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Mathematical modelling and analysis of elite athletes' sprint data to study the rate and regulation of ATP during a maximal exercise of short duration

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Title: Mathematical modelling and analysis of elite athletes' sprint data to study the rate and regulation of ATP during a maximal exercise of short duration

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Abstract: According to the energy supply/energy depletion model, it is not clear how the depletion of substrates (adenosine triphosphate) affects sprint performance. Therefore, this research was conducted to find out how the human organism regulates the amount and the rate of adenosine triphosphate to observe how these factors affect performance specifically during a maximal exercise of short duration.It was found there was a causal relationship between percentage of PCr and speed which might affect sprint performance.The percentage of chemical energy derived from the anaerobic energy system was found to be 95% for 100-m sprint running. The rate constant for the PCr anaerobic metabolic energy process ($0.31s^{-1}$) was found to be greater than that of the oxygen-independent glycolysis metabolic process ($0.11s^{-1}$) and these rate constants affect sprint performance.

Keywords: Anaerobic Subsystems, ATP, Mathematical Modelling, Optimisation, PCr, Sprinters

1 Introduction

The energy which is produced during anaerobic metabolism has been of growing interest to exercise physiologists to understand how this specific energy pathway affects the high-intensity exercise of short duration (Shulman & Rothman, 2001; de Koning JJ et al., 2011; Hill et al., 1923; Lambert et al., 2004). Despite the physiological laboratory works during short-duration maximal exercise (Bogdanis, 1996; Gaitanos et al. 1993), most proposed theoretical models (Edwards, 1983; Hill et al., 1923; Hill, 1927; Noakes, 2000; Ulmer, 1996), however, cannot explain how the human body controls the rate of adenosine triphosphate (ATP) production to prevent a severe fall in ATP concentration in the active muscles.

Therefore, in this study, mathematical modelling and analysis was used in an attempt to validate certain physiological theories of control fatigue. In fact, a large fall in ATP levels occurs during times of maximal exercise of short duration such as sprinting (Fox et al., 1993; Matthews et al., 1971). To date, various research studies have incorporated single equation models to analyse the anaerobic metabolism (Di Prampero et al., 1993; Laurent and Locatelli, 2002; Lloyd, 1967; Peronnet and Thibault, 1989; Ward-Smith, 1985; Ward-Smith and Mobey, 1995). However, one study (Ward-Smith and Radford, 2000), has tentatively developed a mathematical model to represent the biochemical processes during the anaerobic metabolism based on several assumptions. They considered the total finishing times or duration of the sprints using a fourth-order Runge-Kutta mathematical method whereby temporal information of what is happening at discrete time intervals was lost and, by taking the height of the centre of mass of all sprinters who participated in the sprinting event to be equal, which was not the case according to their different weights and heights (Ferro et al., 2001; IAAF, 2008). Based on these assumptions, they found that the overall maximum anaerobic power of the sprinters for the 100m event at the 1987 World Championships was 51.6 Wkg-1. The oxygen independent glycolysis, being the highest contributor of energy, was 11.7% greater than the energy derived from phosphocreatine (PCr) utilisation anaerobic energy subsystem. In order to extend the previous models, the aims of this experimental study were, therefore, firstly to develop mathematical models to determine indirectly the rate of ATP production and utilisation through the anaerobic subsystems that are endogenous ATP (i.e. ATP initially stored in the exercising muscles), PCr utilization and oxygen-independent glycolysis. Secondlythis research aimed to assess how the anaerobic subsystems can be exploited further to improve high-intensity short duration sporting activities such as sprint performance.

2 Method

2.1 Data collection and preliminary calculations

In this experimental case study, the International Association of Athletics Federations (IAAF) 10-m split times, for the Men's 100-m Final at the 1999 world championships, in Sevilla Spain, were used to model mathematically high-intensity exercise of short duration (Table 1) to investigate the elite athletes' sprint performance. In addition, the professional level of these sprinters would represent a good baseline for comparison purposes of the anaerobic subsystems and aerobic system at a track and field event. This mathematical model was then used to investigate the availability of metabolic resources, as well as the rate of energy production among the elite athletes. The mean (±standard deviation) height, mass and body mass index (BMI) of the athletes were 1.78 (±0.03) m, 75.8 (±6.6) kg and 23.8 (±1.5) kgm^{-2} respectively (IAAF, 2008). Each elite sprinter's height, weight and reaction times was used to mathematically model the energy systems. The wind speed was +0.2ms^{-1}, the air temperature was 27^{o}C (300.15 K), air density was 1.179 kgm^{-3} and the mean reaction time of the sprinters was 0.141 (±0.01) s.

