Mixed integer nonlinear programming for three-dimensional aircraft conflict avoidance

Junling Cai Corresp., , **Ning Zhang**

Corresponding Author: Junling Cai
Email address: cjunling@buaa.edu.cn

The problem of aircraft conflict avoidance for Air Traffic Management systems is studied. In the scenario, aircraft are considered to fly within a shared three-dimensional airspace and not allowed to approach close less than a minimum safe separation during their flights in order to avoid various conflicts. This paper proposes a formulation of the three-dimensional conflict avoidance problem as a Mixed Integer Non-Linear Programming (MINLP) model where aircraft are allowed to change both their heading angle and velocity simultaneously to keep the separation. The validity of the proposed model is demonstrated by a comparison of the results from the MINLP model and the previous conflict avoidance models with one maneuver of the heading angle or the velocity. The numerical studies show that the MINLP model improves the efficiency of computation and maintain the safety of flights even by using a standard global optimization solver

PeerJ Preprints | https://doi.org/10.7287/peerj.preprints.27410v1 | CC BY 4.0 Open Access | rec: 5 Dec 2018, publ: 5 Dec 2018

Mixed integer nonlinear programming for three-dimensional aircraft conflict avoidance

Junling Cai[1], Ning Zhang[1]

[1] School of Economics and Management, Beihang University, Beijing, 100083, P.R. China

Corresponding Author:
Junling Cai[1]
No. 37 Xueyuan Road, Haidian District, Beijing, 100083, P.R. China, 100083.
Email address: cjunling@buaa.edu.cn

Abstract

The problem of aircraft conflict avoidance for Air Traffic Management systems is studied. In the scenario, aircraft are considered to fly within a shared three-dimensional airspace and not allowed to approach close less than a minimum safe separation during their flights in order to avoid various conflicts. This paper proposes a formulation of the three-dimensional conflict avoidance problem as a Mixed Integer Non-Linear Programming (MINLP) model where aircraft are allowed to change both their heading angle and velocity simultaneously to keep the separation. The validity of the proposed model is demonstrated by a comparison of the results from the MINLP model and the previous conflict avoidance models with one maneuver of the heading angle or the velocity. The numerical studies show that the MINLP model improves the efficiency of computation and maintain the safety of flights even by using a standard global optimization solver.

1 Introduction

In recent years air traffic volume has tremendously increased worldwide. The flight delay and congestion become more serious for the challenge of flight safety in air traffic management. The consequences are the worse of workload fatigue in air traffic controllers and the flight conflict of aircraft in airspace. In this context, it is necessary to develop more efficient and reliable optimization tools for enhancing Air Traffic Management (ATM). Presently, numerous researches have been concentrated on air traffic safety with wide topics in the problems of aircraft conflict detection and resolution (CDR). One of the significant solutions is focused on the aircraft conflict avoidance.

1.1 Literature review

PeerJ Preprints | https://doi.org/10.7287/peerj.preprints.27410v1 | CC BY 4.0 Open Access | rec: 5 Dec 2018, publ: 5 Dec 2018

A conflict occurs when the distance between any pairs of aircraft involved is less than the minimum safe separation. In general, the standard separation norms are 5 nm (nautical miles) for the horizontal separation and 2000 ft (feet) for the vertical separation in en-route airspace [1]. To avoid various possible conflicts, three types of maneuvers are used to separate aircraft. Altitude change (AC) is the most efficient maneuver for resolving short-term conflicts, even if it is not frequently recommended due to the discomfort of passengers and the large consumption of fuel. The other two maneuvers are the velocity change (VC) and heading angle change (HAC), which are used as separation maneuvers in most of mathematical optimization approaches. Given the main parameters of all aircraft in the airspace including the initial position, velocities, angles of direction, and predicted trajectory, the aircraft conflict avoidance problem consists in identifying, starting from such initial configuration, a new one such that all conflict situations are avoided.

Various mathematical optimization approaches to address the problem of aircraft conflict avoidance were proposed in the relevant literature. Kuchar et al. [2] presented a comprehensive literature review on conflict avoidance approaches from 1900s to 2000s. In recent years, the Mixed Integer Linear Programming (MILP) and Mixed Integer Nonlinear Programming (MINLP) models based on the above three maneuvers provide powerful theoretical frameworks for the aircraft conflict avoidance [3]. Pallottino et al. [4] developed a mathematical model based on the geometric construction with the formulation of multi-aircraft conflict avoidance by using two different MILP models, one with adjusting velocity and another with changing heading angle, solved by a standard MILP software in a very short computational time. Christodoulou et al. [5] proposed an approach by combining aircraft velocities and heading angles maneuvers to resolve the aircraft conflicts in a formulation of the MINLP, which was applied to small-scale aircraft conflict problem with more computational effort. Schouwenaars [6] developed a framework for safe online trajectory planning that is formulated as a receding horizon optimization problem using MILP to incorporate kino-dynamic, obstacle avoidance and conflict avoidance constraints. Vela et al. [7] presented a MILP model in which conflict situations are avoided by performing velocity change and altitude change. Subsequently, Vela et al. [8] proposed another MILP model with the objective function of minimizing fuel costs to determine the required heading angle variation and velocity variation of each aircraft for avoidance of conflict. In 2011s, Alonso-Ayuso et al. [9] presented some different mathematical approaches by extending and improving the VC and HAC models proposed in [4]. Alonso-Ayuso et al. [10] developed a mixed 0-1 linear optimization model based on VC for conflict avoidance between aircraft in the airspace. Alonso-Ayuso et al. [11] presented a mixed 0-1 nonlinear nonconvex model for resolving the conflict avoidance problem, and used an approximate sequential integer linear optimization approach to solve heuristically the problem by a MIP solver at each iteration. Two MIP models where velocity and altitude changes are considered respectively as maneuvers to avoid conflicts are proposed in Alonso-Ayuso et al. [12]. Omer et al. [13] developed a hybrid algorithm by using the optimal solution of a MILP as a starting point when solving a nonlinear formulation of the aircraft conflict avoidance. Otherwise, Cafieri et al. [14] presented a MINLP model for resolution of two-dimensional aircraft conflict with the maneuver of accelerating or decelerating aircraft to avoid conflict. Omer [15] proposed a space-discretized MILP model for air conflict resolution with a combination of velocity and heading angle maneuvers and the space discretization of the aircraft trajectories. Alonso-Ayuso et al. [16] developed a non-convex MINLP model based on HAC to detect and resolve the potential conflicts by using an unconstrained quadratic program to force each aircraft to return to the original flight configuration sequentially. Alonso-Ayuso et al. [17] presented an exact MINLP model for tackling the aircraft conflict detection and resolution problem by allowing aircraft to perform both horizontal (VC and HAC) and vertical (AC) maneuvers which were determined by three multicriteria approaches (the lexicographic ordering, the compromise criterion and a mixture of minimizing the largest deviation of the whole set of maneuvers to be performed from the ideal value of each maneuver). Alonso-Ayuso et al. [18] developed an approximating sequential MINLP approach to deal with the aircraft conflict problem depended on the changes of heading angle, velocity, and altitude which were well arranged by using the goal programming scheme. Hong et al. [19] adopted a concept of airspace traffic complexity to develop a conflict resolution model that is formulated as a MILP allowing all aircraft to perform either heading angle change or velocity change, but not both at the same time. This model reduces not only the number of required maneuvers for aircraft to resolve conflict, but also the number of aircraft involved during the process of conflict resolution. In the case of effective aircraft separation, Cafieri et al. [20] proposed a two-step MINLP model for the aircraft deconfliction by sequentially performing velocity and heading angle changes. Cafieri et al. [21] presented aircraft conflict resolution models by combining MINLP model with greedy algorithm in which aircraft relied solely on velocity changes to achieve separation. The model provides exact flight trajectory adjustments for each aircraft. An up-to-date survey on MINLP modeling methods was presented by Cafieri et al. [22].

