

# An improved tree-based statistical method for genome-wide association study

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## ABSTRACT

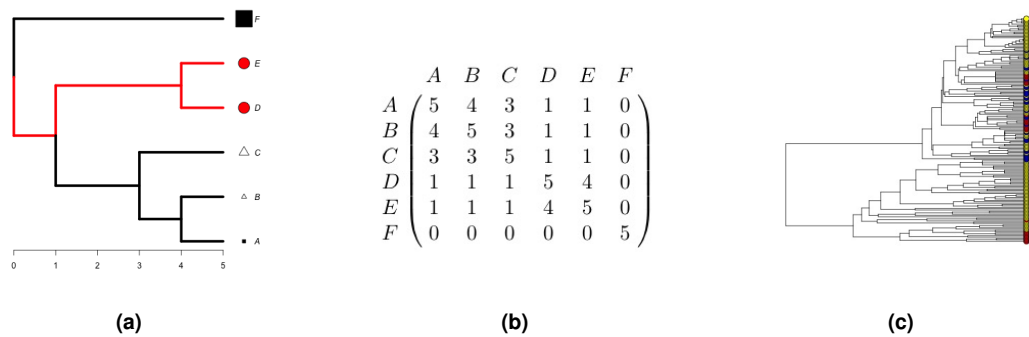
In genetic studies, quantitative traits are found possibly associated with genetic data. Due to advanced sequencing technology, many methods have been proposed in genome wide association study (GWAS) to search the single nucleotide polymorphism (SNP) associated with the traits. Currently several methods that account for the evolutionary relatedness among individuals were developed. When comparing with conventional methods without evolutionary relatedness among individuals, tree based methods are found to have better performance when the population structure increases. In this work, we extend a tree based method in previous studies by varying the magnitude of relatedness. The magnitude of relatedness of the evolutionary history is controlled by an Ornstein-Uhlenbeck (OU) process through its parameters. Our method combines a pertinent process and phylogenetic comparative method where the incorporated evolutionary history is built by SNP data. We perform simulation as well as analyze drosophila longevity data set.

## INTRODUCTION

In genetics, searching single nucleotide polymorphism (SNPs) associated with traits helps people to identify and localize the possible origin of disease. In the past, scientists made effort to develop methods that aim to utilize SNPs for seeking connection with relevant trait. As the main goal is to find possible association, SNP from sequencing technology as well as traits measured from various ways are collected from controlled group and case group. However, currently it is still quite challenge to find statistical significance in association study. One difficulty is to successfully link the SNPs with the traits. As those studies were required to meet sufficient rigorous statistical tests during the process. From genetic basis, SNP makers are scanned into analysis using a couple of thousand individuals each with certain long sequence length (around 0.5 million in general).

The statistical methods developed for association studies in literature can be divided into two main categories: the one assumed the evolutionary independence of individuals without relatedness and the other incorporates the evolutionary relatedness of individuals into analysis. For the case of independence assumption among individuals, the observed trait of  $n$  individuals with values  $y_1, y_2, \dots, y_n$  are assumed as independent identical distributed random variable from identical statistical distribution. To detect association between trait and SNP datasets, under the evolutionary *independence* assumption, typically a paired  $t$ -test is conducted for investigating the significance of the SNPs associated with the trait of interest on the controlled group  $y_{i_1}, y_{i_2}, \dots, y_{i_{n_1}}$ , and with the trait on the case group  $y_{j_1}, y_{j_2}, \dots, y_{j_{n_2}}$  where  $n_1 + n_2 = n$  (McClurg et al., 2006). The paired  $t$ -test method serves a fast and efficient way in association study (Thompson and Fardo, 2016).

In the other category of study for linking SNP and traits, people incorporated evolutionary relatedness represented by a tree for association analysis (Pan et al., 2009; Zhang et al., 2012). A previous work (Thompson and Kubatko, 2013) demonstrated that the tree based method for linking the association between traits and gene can be improved when the covariance structure  $\mathbf{V}$  among randomly-sampled individuals is estimated from the evolutionary history within each SNP. To initiate the analysis, those methods make use of SNP data to build a phylogenetic tree  $\mathbb{T}$  which is a rooted, bifurcated (or multifurcated)



**Figure 1.** (a) A demonstrated example of evolutionary tree. The horizontal axis represents a pseudo-evolutionary time from past to current. The evolutionary time started with  $t = 0$  and stopped at current at  $t = 5$ . (b) The matrix representation  $V$  for the evolutionary tree. (c) A phylogenetic tree built from SNP of 164 diploid observations. The corresponding traits are represented by colored circles. Three colors (red, yellow, and blue) represent the magnitude of the trait value with hypothetical low values of yellow, high value in red and intermediate values colored blue. Tree and traits were obtained from (Schmitz, 2017).