Table 1: 10-m split data intervals for the 100m sprint race in Sevilla Spain 1999. (Rivera, A.; Pagola, I.; Ferreruela, M.; Martín, A.; Rocandio, "Biomechanical Analysis of the World Championships in Athletics Sevilla'99: 100, 200, 400m sprint events". New Studies in Athletics, 16 1\2 (2001)).

2.1 Data analysis

All computations were performed using Matlab software R2008a as the programming platform as well as optimization toolbox together with Microsoft Excel 2007 for data handling purposes.

2.1.1 First Law of Thermodynamics

The mathematical equations used in this study were based on the First Law of Thermodynamics (Lehninger, 1971). At the start of a sprint, the rate of the chemical energy production is converted to heat energy (H) and external work energy (W). The rate of change of energy is expressed per unit body mass (Wkg^{-1})and it is written in thedifferential form (Equ. 2.1) where the left term represents the rate of chemical energy conversion (C), and the first and second terms on the right-hand side are the rate of heat energy (H) and external mechanical work (W) respectively. The rate of heat energy is proportional to the instantaneous velocity $v(t)$ (Ward-Smith and Radford, 2000).

$$\frac{dC}{dt} = \frac{dH}{dt} + \frac{dW}{dt} \quad \text{...\textbf{Equ 2.1}}$$

Furthermore, the rate of external mechanical work is expressed as the sum of the rate of change of kinetic energy of the sprinter to move forward; the potential energy of the sprinter relative to his crouching state at the beginning of the race; and the work-done to overcome aerodynamic drag. The parameters for each of the energy components for the external mechanical work can be determined using already developed equations (Laurent and Locatelli, 2002; Ward-Smith and Radford, 2000).

2.1.2Rate of change of potential energy relative to crouching state

For a typical athlete (Baumann, 1976), the centre of mass is raised from its initial position (h_0) of 0.65m in the blocks to about 1.0 m which was assumed to be the centre-of-mass height (h_{cm}) of a standing athlete, and was used same for all athletes for analysis (Laurent and Locatelli., 2002; Ward-Smith and Radford, 2000). Therefore, the change in height (Δh) of the centre-of-mass of the sprinter (Baumann, 1976) above the horizontal running surface relative to his crouching state position is given by equation 2.2.

$$\Delta h = (h_{cm} - h_0)\sin\theta \text{ (where } \theta \neq 0) \quad \text{...\textbf{Equ.2.2}}$$

In equation 2.2, the angle, measured in radians, can be expressed further (Mitra, 2006) as $2\pi ft$, where f, in this case, is the stride frequency which is equal to the number of stride cycles per second, and variable t is the time measured in seconds. It was also shown that the stride frequency is well estimated by taking the inverse of the stride period (frequency is inversely proportional to time) (Stokes, 1998), and hence, the stride velocity is the product of stride length and stride frequency (Kamen, 2002). The centre-of-mass heightfor each sprinter is $0.57hs$ for healthy men (Grimshaw et al., 2004; McGinnis, 2005) and the stride length is given by $1.35hs$, where$hs$is the standing height of the athlete (Hoffman, 1971; Rompottie, 1972).