Additionally, other methods of three-dimensional conflict avoidance that do not rely on the MINP and MINLP models have been also proposed by some researchers. For instance, Hu et al. [23] proposed the optimal three-dimensional conflict-free maneuvers for multiple aircraft by selecting one of three maneuvers to minimize a certain energy function. Geser et al. [24] utilized geometric optimal approach to present a conflict resolution and recovery algorithm for two aircraft in three-dimensional airspace, which produces conflict-free flight plans for the ownship and intruder aircraft. Raghunathan et al. [25] utilized the rigorous numerical trajectory optimization method to address the problem of optimal cooperative three-dimensional conflict resolution involving multi-aircraft. Malaek et al. [26] proposed a decentralized conflict resolution algorithm for multiple aircraft encounters based on a probabilistic model of the aircraft motion. The method cannot guarantee the safety of flight and handle the explosively increasing number of resolution types when the number of aircraft involved is large. A Variable Neighborhood Search (VNS) for three-dimensional conflict resolution model is presented by Shi [27] in which only velocity change is used as a maneuver. This approach resolves both horizontal and vertical conflicts by performing various maneuvers and improves the computational efficiency. Matsuno et al. [28] proposed a stochastic optimal control method for handling the three-dimensional aircraft conflict detection and resolution with multiple heterogeneous aircraft under wind uncertainty by combining with the conflict detection algorithm. Based on dynamic programming, Vasyliev [29] proposed a method of multi-objective conflict resolution between two aircraft using heading angle, velocity, and altitude changes. Chen et al. [30] developed a model with the non-differentiable disjunctive conflict avoidance constraints constructed by integrating a probability density function to address the three-dimensional aircraft conflict resolution. The approach produces conflict-free maneuvers and minimal and negligible approximation errors. Vasyliev [31] presented the conflict-free flight trajectories in three-dimensional airspace developed through multi-objective dynamic programming and selection of the optimal combination with the convolution of optimality criteria. Lehouillier et al. [32] tackled the conflict resolution problem using a new variant of the minimum-weight maximum-cardinality clique model. Cafieri et al. [33] proposed an optimal control model with the minimization of the integral over a time window to solve the aircraft conflict avoidance problem, where aircraft separation is achieved by changing the velocity of aircraft. The model provides smooth solutions in terms of computational time. Most of the previous methods based on mathematical programming to tackle the conflict avoidance problem are mainly focused on the situation of two-dimensional airspace, and the three-dimensional conflict avoidance problems solved by using the MIP model are studied less.

1.2 Contribution statement and paper structure

Two different assumptions are made respectively in most previous models. One is that each aircraft is only allowed to perform instantaneously velocity change or heading angle change for the conflict avoidance, and the other is that no conflict between aircraft occurs at initial time. This is unrealistic for aircraft to make only one specific type of maneuver (heading angle or velocity) to avoid conflict. So in this paper, we consider aircraft separation achieved by a combinational maneuver of heading angle and velocity changes. Specifically, the three-dimensional aircraft conflict avoidance problem is formulated as Mixed Integer Non-Linear Programming (MINLP) model by allowing aircraft to change simultaneously both heading angle and velocity for avoiding various possible conflicts.

The MINLP model is solved using a state-of-the-art global optimization solver. Numerical studies verify the benefit of the proposed combination of the two considered aircraft separation maneuvers, thus validating the proposed approach. In addition, the model can obtain the optimal solution of the problem in a short computational time to resolve effectively the conflict between aircraft, and its performance is superior to those of the previous conflict avoidance models with one maneuver in terms of time and quality of solution.

The rest of the paper is organized as follows. Section 2 presents the MINLP model for the aircraft conflict avoidance problem in three-dimensional airspace, where the potential conflict is avoided by simultaneously performing the heading angle and velocity changes. Section 3 reports numerical experiments as well as the main results obtained by solving the MINLP model versus the previous models which focus on one maneuver. And, finally, conclusion and outlines for future research are drawn in Section 4.

2 Modeling three-dimensional aircraft conflict avoidance

2.1 Main definitions

Given a finite set F of aircraft sharing the same three-dimensional airspace and flying in it. Each aircraft f in F is identified in a three-dimensional coordinate system by the quintet $(x_f, y_f, z_f, \varphi_f, \theta_f)$, which gives its position and space direction (shown in Fig 1). The main elements of the MINLP model are as follows:

Sets

F , the set of aircraft flying in the three-dimensional airspace.

Parameters

For all aircraft $f \in F$:

x_f^o, y_f^o, z_f^o, initial position of aircraft f in a three-dimensional coordinate system including two horizontal coordinates (abscissa, ordinate) and a vertical coordinate, respectively.

v_f , initial velocity of aircraft f .

φ_f , θ_f , two initial angles representing the space direction, i.e., heading angle and track angle of aircraft f .

v_{min} , v_{max} , minimum and maximum velocities variations imposed for aircraft f , respectively, and should satisfy the form of $(v_{max} - v_{min})/v_{min} < 0.1$ from the Federal Aviation Administration [2].

φ_{min} , φ_{max} , minimum and maximum heading angle variations imposed for aircraft f , respectively. Being 30º of initial angle according to the guidelines from the ERASMUS [15].

$c_f^{p^+}$, $c_f^{p^-}$, unit costs for heading angle changes for aircraft f including positive (turn) and negative (right) heading changes variations.

$c_f^{q^+}$, $c_f^{q^-}$, unit costs for velocity changes for aircraft f including positive (acceleration) and negative (deceleration) velocity variations.

d , the minimum safe separation between any two aircraft.

Variables

For each aircraft $f \in F$ **:**

p_f , heading angle variation of aircraft f for separating aircraft. This variable is real and can be split into two nonnegative variables, say, p_f^+ and p_f^-, such that $p_f = p_f^+ - p_f^-$.

q_f, velocity variation of aircraft f for separating aircraft. This variable is real and can be split into two nonnegative variables, say, q_f^+ and q_f^-, such that $q_f = q_f^+ - q_f^-$.

For all aircraft $i, j \in F : i < j$

$t_{i,j}$, the time.

$b_{i,j}$, $k_{i,j}$, auxiliary 0-1 variables such that the $t_{i,j} \geq 0$ when $b_{i,j} = 1$ and $k_{i,j} = 0$.

$x_f(t)$, $y_f(t)$, $z_f(t)$, position of aircraft f in three-dimensional coordinate system at time $t_{i,j}$.

2.2 The MINLP model for HAC and VC

Let us consider an arbitrary pair of aircraft i and j $(i, j \in F : i < j)$ to obtain the conflict avoidance constraints of the MINLP model. The minimum separation distance between aircraft i and j in the three-dimensional airspace is as follows:

$$\sqrt{(x_i(t)-x_j(t))^2 + (y_i(t)-y_j(t))^2 + (z_i(t)-z_j(t))^2} \geq d \tag{1}$$

A conflict between aircraft i and j occurs if the above condition is not satisfied for some time $t_{i,j}$, and otherwise, no conflict. To achieve the separation, each aircraft is allowed to change simultaneously its heading angle and velocity in our model for the conflict resolution.

Assume that the theory of uniform motion law and relative motion are applied in the heading angle and velocity changes for the three-dimensional aircraft conflict avoidance problem, whose equations for position of aircraft i and j at time t are as follows:

$$x_i(t) = x_i^o + (v_i + q_i)t\sin(\varphi_i + p_i)\cos\theta_i \tag{2}$$

$$y_i(t) = y_i^o + (v_i + q_i)t\sin(\varphi_i + p_i)\sin\theta_i \tag{3}$$

$$z_i(t) = z_i^o + (v_i + q_i)t\cos(\varphi_i + p_i) \tag{4}$$

$$x_j(t) = x_j^o + (v_j + q_j)t\sin(\varphi_j + p_j)\cos\theta_j \tag{5}$$

$$y_j(t) = y_j^o + (v_i + q_j)t\sin(\varphi_j + p_j)\sin\theta_j \tag{6}$$

$$z_j(t) = z_j^o + (v_j + q_j)t\cos(\varphi_j + p_j) \tag{7}$$

According to Fig 1, the velocity vector of aircraft i and j can be described, respectively:

$$v_i = \begin{bmatrix} (v_i + q_i)\sin(\varphi_i + p_i)\cos\theta_i \\ (v_i + q_i)\sin(\varphi_i + p_i)\sin\theta_i \\ (v_i + q_i)\cos(\varphi_i + p_i) \end{bmatrix} \tag{8}$$

$$v_j = \begin{bmatrix} (v_j + q_j)\sin(\varphi_j + p_j)\cos\theta_j \\ (v_j + q_j)\sin(\varphi_j + p_j)\sin\theta_j \\ (v_j + q_j)\cos(\varphi_j + p_j) \end{bmatrix} \tag{9}$$

where q_i (q_j) and p_i (p_j) for all aircraft i and j are the decision variables of MINLP model that we are going to build.

If aircraft j is seen as a reference object, then the velocity of aircraft i relative to aircraft j is $V_{i|j}^r$, i.e., the relative velocity between aircraft i and j.

$$V_{i|j}^r(t) = \begin{pmatrix} (v_i + q_i)\sin(\varphi_i + p_i)\cos\theta_i - (v_j + q_j)\sin(\varphi_j + p_j)\cos\theta_j \\ (v_i + q_i)\sin(\varphi_i + p_j)\sin\theta_i - (v_j + q_j)\sin(\varphi_j + p_j)\sin\theta_j \\ (v_i + q_i)\cos(\varphi_i + p_i) - (v_j + q_j)\cos(\varphi_j + p_j) \end{pmatrix} \tag{10}$$

The initial relative position of aircraft i with respect to aircraft j is $C_{i|j}^{ro}$:

$$C_{i|j}^{ro} = \begin{pmatrix} x_i^o - x_j^o \\ y_i^o - y_j^o \\ z_i^o - z_j^o \end{pmatrix} \tag{11}$$

The parametric equation of any line with respect to time $t_{i,j}$ is defined as $r(t_{i,j}) = C_{i|j}^{ro} + V_{i|j}^r t_{i,j}$, as shown in Fig 2. If the closest point on the line to the origin is $r(t_{i,j}^m)$, then the line segment connecting the origin to $r(t_{i,j}^m)$ is perpendicular to $V_{i|j}^r$, that is, the inner product between them is 0.

$$(C_{i|j}^{ro} + V_{i|j}^r t_{i,j}^m) \bullet V_{i|j}^r = 0 \tag{12}$$

The value of $t_{i,j}^m$ is then obtained:

$$t_{i,j}^m = -\frac{C_{i|j}^{ro} \bullet V_{i|j}^r}{V_{i|j}^r \bullet V_{i|j}^r} \tag{13}$$

The relative shortest distance between aircraft i and j is $\| r(t_{i,j}^m) \|$ which is the distance from the origin to line $r(t_{i,j})$ (or point $r(t_{i,j}^m)$).

$$\begin{aligned} \| r(t_{i,j}^m) \|^2 &= r(t_{i,j}^m) \bullet r(t_{i,j}^m) \\ &= C_{i|j}^{ro} \bullet C_{i|j}^{ro} - \frac{(C_{i|j}^{ro} \bullet V_{i|j}^r)^2}{V_{i|j}^r \bullet V_{i|j}^r} \end{aligned} \tag{14}$$

There is no conflict between aircraft i and j if $\| r(t_{i,j}^{m}) \|$ is greater than the minimum safe separation d. So the conflict avoidance constraint of any pair of aircraft i and j is expressed as follows:

$$\| r(t_{i,j}^{m}) \|^2 \geq d^2 \tag{15}$$

$$(V_{i|j}^{r} \bullet V_{i|j}^{r})(C_{i|j}^{ro} \bullet C_{i|j}^{ro} - d^2) - (C_{i|j}^{ro} \bullet V_{i|j}^{r})^2 \geq 0 \tag{16}$$

Apparently, the left-hand side of the constraint (16) is a function of time $t_{i,j}^{m}$. The constraint (16) can be satisfied only when $t_{i,j}^{m} \geq 0$. Two auxiliary 0-1 variables $b_{i,j}$ and $k_{i,j}$ are introduced for each pair of aircraft i and j to check sign of $t_{i,j}^{m}$, and must satisfy the following condition.

$$b_{i,j} + k_{i,j} = 1 \tag{17}$$

where $t_{i,j}^{m} \geq 0$ only if $b_{i,j} = 1$ and $k_{i,j} = 0$, the constraint (16) will be imposed in MINLP model.

The new conflict avoidance constraint for each pair of aircraft i and j is reformulated, which is nonlinear in those q_i (q_j) and p_i (p_j).

$$b_{i,j}((C_{i|j}^{ro} \bullet V_{i|j}^{r})^2 - (V_{i|j}^{r} \bullet V_{i|j}^{r})(C_{i|j}^{ro} \bullet C_{i|j}^{ro} - d^2)) \leq 0 \tag{18}$$

As for the objective function, our choice is to minimize the total cost of the sum of the positive and negative variations for the heading angle and velocity maneuver, that is

$$\min \sum_{f \in F} (| c_f^{p^+} p_f^+ - c_f^{p^-} p_f^- | + | c_f^{p^+} q_f^+ - c_f^{q^-} q_f^- |) \tag{19}$$

The full formulation for the MINLP model is summarized below, including all the aspects that have been studied above.

$$\min \sum_{f \in F} (| c_f^{p^+} p_f^+ - c_f^{p^-} p_f^- | + | c_f^{p^+} q_f^+ - c_f^{q^-} q_f^- |) \tag{20}$$

subject to

$$\forall i, j, f \in F : i < j$$

$$b_{i,j}((C_{i|j}^{ro} \bullet V_{i|j}^{r})^2 - (V_{i|j}^{r} \bullet V_{i|j}^{r})(C_{i|j}^{ro} \bullet C_{i|j}^{ro} - d^2)) \leq 0 \tag{20}$$

$$t_{i,j}^{m} = -\frac{C_{i|j}^{ro} \bullet V_{i|j}^{r}}{V_{i|j}^{r} \bullet V_{i|j}^{r}} \tag{21}$$

$$(2k_{i,j} - 1) t_{i,j}^{m} \leq 0 \tag{22}$$

$$b_{i,j} + k_{i,j} = 1 \tag{23}$$

$$v_{\min} \leq v_f + q_f \leq v_{\max} \tag{24}$$

$$\varphi_{min} \leq \varphi_f + p_f \leq \varphi_{max} \tag{25}$$

$$b_{i,j}, k_{i,j} \in \{0,1\} \tag{26}$$

PeerJ Preprints | https://doi.org/10.7287/peerj.preprints.27410v1 | CC BY 4.0 Open Access | rec: 5 Dec 2018, publ: 5 Dec 2018

Constraints (20) are the separation condition for each pair of aircraft. Constraints (21) define the value of $t_{i,j}^m$. Constraints (22) is set to check the sign of time $t_{i,j}^m$. Constraints (23) give the conditions that variables $b_{i,j}$ and $k_{i,j}$ should satisfy. Constraints (24) - (25) are the minimum and maximum value of velocity and heading angle variations for each aircraft f, respectively. Constraints (26) define the type of variables in the model.

After the conflict solved using the MINLP model above, new changes in the heading angles have to be made in order to return aircraft to their initial trajectories. So we need to determine the optimal time for which each aircraft can return to its initial trajectory after the conflict resolution, obtained by solving unconstrained quadratic programming (QP) problem [15] for each pair of aircraft i and j $(i, j \in F)$. The objective function of the problem consists of minimizing the relative Euclidean distance, which is computed using new three dimensional coordinates obtained by using heading angles from the proposed MINLP model, between aircraft i and j with respect to time t. Specifically, knowing p_f and q_f for each aircraft $f \in F$ from the solution of MINLP model, the new position of aircraft f in the three-dimension coordinate system is expressed as

$$x_f(t) = x_f^o + (v_f + q_f)t\sin(\varphi_f + p_f)\cos\varphi_f \tag{27}$$

$$y_f(t) = y_f^o + (v_f + q_f)t\sin(\varphi_f + p_f)\sin\varphi_f \tag{28}$$

$$z_f(t) = z_f^o + (v_f + q_f)t\cos(\varphi_f + p_f) \tag{29}$$

For each pair of aircraft i and j $(i, j \in F)$, the objective function for the QP to be solved is

$$\min_{t_{i,j}} \left\| \begin{pmatrix} x_i(t_{i,j}) - x_j(t_{i,j}) \\ y_i(t_{i,j}) - y_j(t_{i,j}) \\ z_i(t_{i,j}) - z_j(t_{i,j}) \end{pmatrix} \right\|^2 \tag{30}$$

that can calculate the optimal time for each pair of aircraft i and j such that they are separated when their new heading angles are used. When the optimal solution $t_{i,j}^{m1}$ for above problem is obtained, the optimal time $T_{i,j}^m := \max_{i<j(i,j\in F)} t_{i,j}^{m1}$ for which aircraft f returns to its initial trajectory after the conflict resolution can be computed. The new trajectory of each aircraft to come back to its initial trajectory is then determined easily by connecting $(x_f(T_{i,j}^m), y_f(T_{i,j}^m), z_f(T_{i,j}^m))$ and finial position coordinate of aircraft f. There is detailed description of the method in [16].

3 Numerical experiments

This section presents the main results of a broad numerical experiments that have been performed for assessing the validity of the MINLP model, where the AMPL modeling language [34] is used to implement all considered mathematical programming models, and the state-of-the-art solver of choice is COUENNE, see Belotti [35].

Two types of initial aircraft configurations are considered. First, n aircraft are occupied the three-dimensional space enclosed by the surface of the outer sphere of radius 100 nm. Second, n aircraft are randomly distributed on a cube of edge 100 nm and fly in it according to the pre - randomly set path. The initial velocity of each aircraft is set to 0.55 nm/s (nautical miles per second) and the two angles of direction including the heading angle and track angle between $-\pi$ and π. The minimum safe separation is set to 5 nm. The cases with different numbers of aircraft are randomly generated based on the above three initial configurations, and each case is composed of five instances in order to simulate a more realistic scenario.

For ease of simulation, the cost-related parameters for the objective functions are fixed at 1 in all of the instances:

$$c_f^{p^+}, c_f^{p^-}, c_f^{q^+}, c_f^{q^-} = 1$$

The headings of the tables for case studies are as follows: n, number of aircraft; n_{cons}, number of constraints; n_{cont}, number of continuous variables; n_{int}, number of integer variables contained in continuous variables; n_{acont} and n_{aint}, number of continuous and integer variables contained in auxiliary variables; Ci_n, i =1, 2, case where i and n denote the two different initial configurations described above and the number of aircraft in consideration, respectively; n_c, number of potential conflict; n_{hth}, number of head-to-head conflicts; n_{nc}, number of unresolved conflicts; time, computational time (s) to obtain the optimal solution; obj, objective function value of the model; M_{VC}, the previous conflict avoidance model with only velocity change proposed in [14]; M_{HAC}, the previous conflict avoidance model with only heading angle change proposed in [20]; M_{VC+HAC}, the MINLP model in our study; $rg_{v/vc,}$ relative gap of the absolute values of the solution for the velocity changes obtained by M_{vc} versus M_{vc+hac} model; $rg_{ha/hac}$, relative gap of the absolute values of the solution for the heading angle changes obtained by M_{hac} versus M_{vc+hac} model.

Table 1 shows the problem dimensions of the MINLP model. It is obvious that increasing the number of aircraft, constraints and variables, in particular that of continuous variables contained in auxiliary variables used to solve the MINLP model increases largely.

Table 2 and 3 report the objective function values obtained by solving models M_{vc}, M_{hac} and M_{vc+hac} as well as the required computational time. Additionally, the tables show the number of potential conflict situations that take place. Finally, the number of head-to-head conflicts and unresolved conflicts are also reported, respectively. Notice that for each case, the results for time and obj are averages of five instances.

It can be observed in Table 2 and 3 that for all the cases we performed, the proposed MINLP model significantly improves the solution in terms of computational time and quality of solution. On the one hand, as expected, most of conflicts can be solved by adjusting slightly velocity in a shorter computational time, but the velocity change only allowed for the aircraft is insufficient to resolve some difficult conflict situations like head-to-head conflict (see column n_{nc} in Table 2 and 3, respectively), since they must be avoided by performing the HAC maneuver. On the other hand, M_{hac} model have a good performance for solving various possible conflicts. However, the magnitude of the variations is far larger in the heading angle of all aircraft than in the one of M_{vc+hac} model (see column obj in Table 2 and 3, respectively). This means that the solution provided by the M_{hac} model could cause aircraft to consume enormous amounts of fuel to adjust heading angle for keeping the separation. The M_{vc+hac} model with combined maneuvers of velocity and heading angle, by contrast, is easier to avoid conflicts by slightly making heading angle and velocity variations, thus saving fuel and ensuring the safety of flights. Additionally, M_{vc+hac} model requires less the computational time for obtaining the optimal solution compared with M_{ha}c model. This makes it more suitable for solving three-dimensional aircraft conflict avoidance problem in real life, providing timely decision for pilots.

The relative gap is employed to further testify the improvement of the MINLP model. Table 4 and 5 report the relative gap between the absolute values of the solution for the maneuver changes obtained by M_{vc} (or M_{hac}) versus M_{vc+hac} models, being computed as $\frac{sv_{orig} - sv_{ne}}{sv_{ne}}\%$, where subscripts orig and ne refer to M_{vc} (or M_{hac}) and M_{vc+hac} models, respectively. Notice that for each case the results reported are averages of five instances performed. It can be observed clearly that in all cases the relative gaps obtained by M_{vc} (or M_{hac}) versus M_{vc+hac} models are very small and always remain below 0 percent. This shows that there are small difference between the solutions for maneuver variations obtained by solving models M_{vc} (or M_{hac}) and M_{vc+hac}. But, the VC and HAC variations obtained by models M_{vc} and M_{hac} are high than the ones obtained by M_{vc+hac}, respectively. In a word, the MINLP model is superior than the previous aircraft conflict avoidance models with one maneuver in solving aircraft conflict avoidance problem.

Two instances included in C1-8 and C2-8 cases are taken as examples to discuss the conflict resolution of the MINLP model in detail. Here, two types of figures are presented to describe the process of aircraft conflict avoidance, respectively. One shows the initial flight trajectories of eight aircraft before maneuvers are performed, and the other is the corresponding measures of conflict resolution after changing both heading angle and velocity simultaneously, as shown in Fig 3 and Fig 4.

Fig 3 and 4 give eight aircraft flying in a sphere and cube, respectively. Starting from time $t \geq 0$, whether a conflict occurs between any pairs of aircraft could be detected by using the MINLP model. If the answer is affirmative, all aircraft in conflict will be forced to change simultaneously their heading angle and velocity to achieve the separation by accelerating or decelerating and turning left or right, according to the results obtained by solving the MINLP model. It should be pointed out that the assumption of this model is that aircraft are allowed to make heading angle and velocity changes at time $t = 0$ in order to avoid conflicts.

Specifically, in case C1-8, aircraft (1) and (6) have a head-to-head conflict situation that can be solved by simultaneously performing small deceleration and right turn, and aircraft (2) and (7) are in a similar situation. However, these aircraft also have a multiple conflict situation with other aircraft in the current airspace, which are solved by adjusting slightly velocity and heading angle. For pairs of aircraft (2)-(8), (4)-(7) in case C2-8, there are head-to-head conflict between them, respectively, solved by simultaneously changing their velocity and heading angle. Additionally, aircraft (1)-(2)-(3) and (5)-(6)-(7) have multiple conflict situation, respectively, which can be solved effectively by decelerating and turning right.

After the conflict resolution, all aircraft keep on traveling with new velocities $v+q$ and new heading angles $\varphi+p$, respectively, and return to their initial trajectory at the optimal time determined by solving unconstrained QP problem (as section 2.2).

4 Conclusion

A Mixed Integer Nonlinear Programming (MINLP) model for solving the three-dimensional aircraft conflict avoidance problem by allowing aircraft to perform simultaneously heading angle and velocity changes is presented in this paper. The model is solved with a global optimization solver COUENNE even for large-scale instances in a short computational time. The results from numerical experiments show that the MINLP model improves significantly the quality of solution and ensures the safety of flight compared with the previous aircraft conflict models with one maneuver. Additionally, the computational time required for obtaining the problem is small that the MINLP model can be applied in realistic applications to identify the optimal conflict avoidance maneuvers of aircraft.

As a follow-up to this paper, the altitude change will be introduced as new maneuver in our model to avoid various possible conflicts by combining the heading angle and velocity changes. Additionally, all aircraft equally accelerate (or decelerate) and turn right (or left) to achieve the separation. This also needs further improvement in future research.

References

[1] Yang, L C and Kuchar, J K (1997). Prototype conflict alerting system for free flight. *Journal of Guidance, Control, and Dynamics*, 20(4), 768-773.

[2] Kuchar, J K and Yang, L C (2000). A review of conflict detection and resolution modeling methods. *Intelligent Transportation Systems IEEE Transactions on*, 1(4), 179-189.

[3] Lee, J and Leyffer, S (2011). Mixed integer nonlinear programming. *Springer Science & Business Media*.

[4] Pallottino, L, Feron, E M, Bicchi, A (2002). Conflict resolution problems for air traffic management systems solved with mixed integer programming. *IEEE Transactions on Intelligent Transportation Systems*, 3(1), 3-11.

[5] Christodoulou, M and Costoulakis, C (2004). Nonlinear mixed integer programming for aircraft collision avoidance in free flight. *In Proceedings of the 12th IEEE Mediterranean electrotechnical conference, MELECON*. 1, 327-330.

[6] Schouwenaars, T (2006). Safe trajectory planning of autonomous vehicles, *Ph.D. thesis. Massachusetts Institute of Technology*.

[7] Vela, A, Solak, S, Singhose, W, Clarke, J P (2009). A mixed integer program for flight-level assignment and speed control for conflict resolution. *In Proceedings of the 48th IEEE Conference held jointly with the the 2009 28th Chinese Control Conference, CDC/CCC*. 5219-5226.

[8] Vela, A E, Solak, S, Clarke, J P B (2010). Singhose, W. E.; Barnes, E. R.; Johnson, E. L.: Near real-time fuel-optimal en route conflict resolution. *IEEE Transactions on Intelligent Transportation Systems*, 11(4), 826-837.

[9] Alonso-Ayuso A, Escudero L F, Martin-Campo F J (2011). Collision avoidance in Air Traffic Management: A Mixed-Integer Linear Optimization approach. *IEEE Transactions on Intelligent Transportation Systems*, 12(1), 47-57.

[10] Alonso-Ayuso, A, Escudero, L F, Martín-Campo, F J (2012). A mixed 0–1 nonlinear optimization model and algorithmic approach for the collision avoidance in ATM: Velocity changes through a time horizon. *Computers & Operations Research*, 40(1), 3136-3146.

[11] Alonso-Ayuso A, Escudero L F, Martín-Campo F J, et al. (2012). VNS based algorithm for solving a 0–1 nonlinear nonconvex model for the Collision Avoidance in Air Traffic Management. *Electronic Notes in Discrete Mathematics*, 39(1),115-120.

[12] Alonso-Ayuso, A (2013). Conflict avoidance: 0-1 linear models for conflict detection & resolution. *Top*, 21(3), 485-504.

[13] Omer, J and Farges, J L (2013). Hybridization of nonlinear and mixed-integer linear programming for aircraft separation with trajectory recovery. *IEEE Transactions on Intelligent Transportation Systems*, 14(3), 1218–1230.

[14] Cafieri, S, Durand, N (2014). Aircraft deconfliction with speed regulation: new models from mixed-integer optimization. *Journal of Global Optimization*, 58(4), 613-629.

[15] Omer, J (2015). A space-discretized Mixed-Integer Linear Model for air-conflict resolution with speed and heading Maneuvers. *Computers & Operations Research*, 58(C), 75-86.

[16] Alonso-Ayuso, A, Escudero, L F, Martín-Campo, F J (2016). Exact and approximate solving of the aircraft collision resolution problem via turn changes. *Transportation Science*, 50(1), 263-274.

[17] Alonso-Ayuso, A, Escudero, L F, Martín-Campo, F J (2016). An exact multiobjective mixed integer nonlinear optimization approach for aircraft conflict resolution. *TOP*, 24 (2), 381–408.

[18] Alonso-Ayuso, A, Escudero, L F, Martín-Campo, F J (2016). Multiobjective optimization for aircraft conflict resolution. A metaheuristic approach. *European Journal of Operational Research*, 248(2), 691-702.

[19] Hong, Y, Choi, B, Lee, K, Kim, Y (2016). Conflict management considering a smooth transition of aircraft into adjacent airspace. *IEEE Transactions on Intelligent Transportation System*s, 17(9), 2490-2501.

[20] Cafieri, S and Omheni, R (2017). Mixed-integer nonlinear programming for aircraft conflict avoidance by sequentially applying velocity and heading angle changes. *European Journal of Operational Research*, 260, 283-290.

[21] Cafieri S and Rey D (2017). Maximizing the number of conflict-free aircraft using mixed-integer nonlinear programming. *Computers & Operations Research*, 80, 147-158.

[22] Cafieri, S (2017). MINLP in Air Traffic Management: Aircraft conflict avoidance. in Advance and Trends in Optimization with Engineering Applications. *In T. Terlaky, M. F. Anjos, & S. Ahmed (Eds.), MOS-SIAM Book Series on Optimization. Philadelphia: SIAM*.

[23] Hu, J, Pradini, M, Sastry, S (2002). Optimal coordinated maneuvers for three-dimensional aircraft conflict resolution. *Journal of Guidance Control and Dynamics*, 25 (5), 888-900.

[24] Geser, A and Munoz, C (2002). A geometric approach to strategic conflict detection and resolution// *Digital Avionics Systems Conference, 2002. Proceedings. the. IEEE*, 1.

[25] Raghunathan, A U, Gopal, V, Subramanian, D, Biegler, L T, Samad, T (2004). Dynamic optimization strategies for three-dimensional conflict resolution of multiple aircraft. *Journal of Guidance Control and Dynamics*, 27(4), 586-594.

[26] Malaek, M B and Parastari, J (2004). Optimal circular 3-arced with constant speed coordinated maneuvers for planar multi aircraft conflict resolution// *Digital Avionics Systems Conference*, 2004. Dasc 04. the. IEEE, 2004:3.E.4-31-10,1.

[27] Shi, W (2014). A Variable Neighborhood Search Heuristic for three-dimensional conflict resolution model. *Journal of Information & Computational Science*, 11(16), 5811-5824.

[28] Matsuno, Y, Tsuchiya, T, Wei, J, et al. (2015). Stochastic optimal control for aircraft conflict resolution under wind uncertainty. *Aerospace Science & Technology*, 43, 77-88.

[29] Vasyliev, D (2016). Method of multi-objective resolution of two-aircraft conflict in three-dimensional space based on dynamic programming. *Proceedings of the National aviation university*, 68(3), 35-45.

[30] Chen, W, Chen, J, Shao, Z, et al. (2016). Three-Dimensional Aircraft Conflict Resolution Based on Smoothing Methods. *Journal of Guidance Control & Dynamics*, 39(7), 1481-1490.

[31] Vasyliev, D (2017). Method for optimal resolution of multi-aircraft conflicts in three-dimensional space. *Proceedings of the National aviation university*, 70(1), 36-47.

[32] Lehouillier, T, Omer, J, Soumis, F, et al. (2017). Two Decomposition Algorithms for Solving a Minimum Weight Maximum Clique Model for the Air Conflict Resolution Problem. *European Journal of Operational Research,* 256(3), 696-712.

[33] Cafieri, S, Cellier, L, Messine, F, et al. (2018). Combination of optimal control approaches for aircraft conflict avoidance via velocity regulation. *Optimal Control Applications & Methods,* 39(1).

[34] Gay, D M (2015). The AMPL modeling language: An aid to formulating and solving optimization problems. Numerical analysis and optimization. *Springer International Publishing*, 95-116.

[35] Belotti, P (2009). Couenne: a user's manual. *Technical Report, Lehigh University*.

Table 1(on next page)

Figures and tables in this manuscript

PeerJ Preprints | https://doi.org/10.7287/peerj.preprints.27410v1 | CC BY 4.0 Open Access | rec: 5 Dec 2018, publ: 5 Dec 2018

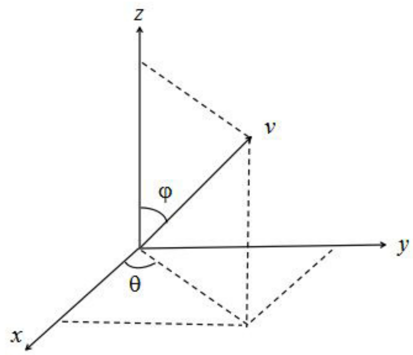

Fig 1. The three-dimensional coordinate

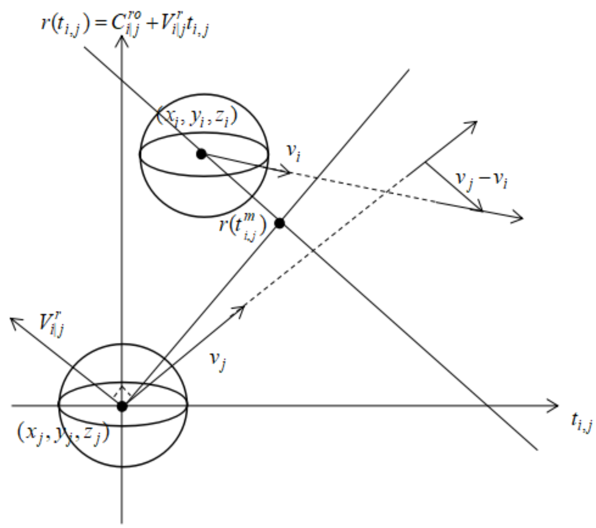

Fig 2. The relative shortest distance between aircraft i and j

Table 1 The dimensions of MINLP model

n	n_{cons}	n_{cont}	n_{int}	n_{acont}	n_{aint}
2	4	7	2	32	1
3	12	15	6	73	3
4	24	26	12	129	6
5	40	40	20	202	10
6	60	57	30	291	15
7	63	56	21	337	21
8	84	72	28	441	28

9	108	90	36	559	36
10	135	110	45	691	45

Table 2 The results for conflict resolution based on the first initial configuration

Ci_n	nc	n_{hth}	M_{VC}			M_{HAC}			M_{VC+HAC}		
			n_{nc}	time	obj	n_{nc}	time	obj	n_{nc}	time	obj
C1-2	1	0	0	0.0074	0.0060	0	0.16	1.1911	0	0.157	1.0526
C1-3	3	0	0	0.0066	0.0090	0	1.473	3.2818	0	0.801	1.5374
C1-4	6	1	1	0.0124	0.0120	0	2.003	3.5704	0	1.834	1.5555
C1-5	10	1	1	0.0537	0.0161	0	9.181	4.5396	0	7.714	1.8357
C1-6	15	1	1	0.7582	0.0190	0	19.57	6.4787	0	15.117	2.3824
C1-7	21	2	2	0.8128	0.0210	0	31.302	7.8561	0	28.201	6.44471
C1-8	28	2	2	0.9227	0.0244	0	52.992	8.4154	0	45.152	7.0606
C1-9	36	2	2	1.5458	0.0270	0	105.496	10.1578	0	84.269	9.2145
C1-10	45	3	3	1.3723	0.0302	0	159.932	12.6042	0	123.439	11.7607
C1-11	55	3	3	1.5123	0.0330	0	206.162	13.4501	0	175.423	12.0839
C1-12	66	3	3	1.8073	0.0384	0	293.928	14.0847	0	220.378	12.8741
C1-13	78	4	4	5.573	0.0410	0	325.207	14.4061	0	272.464	13.3532
C1-14	91	4	4	10.0784	0.0440	0	367.348	15.9226	0	330.496	14.1097
C1-15	105	4	4	23.295	0.0474	0	416.249	16.4039	0	361.371	15.7372
C1-16	120	5	5	46.262	0.0513	0	493.378	17.2777	0	459.78	16.6491
C1-17	135	5	5	56.279	0.0544	0	603.98	18.9531	0	546.38	17.4131
C1-18	153	5	5	87.918	0.0571	0	974.361	20.3727	0	729.56	18.2693
C1-19	171	5	5	120.193	0.0606	0	1409.45	21.5384	0	1135.42	19.4535
C1-20	190	5	5	143.2206	0.0630	0	1615.67	22.1123	0	1415.61	20.3066

Table 3 The results for conflict resolution based on the second initial configuration

Ci_n	nc	n_{hth}	M_{VC}			M_{HAC}			M_{VC+HAC}		
			n_{nc}	time	obj	n_{nc}	time	obj	n_{nc}	time	obj
C2-2	1	0	0	0.0043	0.0061	0	0.0524	1.3742	0	0.0083	1.0527
C2-3	3	0	0	0.0162	0.0090	0	0.9326	2.8481	0	0.0154	1.9593
C2-4	6	1	1	0.0148	0.0120	0	1.8641	4.8425	0	0.5657	3.2732
C2-5	10	1	1	0.0197	0.0150	0	12.9462	5.6656	0	10.5163	5.3814

C2-6	15	1	1	0.0252	0.0182	0	23.3095	6.9528	0	19.4188	6.4592
C2-7	21	2	2	0.0276	0.0210	0	54.3046	8.2900	0	23.3224	6.8075
C2-8	28	2	2	0.0264	0.0240	0	80.8433	10.2269	0	36.4975	9.2348
C2-9	36	2	2	0.0535	0.0270	0	109.2108	11.0395	0	75.7122	9.9473
C2-10	45	3	3	0.0482	0.0300	0	138.0257	12.9305	0	95.3637	10.9088
C2-11	55	3	3	20.805	0.0330	0	157.889	13.7171	0	128.448	11.6258
C2-12	66	3	3	15.942	0.0360	0	191.021	14.0198	0	138.789	12.5448
C2-13	78	4	4	23.242	0.0390	0	206.718	15.1507	0	156.571	13.8266
C2-14	91	4	4	25.952	0.0420	0	375.724	15.4568	0	249.146	14.4896
C2-15	105	4	4	74.774	0.0461	0	471.575	16.9360	0	412.877	15.7679
C2-16	120	5	5	131.421	0.0492	0	696.584	17.8410	0	621.91	16.1682
C2-17	135	5	5	170.648	0.0511	0	835.573	18.2541	0	784.752	17.2029
C2-18	153	5	5	216.082	0.0540	0	1091.816	19.3407	0	957.123	18.7782
C2-19	171	5	5	257.766	0.0570	0	1510.42	20.6085	0	1044.06	19.2773
C2-20	190	5	5	317.408	0.0600	0	1959.672	21.9649	0	1341.63	20.4176

Table 4 The relative gap of solution values (the first configuration)

Ci_n	nc	$rg_{v/vc}$	$rg_{ha/hac}$
C1-2	1	-0.2	-0.1203
C1-3	3	-0.0111	-0.5342
C1-4	6	-0.025	-0.5676
C1-5	10	-0.0062	-0.5991
C1-6	15	-0.0316	-0.6351
C1-7	21	-0.0095	-0.1823
C1-8	28	-0.0082	-0.1639
C1-9	36	-0.0148	-0.0955
C1-10	45	-0.0033	-0.0693
C1-11	55	-0.0061	-0.1040
C1-12	66	-0.0130	-0.0886
C1-13	78	-0.0073	-0.0759
C1-14	91	-0.1045	-0.1163
C1-15	105	-0.0127	-0.0435
C1-16	120	-0.0039	-0.0393
C1-17	135	-0.0074	-0.0841
C1-18	153	-0.0088	-0.1060

C1-19	171	-0.0066	-0.0996
C1-20	190	-0.0016	-0.0845

Table 5 The relative gap of solution values (the second configuration)

Ci_n	nc	$rg_{v/vc}$	$rg_{ha/hac}$
C2-2	1	-0.0164	-0.2384
C2-3	3	-0.0889	-0.3149
C2-4	6	-0.0333	-0.3266
C2-5	10	-0.08	-0.0527
C2-6	15	-0.0055	-0.0736
C2-7	21	-0.0143	-0.1813
C2-8	28	-0.0167	-0.0992
C2-9	36	-0.0556	-0.1013
C2-10	45	0	-0.1586
C2-11	55	-0.1333	-0.1545
C2-12	66	-0.0361	-0.1077
C2-13	78	-0.0154	-0.0899
C2-14	91	-0.0548	-0.0651
C2-15	105	-0.0195	-0.0716
C2-16	120	-0.0264	-0.0964
C2-17	135	-0.0019	-0.0604
C2-18	153	-0.0111	-0.0318
C2-19	171	-0.0211	-0.0673
C2-20	190	-0.0817	-0.0729

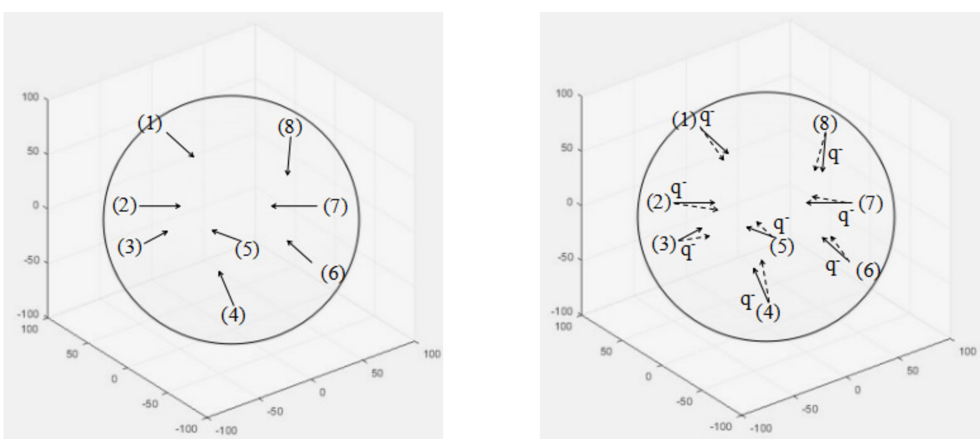

Fig 3. Flight trajectories of eight aircraft before and after conflict resolution (C1-8)

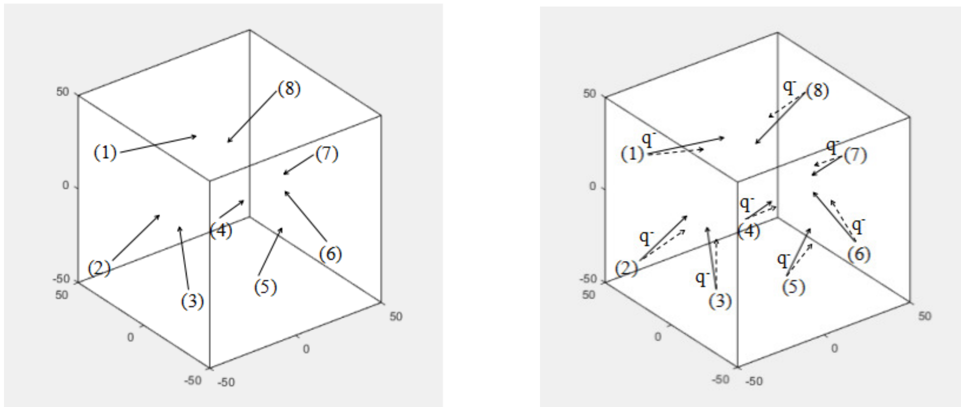

Fig 4. Flight trajectories of eight aircraft before and after conflict resolution (C2-8)