47 directed and ultrametric (each individual has the same height from the root to tip in the tree) graph. To  
 48 given an illustration, we use a simple tree containing a few individuals. The evolutionary relatedness of  
 49 five individuals A, B, C, D, E, F is shown by a tree in Fig. 1a. It is expected that the level of relatedness  
 50 among individuals contain some useful information linking to the trait. For example, as two individuals D  
 51 and E shared more evolutionary history, it is reasonable to think that their trait are more similar (shown  
 52 in red circles ●). While individuals A and F are more evolutionary unrelated (independent), hence their  
 53 characteristics might present more diversity (e.g. the two black squares ■ and ■ in different sizes). The  
 54 matrix  $V$  shown in Fig. 1b is the covariance matrix (an isomorphic transformation) of the tree in Fig. 1a.  
 55 An element  $v_{ij}$  in the matrix  $V$  represents relatedness between a pair of individuals  $i$  and  $j$ . Note that  $v_{ij}$   
 56 is obtained by measuring the shared evolutionary history from root (scaled at 0) of the tree to their most  
 57 recent common ancestor.

58 The tree in Fig. 1c is a larger tree constructed using a SNP data of 164 individuals. It is reasonable to  
 59 view that trait among individuals are more similar when sharing higher relatedness.

In this paper, we intend to expand model in Thompson and Kubatko (2013). We start by briefly  
 introducing the tree based method as following. Considered a cluster of tree where the trait of  $n$  individuals  
 are separated into  $k$  clusters. We can use an  $n$  by  $k$  matrix  $D = [d_{ij}]$  to represent the cluster where  $D$  is  
 defined by

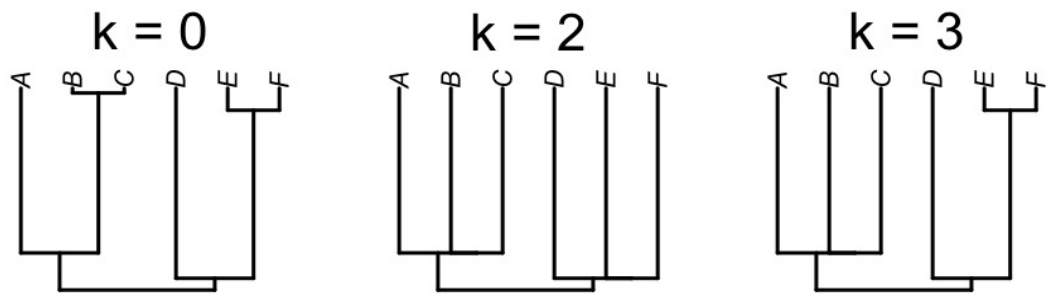
$$d_{ij} = \begin{cases} 1, & \text{if observation } i \text{ falls in cluster } j; \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

60 where  $i = 1, 2, \dots, n$  and  $j = 1, 2, \dots, k$ .

61 The matrix  $D$  will be useful for the next step analysis of studying trait evolution. Let  $Y = (y_1, y_2, \dots, y_n)^t$   
 62 be the trait observed from  $n$  individuals,  $Y$  can be treated as a random variable with expected value  
 63  $E[Y] = D\mu \in \mathbb{R}^n$  where the vector  $\mu = (\mu_1, \mu_2, \dots, \mu_k)^t$  is identified as the mean for the  $k$  distinct groups.

64 We can get cluster trees from setting different number of clusters. The clustered tree can then be  
 65 transformed into the variance covariance matrix  $V$ . We illustrate this by reproducing Fig. 2 in (Thompson  
 66 and Kubatko, 2013). In Fig. 2, three clustered trees for 6 individuals are shown with different cluster  
 67 number  $k = 0, 2, 3$ . The corresponding matrices  $V$ s for the tree of  $k = 0$  and  $k = 2$  are shown in Table 1.

68 Here we use the clustered tree to consider the broad-scale phylogenetic relationships among SNPs, this  
 69 can account for the evolutionary history among genes with using all coalescent relationships where the  
 70 structure of  $V$  is equivalently to the tree topology, and each element in  $V$  is an estimate of the covariance  
 71 structure in the data that is required for estimation of branch lengths along the topology.



**Figure 2.** A demonstration of a six-taxon tree with branch lengths. The overall tree ( $k = 0$ ) is shown in the left panel. The corresponding clustered trees for 2 clusters ( $k = 2$ ) and 3 clusters ( $k = 3$ ) are in middle panel and right panel, respectively.

**Table 1.** The variance covariance matrix  $V$  for the tree in Fig. 2. The left matrix is for the left tree  $k = 0$ , the right matrix is for the middle tree  $k = 2$ . The numbers in bold in the matrix shows the difference between the two clustering results.

	A	B	C	D	E	F		A	B	C	D	E	F
A	100	18	18	0	0	0	A	100	18	18	0	0	0
B	18	100	<b>98</b>	0	0	0	B	18	100	<b>18</b>	0	0	0
C	18	<b>98</b>	100	0	0	0	C	18	<b>18</b>	100	0	0	0
D	0	0	0	100	6	6	D	0	0	0	100	6	6
E	0	0	0	6	100	<b>89</b>	E	0	0	0	6	100	<b>6</b>
F	0	0	0	6	<b>89</b>	100	F	0	0	0	6	<b>6</b>	100

## 72 Model for Haploid Data

73 Haploid of a cell has a single set of unpaired chromosome. For SNPs data of haploid type, the tree can be  
 74 constructed from sequencing reads as well as from assembled genomes or contigs. Thompson and Kubatko  
 75 (2013) used a transformation by considering clustering using tree structure that clusters the individual  
 76 into several subgroups depending on the number of  $k$ . In contrast with work in Besenbacher et al. (2008),  
 77 Thompson and Kubatko (2013) instead assumed that an observation is taken to be a chromosome level  
 78 which offers an alternative to aggregate information. Next, assume a Brownian motion for trait evolution  
 79 on the tree (Felsenstein, 1985), the statistical model given a trait  $Y$  and a tree  $T$  follows a multivariate  
 80 normal distribution with mean vector  $D\mu$  and variance-covariance matrix  $\sigma^2V$

$$Y \sim \mathcal{N}(D\mu, \sigma^2V) \quad (2)$$

81 where the parameter  $\sigma$  measures the rate of evolution during the process.