2.1.3 Rate of change of anaerobic energy

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Moreover, the rate of chemical energy conversion can also be expressed as the sum of the rate of energy produced from the aerobic and anaerobic metabolic pathways (Ward-Smith, 2000). By combining this sum with equation 2.1, therefore, the following formula as shown in equation 2.3 can be derived:

$$\frac{dCan}{dt} = \left(\frac{dH}{dt} + \frac{dW}{dt}\right) - \frac{dCae}{dt} \quad \text{...}\mathbf{Equ2.3}$$

The rate of aerobic energy is subtracted from the sum of the rate of heat energy and mechanical work to determine the rate for the anaerobic energy. The component on the left-hand side (Equ. 2.3) is the rate of change of anaerobic energy (Can) and the components on the right-hand side are the rate of change of heat energy (*H*), mechanical work (*W*) and aerobic energy (C_{ae}) respectively. The associated rate of change of aerobic energy is determined in accordance with theoretical equations previously developed by Van IngenSchenau (1991).

2.1.4 Modelling the rate of energy production for each anaerobic subsystem

The mathematical model that was used to represent the rate of production and decay of each anaerobic energy subsystem, was based on a type of Gamma distribution model since it is a flexible distribution to model biochemical processes that are hypothetically to be exponentially distributed, and a good fit for the sum of independent random variables (Hogg and Craig, 1978; Wlodarczyka and Kierdassuk, 2006). The Gamma mathematical model is expressed and characterised with respect to different parameters in terms of a shape (α) parameter, and a scale (β) parameter which is also known as the rate parameter (Equ. 2.4). For this model, the shape α was taken as 2 in accordance with previous works of Hogg and Craig (1978) so that a first-order in time *t* (Equ. 2.5) was obtained to represent the characteristics of the three anaerobic subsystem power distribution curves and hence, this makes computations faster (Gu et al., 1996). The gamma distribution*G* (Equ. 2.4) comprises of the gamma function which is denoted by$\Gamma$and this mathematical notation is the factorial of an integer number greater or equal to 1. The variables β, α, and *t* represent the scale, shape and time respectively, and the variable *e* represents the exponential value.

$$G(t;\alpha,\beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} t^{\alpha - 1} e^{-\beta t} \quad \text{…}\mathbf{Equ2.4}$$

$$G(t;\beta) = \frac{\beta^{2}}{\Gamma(2)} t^{2 - 1} e^{-\beta t} \quad \text{…}\mathbf{Equ2.5}$$

The initial estimates for the scale parameters β1, β2 and β3 for ATP endogenous, PCrutilization and oxygen-independent glycolysis were determined by finding the time constants corresponding to the respective maximum of the mean anaerobic power distribution curve for all the sprinters. These rate parameters served as initial estimates or inputs to run the computational program.

By using equation 2.5, the rate of change of the anaerobic metabolism was expressed as the sum of multiple gamma distributions to represent the three anaerobic subsystem powers, and this was mathematically represented in equations 2.6, 2.7 and 2.8. The symbol represents the instantaneous powers for each anaerobic subsystem measured in watts per kilogram. The nonlinear parameters are the rate parameters of the respective anaerobic subsystem, and they are initially determined by taking the inverse of the time constant for each anaerobic subsystem. The subscript n is an integer number ranging from 1 to 3 and it represents the three anaerobic subsystems. In equation 2.7, the variable P_1 denotes the rate of energy released from endogenous ATP, the variable P_2 is the rate of energy released from PCr utilisation, and the variable P_3 represents the rate of energy released from the oxygen-independent glycolysis anaerobic subsystem.

$$\frac{dCan}{dt} = {\beta_1}^2.t.e^{-\beta t} + {\beta_2}^2.t.e^{-\beta t} + {\beta_3}^2.t.e^{-\beta t} \quad \text{...Equ.2.6}$$

$$\frac{dCan}{dt} = P_1(t) + P_2(t) + P_3(t) \quad \text{...Equ.2.7}$$

$$\frac{dCan}{dt} = \sum_{n=1}^{3} P_n(t) \quad \text{...Equ.2.8}$$

2.3 Determination of the initial estimates of the nonlinear parameters

The initial estimates of the nonlinear parameters were determined by taking the maxima (3 maxima) from the curve obtained by calculating the mean of the anaerobic powers (i.e. rate of change of anaerobic energy) for all athletes over each 10-m interval, and it is illustrated in Figure 1. It is important to find these first estimates to minimise computational time, and prevent divergence from solutions (Boutayeb and Darouach, 1995; Chen and Fassois, 1992). The computational program was run repeatedly until convergence is reached or until the error (ε) which is the difference between the computed anaerobic power, and the total of the anaerobic subsystem powers at each distance interval for each athlete was minimal. At first, the 10-m split times, the total anaerobic power and the estimated values of the nonlinear parameters (were initial inputs to the computational program to find an estimate of the individual subsystem anaerobic powers. The norm function (norm) was used to find the residual error so that the amplification errors were kept minimum (Kariya and Kurata, 2004; Wolberg, 2005). In addition, the pseudo-inverse function (pinv) was used, in this case, especially for a non-square matrix (6 variables representing the anaerobic subsystem powers and the rate parameters x 10 equations representing the 10 split times) and this function works well when the number of equations are greater than the number of variables (Campbell and Meyer, 1991; Zheng and Bapat, 2004). In this particular case, the initial estimates of the time constants obtained from Figure 1, were determined as 1.1 s; 3.9 s; 7.9 s, and are used subsequently to estimate the rate parameters by taking the inverse of the respective time constants. In Figure 1, the variables m_1, m_2 and m_3 represent the three consecutive maxima of the anaerobic power curve.

Figure 1: Mean anaerobic power of a particular elite athlete and how the maxima (m_1, m_2, and m_3) are being identified.

2.4 Validation of model

The validation of the mathematical modelling was assessed with respect to the root mean square error (RMSE), in determining the total anaerobic powers derived for each athlete. The percentage RMSEwas calculated to find the error between the exact calculated total anaerobic power at each discrete time from the sum of the simulated individual anaerobic subsystem powers at these discrete times. The calculated total anaerobic power was the difference between the total aerobic power and the power lost due to mechanical work and heat. Figure 2 summarises the energy processes involved to mathematically model and analyse the chemical energy produced from the anaerobic energy system pathway. The chemical energy produced from both the anaerobic and aerobic metabolisms was converted into heat energy and mechanical energy.

Consequently, from this relationship, the energy from the anaerobic process can be determined, and then subsequently compared to the sum of the energy produced from the three corresponding anaerobic subsystems (Laurent and Locatelli, 2002; Ward-Smith and Radford, 2000).

Figure 2: The flowchart diagrams summarise the mathematical model in simulating the various anaerobic energy subsystems (Laurent and Locatelli, 2002; Ward-Smith and Radford, 2000).

3 Results

The velocity-time graphs (Fig. 3a and Fig. 3b) of the elite sprinters showed clearly the increase in speed from 0 ms-1 to a maximum speed where, during this period, the acceleration was maximal as shown by the steep slope of the velocity-time curve during the first 2 seconds. Subsequently, around 5 to 8 seconds, the sprinters started to decelerate slowly which continued in the same trend till the completion of this sprinting race

Figure 3: (**A**) Velocity of all the 100m-dash elite sprinters (*n* = 8); **(B)** Velocity of all the 100m-dash elite sprinters (*n* = 8) to show the change in velocity for each sprinter between time = 3s to time = 11s.

3.1 Anaerobic and aerobic power contributions

The total power, anaerobic power and aerobic power per unit body mass for all sprinters were determined (See Figure 5). Respective measured reaction time for each sprinter was excluded from the respective finishing time since during this brief period of about 0.141 (±0.01)s, the sprinters were still at rest, and hence equations 2.1 and 2.3 do not apply as the rate of change of heat energy and mechanical energy were assumed to be zero at time t = 0. It was found that the anaerobic power contributed to approximately 95% of the total power for this 100-m sprint.

3.2 Anaerobic subsystems (ATP endogenous, PCr Utilisation and oxygen independent glycolysis)

Figure 4 shows the normalised maximum rate of energy production for each subsystem for the anaerobic metabolism for a particular athlete to illustrate the difference among the anaerobic subsystems. The time T1 (Figure 5) represents the time when the ATP endogenous curve intersects the oxygen independent glycolysis energy curve measured as 2.71 s, and T2 represents the time when there is anintersection between the PCr utilisation and oxygenindependent glycolysis energy curves measured as 5.17 s. