd, d, t):
    return {elsd['s1'].x1:motion(t), elsd['f2'].f:force(d[m2.v])}
T,R=s.solveDyn(d, timeEnd=2*60/6.5, fnBC=fnBC)
```

The results (Fig. 4) correspond to practical dynamometer cards obtained on real wells. The simulation of the variable structure system (the breakage of the sucker rod string) is implemented in Listing S7. This is done by overriding the `solveDyn` method. The simulation results are shown in Fig. 5.

Conclusions

The Python-classes that allow creating the models in Python without the need to study and apply specialized modeling languages are developed. These classes can also be used to automate the construction of the system of difference equations, describing the behavior of the model, in a symbolic form. To fully describe the behavior of the model, these equations must be supplemented with initial and boundary conditions, which are described by certain functions (`motion`, `force`, `fnBC`). These functions may contain any imperative code and it simplifies integration with the third party software packages. Composing and solving the system of algebraic equations at each discrete time point of the simulation using SymPy and SciPy is quite slow, but it makes easier to implement variable structure systems modeling. For example, by changing the values of system attributes in the `solveDyn` function. The replacement of differential equations by difference equations allows simplifying the implementation of the hybrid modeling and the requirements for the modules for symbolic mathematics and for solving equations. However, the problem in the form of difference equations is usually more difficult to formulate. In the future it is planned to extend the set of the components, optimize the algorithm for solving equations and develop support for hierarchical models and the tools for building models using block diagrams. The source code is available on the GitHub (https://github.com/vkopey/pycodyn).

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Figure 1(on next page)

Block diagram of the oscillator model.

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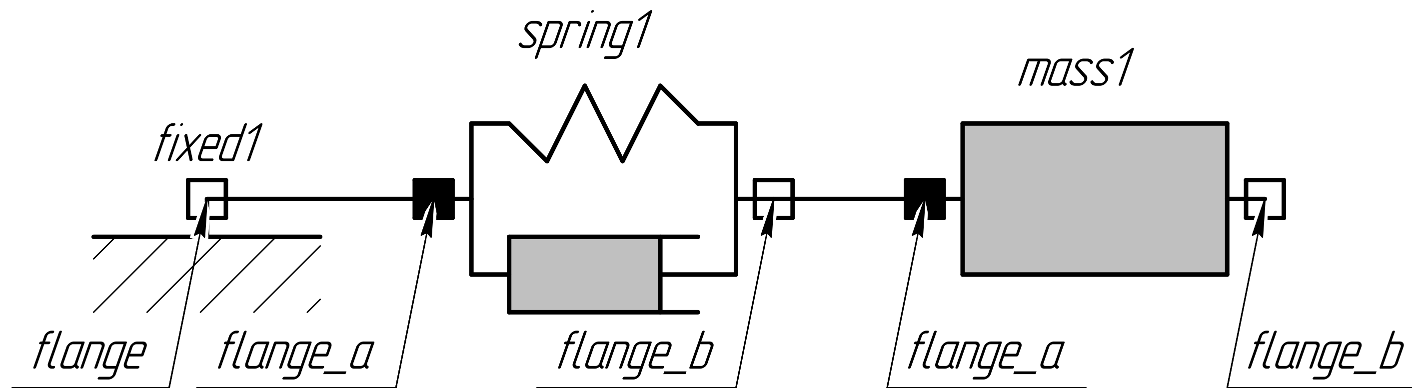

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Figure 2

(on next page)

Plunger position (x) during free oscillation of the string.

(▬) Euler method with time step dt=0.1 s; (—) Euler method with time step dt=0.01 s; (---) Trapezoidal rule with time step dt=0.1 s; (....) Runge–Kutta method, order 4 (Modelica-model).

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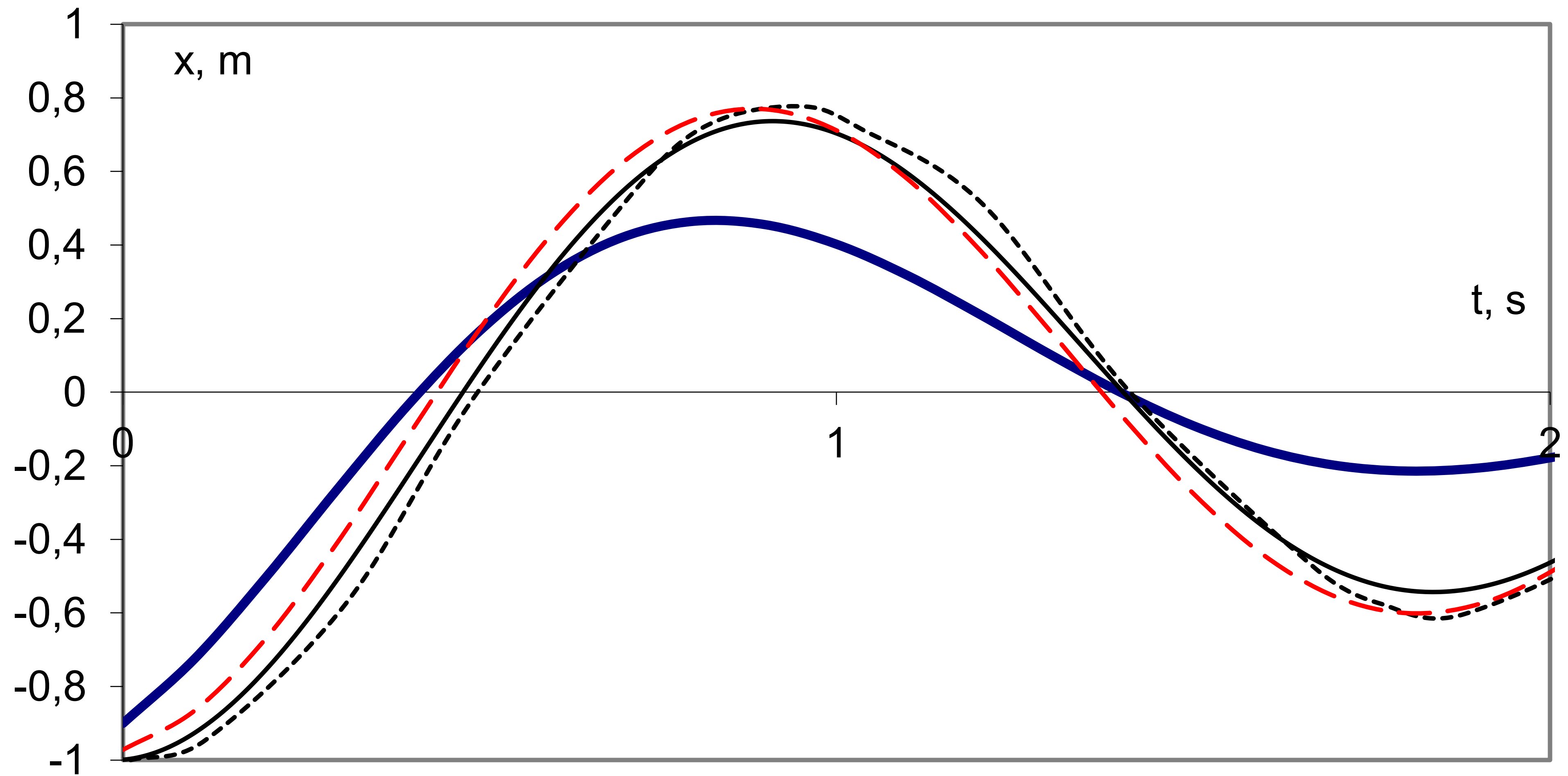

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Figure 3 (on next page)

Block diagram of the model with two sections.

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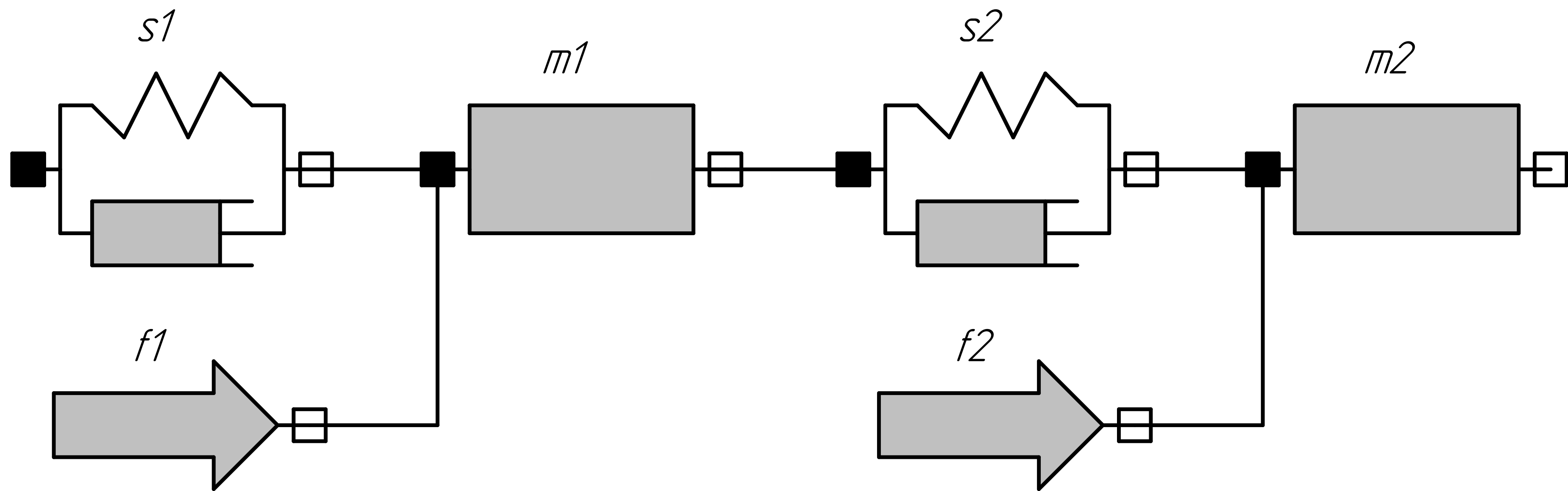
s1
m1
s2
m2
f1
f2

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Figure 4 (on next page)

Simulation results – the wellhead (at the top) and plunger (at the bottom) dynamometer cards.

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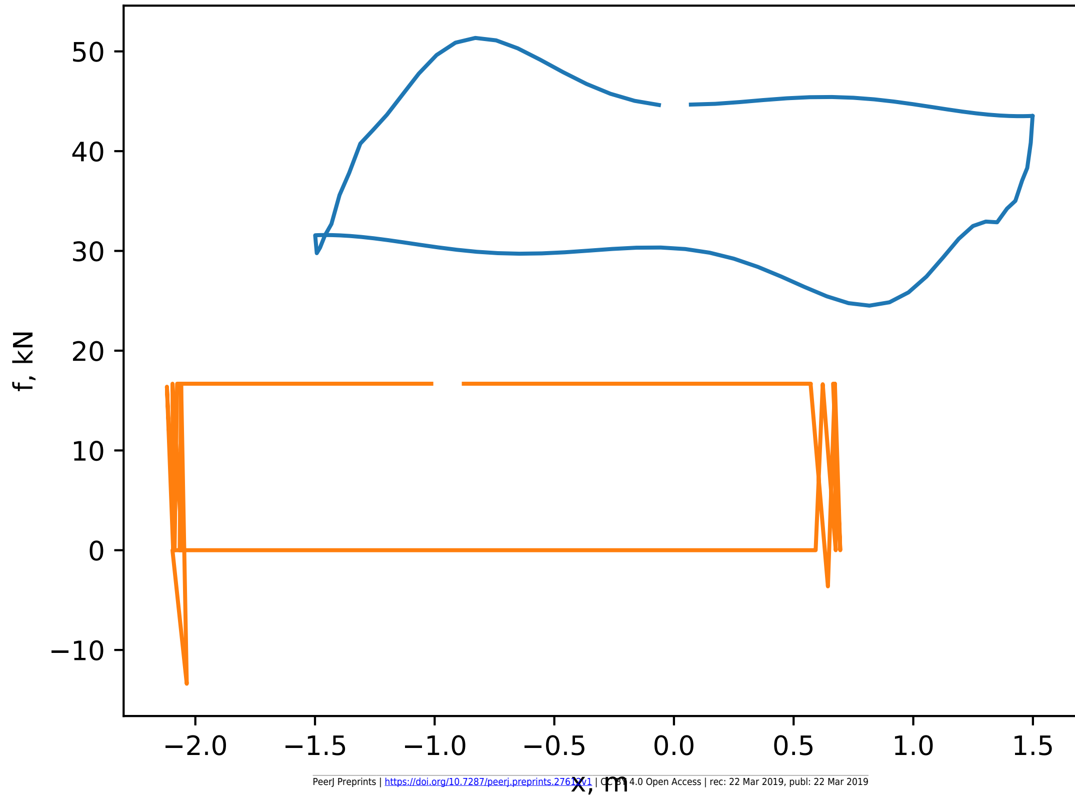

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Figure 5
(on next page)

The simulation of the breakage of the sucker rod string (wellhead dynamometer card).

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