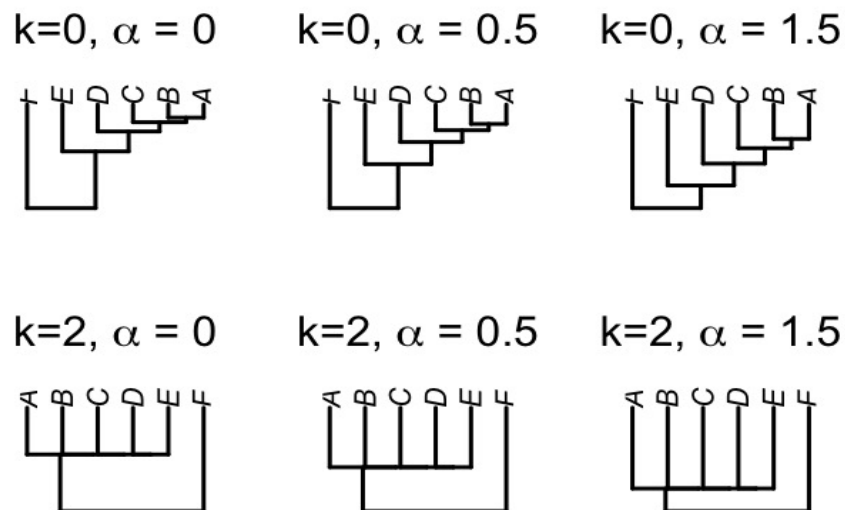
The statistical model in Eq. (2) has analytical formula for the maximum likelihood estimators for the mean  $\hat{\mu} = (D^T V^{-1} D^T)^{-1} D^T V^{-1} Y$  and variance  $\hat{\sigma}^2 = (Y - D\hat{\mu})^T V^{-1} (Y - D\hat{\mu})/n$ , respectively. Therefore, the maximum likelihood can be computed directly once the trait and tree are ready. Thompson and Kubatko (2013) used likelihood score statistics (LSS) score to determine the tree score. LSS is defined as the maximum score over the number of clusters.

$$\text{LSS} = \max_k \{2\ell(\hat{\mu}, \hat{\sigma}^2 | Y, V_k) - k \log n\} \quad (3)$$

82 where  $\ell(\cdot)$  is the log likelihood in Eq. (2). Therefore the hypothesis test for detecting significance  
 83 of association between SNP and trait of a group of individuals can be carried out using a likelihood  
 84 framework.

## 85 Inference

86 In order to identify the detection between SNP and trait, Thompson and Kubatko (2013) use the LSS score  
 87 in Eq. (3) for model in Eq. (2). To access the significance, a permutation test was performed and the LSS  
 88 score for the tree is calculated according to each permuted trait data set at each locus along a chromosome.  
 89 The statistical null hypothesis is setting with no linking between the snp and trait. i.e.  $H_0$ : no association.



**Figure 3.** The magnitude of hierarchical clustering is controlled under the OU process using parameter  $\alpha$ .

90 Then a  $p$ -value is determined by the ranking of score of the observed data set in an ordered score of the  
 91 permuted data set. i.e.  $p\text{-value} = \#\{\text{LSS}_{per} > \text{LSS}_{obs}\} / N$  where  $\text{LSS}_{per}$  is the score for permuted data  
 92 set,  $\text{LSS}_{obs}$  is the score for the observed data set and  $N$  is the number of permutations.

93 In fact, model built in Eq. (2) is under the assumption of Brownian motion for evolution Felsenstein  
 94 (1985) since the variance covariance matrix is specified by utilizing the tree. Observing that currently  
 95 there is still a need of methods that include more biologically-realistic situations, we propose a method  
 96 that extend works in (Thompson and Kubatko, 2013) by introducing Ornstein-Uhlenbeck (OU) process  
 97 Hansen and Martins (1996) for studying the association between traits and SNP data. Our aim is hope to  
 98 provide a robust method in GWAS study.

## 99 METHODS

### 100 OU process for trait evolution

If the trait of the  $i$ th individual is assumed to evolve under an Ornstein-Uhlenbeck process (Hansen and  
 Martins, 1996), then the trait value of the individual at time  $t$ , denoted as a stochastic variable  $y_{i,t}$ , is a  
 solution the following stochastic differential equation

$$dy_{i,t} = \alpha(\mu - y_{i,t})dt + \sigma dB_t, t > 0. \quad (4)$$

101 Eq. (4) expresses the dynamic of  $y_{i,t}$  with respect to time. On the left hand side of Eq. (4), the term  
 102  $dy_{i,t}$  is the change in the character  $y_{i,t}$  over the infinitesimal interval from time  $t$  to  $t + dt$ . The right hand  
 103 side of Eq. (4) contains the sum of two terms: the deterministic term  $\alpha(\mu - y_t)dt$  and the stochastic  
 104 terms  $\sigma dB_t$  where  $B_t$  is a Brownian motion, the real value parameter  $\mu$  represents the optimal value (an  
 105 evolutionary niche) of  $y_{i,t}$ , the positive value parameter  $\sigma$  is the overall rate of evolution, and the positive  
 106 value parameter  $\alpha$  represents the magnitude of force that pulls  $y_{i,t}$  back to the optimum  $\mu$ . When  $y_{i,t}$  is  
 107 far from the optimal  $\mu$ , the force would have stronger effect (larger value of  $\alpha$ ) to pull  $y_{i,t}$  back to the  
 108 optimum  $\mu$  while weaker force (smaller value of  $\alpha$ ) is presented whenever  $y_{i,t}$  is close to the neighborhood  
 109 of  $\mu$ .

110 To implement OU model in tree based genome wide association study, we use  $\alpha$  to control the level  
 111 of clusters. In Fig. 3, larger values of  $\alpha = 0.5$ , or  $\alpha = 1$  provided more independent relatedness among  
 112 clusters than the smaller value of  $\alpha = 0$  given different number of cluster  $k = 0$  (plots in upper panel for  
 113 the raw tree case) or  $k = 2$  (plots in lower panel for 2 cluster case). Implementing OU process could be a  
 114 potential benefit for detecting the association between snp and trait.

115 Currently we focus on studying the model in (Hansen and Martins, 1996) with single force, single  
 116 optimum and single rate as mentioned in Eq. (4) though other more sophisticated models are possible to  
 117 develop for this purpose (see (OMeara et al., 2006; Butler and King, 2004; Beaulieu et al., 2012)).

### 118 OU Model for Haploid Data

The observed trait for the  $i$ th individual  $y_{i,t}$  under the OU process has normal distributions with the mean

$$E(y_{i,t}|y_0) = y_0 \exp(-\alpha t) + \mu(1 - \exp(-\alpha t)) \quad (5)$$

and variance

$$var[y_{i,t}|y_0] = \frac{\sigma^2}{2\alpha}(1 - \exp(-2\alpha t)) \quad (6)$$

119 where  $y_0 = y_{i,0}$  is the trait value at  $t = 0$ .

The method used in Thompson and Kubatko (2013) set the parameter  $\alpha = 0$  which reduces it to the Brownian Motion with means  $E(y_{i,t}|y_0) = y_0$  and  $var(y_{i,t}|y_0) = \sigma^2 t$ . For OU model with  $n$  individuals, the observed trait  $Y = (y_1, y_2, \dots, y_n)^T$  is treated as a random vector that following a multivariate normal distribution with mean vector  $\mu = (\mu_1, \mu_2, \dots, \mu_n)^T_{n \times 1}$  and variance-covariance matrix  $V_{n \times n}$  where  $V[i, j] = cov[y_{i,t}, y_{j,t}]$  is the covariance between species  $i$  and species  $j$  of the form

$$cov[y_{i,t}, y_{j,t}] = \sigma^2 V_{\alpha ij} = \frac{\sigma^2}{2\alpha} e^{-\alpha d_{ij}} e^{-2\alpha t_{ij}} \quad (7)$$

120 where  $t_{ij}$  is the branch length shared by the  $i$ th and the  $j$ th individual and  $d_{ij}$  is the distance between the  
 121  $i$ th and the  $j$ th individual on the tree.

Under the OU process, the trait vector observed at the tip denoted as  $Y = (y_1, y_2, \dots, y_n)^T$  would follow a joint multivariate normal distribution

$$Y \sim \mathbf{MVN}(D\mu, \sigma^2 V_\alpha) \quad (8)$$

The mean vector and variance can be expressed as a function of  $\alpha$

$$\hat{\mu}(\alpha|\mathbb{T}, Y, D) = (D^T V_\alpha^{-1} D^T)^{-1} D^T V_\alpha^{-1} Y, \quad (9)$$

$$\hat{\sigma}^2(\alpha|\mathbb{T}, Y, D, \hat{\mu}) = \frac{(Y - D\hat{\mu})^T V_\alpha^{-1} (Y - D\hat{\mu})}{n}. \quad (10)$$

By Eq. (9) and Eq. (10), the negative log likelihood function for OU model can be written as a function of  $\alpha$  :

$$\ell(\alpha|Y, \mathbb{T}, \hat{\mu}, \hat{\sigma}^2) = \frac{n}{2} \log(2\pi) + \frac{n}{2} \log \hat{\sigma}^2 + \frac{1}{2} \log |V_\alpha| + \frac{1}{2\hat{\sigma}^2} (Y - D\hat{\mu})^T V_\alpha^{-1} (Y - D\hat{\mu}). \quad (11)$$

### 122 Inference

From the model in Eq. (8), the hypothesis testing for significance between SNPs and trait can be proceeded through a likelihood framework. To choose the best cluster, we modify the penalized likelihood approach in Thompson and Kubatko (2013) where the likelihood score statistics is calculated as

$$LSS = \max_{0 \leq k \leq m} \{2 \log(\hat{\alpha}, \hat{\mu}, \hat{\sigma}^2 | Y, V) - k \log n\} \quad (12)$$

123 where  $m$  is the maximum number of clusters that used for analysis.

124 To access the statistical significance, we further consider to use an upper bound defined by the  
 125 maximum of the observed LSS value plus the standard error of the permuted maximum LSS valued  
 126 multiplied by the  $(1 - \alpha)$  quantile of  $t$  distribution with degree of freedom of  $n - 1$  where  $n$  is the number  
 127 of individuals. i.e.

$$b = \max_{0 \leq k \leq m} LSS_{\text{obs}} + qt_{\alpha/2, df=n-1} \frac{sd_{LSS_{\text{per}}}}{\sqrt{n}} \quad (13)$$

128 where  $sd_{LSS_{\text{per}}} = sd(\{\max_{0 \leq k \leq m} LSS_{\text{per}}\}_{i=1}^m)$ . And the p-value is calculated by the number of permuted  
 129 LSS score greater than this bound  $b$ . This provide a more conservative alternative in detecting the  
 130 significance.

## 131 SIMULATION

### 132 Haploid Data

133 In order to assess the performance of the proposed techniques, we simulating the data sets under specific  
 134 parameter values, the local phylogenetic tree at each SNP is estimated using SVDquatets (Chifman and  
 135 Kubatko, 2014). The SVDquatets is currently implemented in PAUP (Swofford, 2011) and computes a  
 136 score based on singular value decomposition of a matrix of site pattern frequencies corresponding to a  
 137 split on a phylogenetic tree. These quartet scores can be used to select the best supported topology for  
 138 quartets of taxa, which in turns can be used to infer the species phylogeny using quartet methods where  
 139 branch lengths are estimated.

140 Given an estimated  $\mathbb{T}$ , the next step to complete the association study is to conduct the phylogenetic  
 141 comparative analysis to computing the LSS score. Tree from PAUP analysis is a non-ultrametric tree.  
 142 Since the expected quantity of trait change (the variance  $v_{ii}$  in the variance-covariance matrix  $V$ ) in  
 143 comparative analysis under Brownian motion is given by the product of the rate of evolution  $\sigma$  of the  
 144 trait with branch length and under OU process the  $v_{ii}(\alpha)$  is given by the deterministic change inherit  
 145 from ancestor plus the Brownian motion for random change. So using a non-ultrametric tree is a way to  
 146 assume different rates of evolution for each branch which leads to a more sophisticated and complex case.  
 147 To alleviate this, we convert the non-ultrametric tree to an ultrametric tree using the mean path lengths  
 148 (MPL)(Britton et al., 2002) method where the age of a node is estimated with the mean of the distances  
 149 from this node to all tips descending from it. Hence we can assume a clock-like trait evolution which  
 150 means the quantity of change from the root to the tips is the same.

151 To calculate score in Eq. (12), we current use the number of cluster from  $k = 3$  to  $k = 5$ . Algorithm 1  
 152 provides a step-by-step procedure for calculating the  $p$ -value.

153 For each size, we use `ms`(Hudson, 2004) to simulate sequence of length 1000. We use `paup` to analyze  
 154 the sequence and get the tree by `SVDquartets`. For each haploid size, we simulate 100 replicates of  
 155 sequence to get 5 trees respectively. To simulate trait, given a tree with known topology and branch length,  
 156 we consider to use two stages OU model with parameters  $\Theta = (\alpha_1, \alpha_2, \sigma_1, \sigma_2, \theta_0, \theta_1, \theta_2)$  where for BM  
 157 data simulating using  $\alpha_1 = \alpha_2 = 1e - 6$ ;  $\theta_0 = 90, \theta_1 = 80, \theta_2 = 100$ , we set three different rate evolution  
 158  $\sigma_1 = \sigma_2 = 1, 5$  and 10, respectively.

159 We use 100 replicates where for each replicates we simulate traits using the true parameters  $\Theta$ . We  
 160 then consider to estimate the parameters using the 100 replicates. Since there are various clusters, we  
 161 use the parameter estimate from the best selected cluster  $k^*$ . For each replicate, we consider to assess the  
 162 significance of the trait associated with the simulated snp. We use the permutation method in algorithm  
 163 Thompson and Kubatko (2013) to permute the trait for 500 times. We present our algorithm in Algorithm  
 164 1.

## 165 RESULT

166 We present out simulation results for BM model and OU model in the following subsection.

### 167 Haploid: OU vs BM

168 We first simulate snp sequence using `ms`(Hudson, 2002). The `ms` setting

```
169 ms 10 1 -T -s 1000
```

170 would generate 10 individuals each is with sequence length 1000. We then use this data and `paup`(Swofford,  
 171 2011) to obtain the estimated tree. We simulate traits under BM model using  $\sigma^2 = 1$  and treated it as the  
 172 true data set. To evaluate the  $p$ -value for this data set, we use 500 replicates, and a  $p$ -value is computed by  
 173 the ratio of count of the maximum LSS statistics greater than the max LSS of the true trait over 500. For  
 174 each replicates, we compare the LSS statistics of  $k = 3, 4, 5$  cluster to get the maximum LSS statistics. We  
 175 repeat above procedure 50 times for each tree estimated using of sequence length 1000. The following  
 176 table is the overall average of parameter estimates using 10 trees. We consider the taxa size of 10, 30, 50  
 177 Table 2 shows the median estimate and the 95% confidence interval of the  $p$ -value under BM and OU  
 178 model

179 Currently we found for BM model, the overall  $p$ -value bandwidth is narrower when compare to OU  
 180 model. This might indicates that OU model is more conservative to detect the significance than the BM

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**Algorithm 1** Model Inference
 

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- 1: simulate snp sequence data set from  $ms$  and treat  $Y$  as the true data set.
  - 2: use PAUP and SVDquartet to analyze the data sets and return an estimate tree  $\mathbb{T}$  with topology and branch length information.
  - 3: **for**  $j = 1 : k$  **do**
  - 4: cluster the tree  $\mathbb{T}$  and get  $\mathbb{T}_j$  of  $j$  cluster and store matrix  $D$  and variance covariance matrix  $V_j$ .
  - 5:   **if** model is **BM**
  - 6:     simulate trait data  $Y$  under multivariate normal distribution.
  - 7:     compute  $\log(\hat{\mu}, \hat{\sigma}^2 | Y, \mathbb{T}, V)$ ;
  - 8:   **if** model is **OU**
  - 9:     simulate trait data  $Y$  under multivariate normal distribution.
  - 10:     transform  $V_j$  into  $V_{\alpha,j}$
  - 11:     optimize the log function
  - 12: compute the LSS statistics using formula
  - 13: choose the largest value of  $LSS$  and return the best cluster index  $j^*$ .
  - 14: **endfor**
  - 15: **for**  $i = 1 : b$  **do**
  - 16:   obtain sample  $Y_i$  by permuting  $Y$ .
  - 17:   repeat step 2 to step 10 to obtain  $LSS_i$ .
  - 18: **endfor**
  - 19: compare  $LSS_i$  with  $LSS$  and report  $p$  value.
- 

**Table 2.** quantile for the  $\sigma^2$  from simulation, the true value is 1.

Taxa	10	30	50
BM	0.46(0.13,1.16)	0.64(0.35,1.06)	0.63(0.32,1.02)
OU	1.03(0.98,1.17)	1.01(1.00,1.09)	1.01(1.00,1.05)

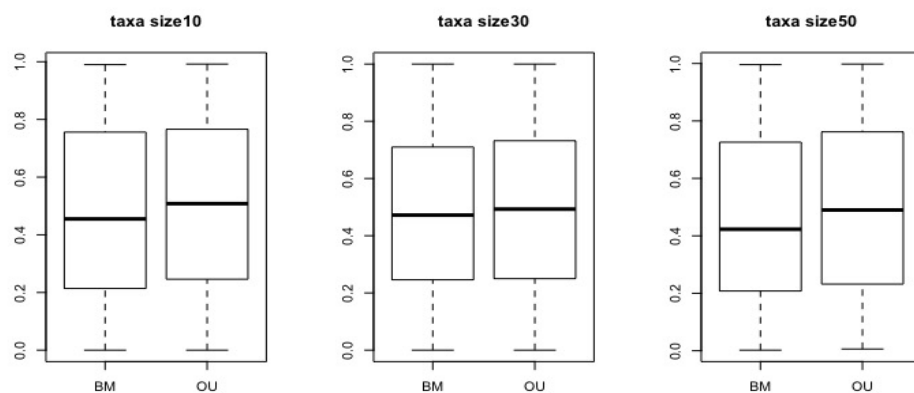


**Table 3.** quantile for the  $\alpha$  from simulation, the true value is 0.25.

Taxa	10	30	50
OU $\alpha$	1.07 (0.01,4.65)	0.53(0.01,1.46)	0.57(0.01,1.80)

181 model. As BM model is a submodel of OU, it is likely that this phenomenon come from data is simulated  
 182 from OU model with a special case of  $\alpha = 0$ .

183 Table 3 shows the median estimate and the 95% confidence interval of the p-value under the OU  
 184 model for parameter  $\alpha$



**Figure 4.** Assess significance through simulation study under OU model. compare to BM, OU has higher p-value which indicates that OU model is more conservative than BM model.

185 Figure 4 compare the result of significance under different number of taxa for OU model. Trait data is  
 186 simulated under OU model, OU has a bit higher p-value than the BM model.

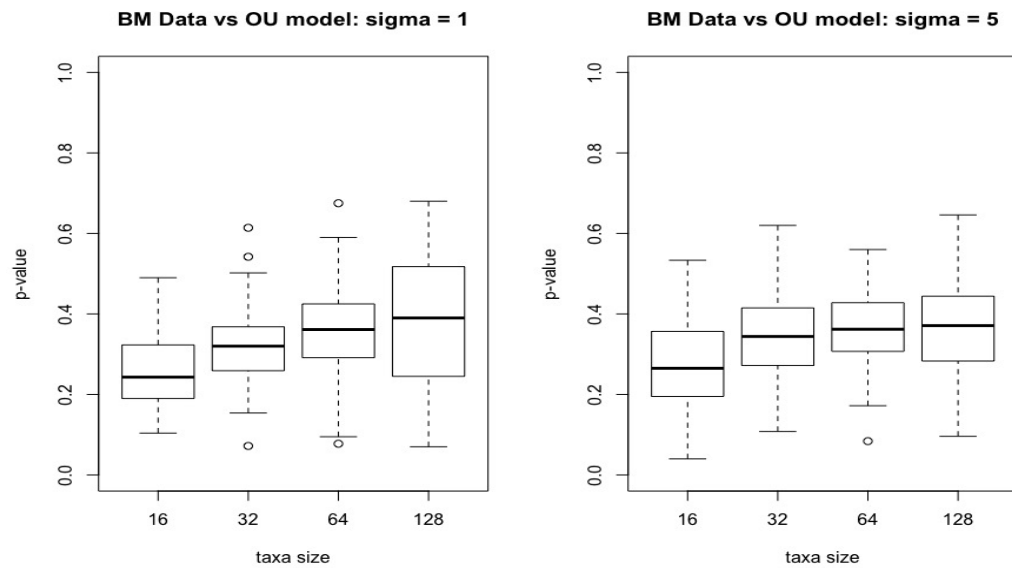
### 187 Power Analysis

188 We access the power of the OU model. Currently we use 100 trials where for each trial a  $p$ -value is  
 189 obtained using algorithm 1. The power is computed by counting the frequencies of  $p$ -value smaller than a  
 190 given significant level (here we set the level to 0.1).

#### 191 0.0.1 Haploid data from BM model

192 For the power of OU model, We look at the  $p$ -value of OU model when data are simulated from BM  
 193 model. We show the boxplot in Figure 5



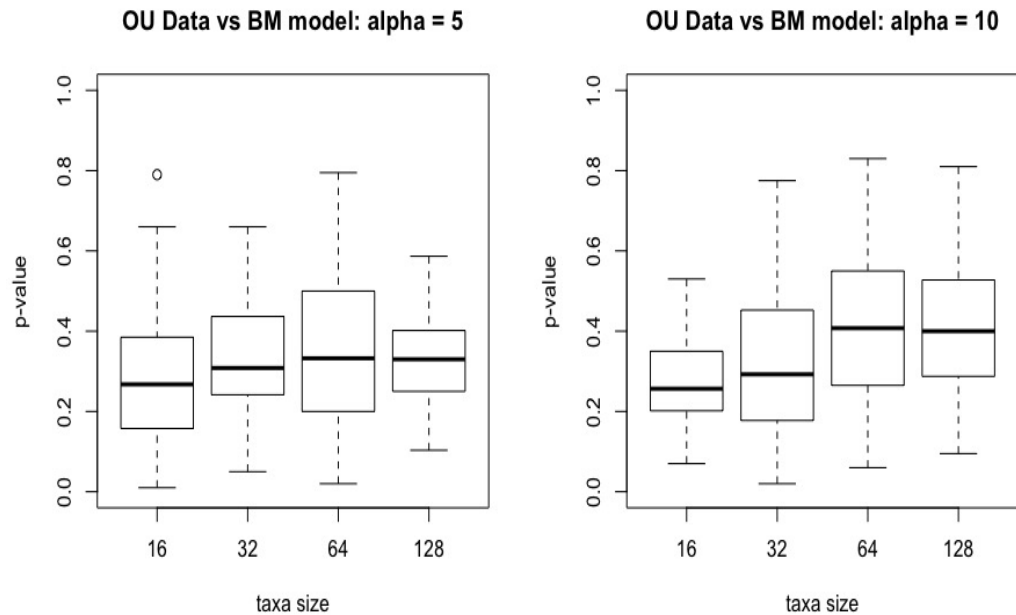


**Figure 5.** Data are simulated from BM model and analyzed under the OU model. The p-value increases with sample size and hence decrease the power. Overall the p-values does not change in different  $\sigma$  ( $\sigma = 1, 5$ ) The results is summarized using 5 trees.

194 The p-value increases with sample size and hence decrease the power. One possible rationale behind  
 195 this plot might due to equation (13), when sample size  $n$  increases, the bound for determining the p-value  
 196 shrinks which increases the number of permuted maximized LSS score that exceed this bound, hence  
 197 increasing the p-value.

#### 198 *Haploid data from OU model*

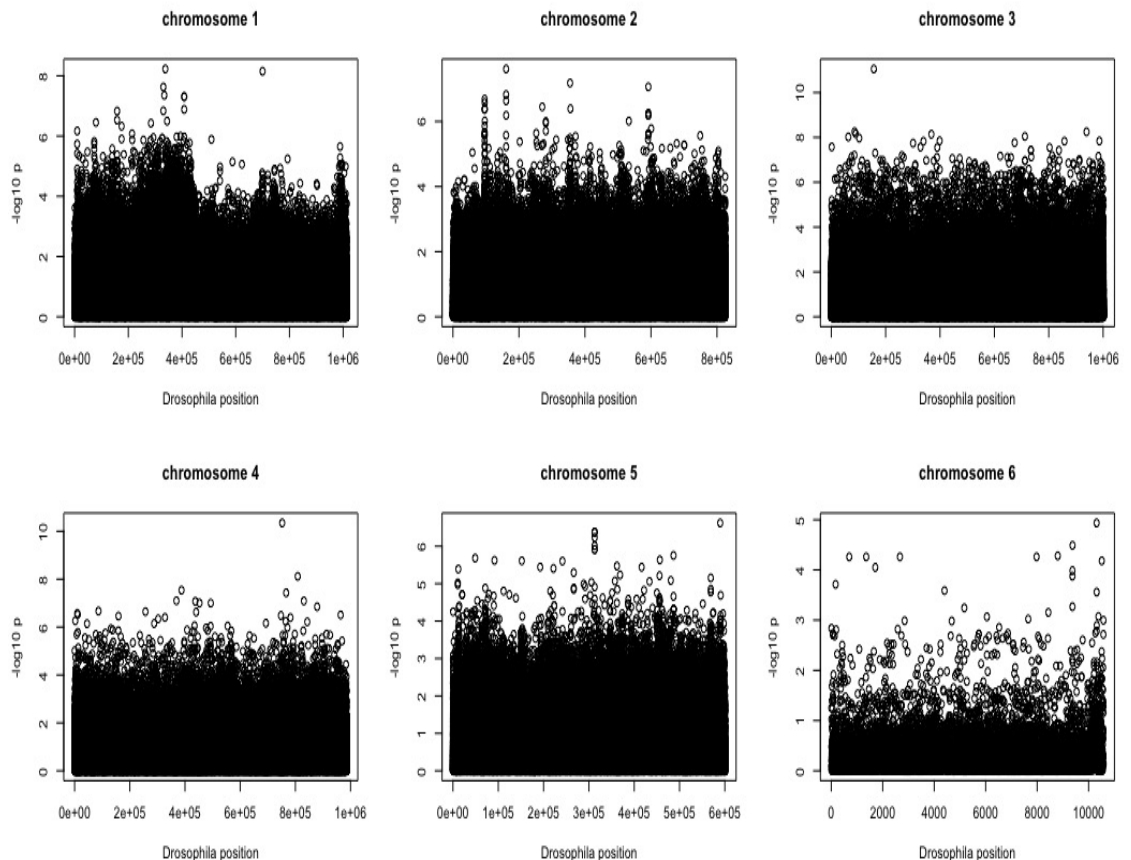
199 We also look at the power of BM model when data are simulated from OU model We show the box-plot  
 200 in Figure 6



**Figure 6.** Data are simulated from OU model and analyzed under the BM model. The p-value increases with sample size and hence decrease the power. Overall the p-values does not change in different  $\alpha$  ( $\alpha = 5, 10$ ) The results is summarized using 1 tree. Result using summarizing 5 trees has similar pattern with 5 tree but slightly lower in the 128 taxa case.

## 201 *Drosophila melanogaster*

202 Fruit flies (*Drosophila melanogaster*) have haploid cell. In literature, there are studies about the longevity of  
 203 fruit flies. Durham et al. (2014) identified that the senescence (a decline in physiological function in age)  
 204 trait is related to the longevity of fly. They provided evidences that individuals alleles influence fecundity  
 205 in an age specific manner and so the genetic basis of natural variation in fecundity changes dramatically  
 206 with age. They complete a genome-wide association to identify single-nucleotide polymorphism (SNPs)  
 207 affecting lifespan and age-specific fecundity using the *Drosophila melanogaster* genetic Reference panel.  
 208 They identified 1,031 SNPs affecting fecundity and 52 influencing lifespan. Only one SNP is associated  
 209 with both early and late-age fecundity. The age-specific effect of candidate genes on fecundity is validated  
 210 using RNA interference. Their result provides support for the mutation accumulation theory of aging.



**Figure 7.** Genotype-phenotype association for 6 chromosomes in 205 drosophila and calculated using single-SNP linear regression, while controlling for genetic structure(tree was built under SNP dataset).  $-\log_{10}(p\text{-value}) > 4$  or  $p\text{-value} < 0.0001$  are regarded as SNPs significant.

## 211 DISCUSSION AND CONCLUSION

212 In this work, we extend the tree-based methods described in Thompson and Kubatko (2013) for genome-  
 213 wide association study(GWAS) for the haploid case. Our method considers incorporating phylogenetic  
 214 tree built under the SNP dataset and then use the tree as a dependent evidence among individuals. We then  
 215 use clustering technique in order to identify any possible associations between a trait and SNP maker. To  
 216 cluster tree, we consider to alter the strengths of affinity among individuals but not change the topology.  
 217 To do this, we apply a Gaussian process called Ornstein-Uhlenbeck process to stretch/lengthen/shrink the  
 218 branch lengths in the tree.

219 We evaluate the performance of our model as well as compare the existing tree-based model via  
 220 through accessing their statistical power. Currently, we found that the overall statistical performance for  
 221 our model is with lower powers when true data are simulated from the alternative models (data simulated  
 222 from BM model). This might due to the tree is estimated from the SNP data. However, the major issue  
 223 that contributes to this lower power of OU model could be the clustering procedure which changes the  
 224 structure of the affinity among the individuals. Hence true data loses some information inherited from the  
 225 model. In particular, this might due to the clustering  $k$  index and the matrix  $D$  transform the mean and  
 226 variance among individual  $V$  which might cause the different result of estimation from the true value. For  
 227 OU model, we find that  $\alpha$  and  $\sigma^2$  cannot be estimated well simultaneously.

228 It is possible to report the false discovery rate for both BM and OU model, in that case we can compare  
 229 both models. We also can compare the model by determining the sample size at a threshold power level.  
 230 Smaller size would report a higher power of the model. Finally, we hope that our model can benefit the  
 231 research community in GWAS research area. While our model is planned to analyze the haploid dataset,  
 232 we also wish to extend it to apply to association study in primate or human.

233 R script as well as other analysis result can be accessed at <https://github.com/djhwueng/>  
234 OUSnp.

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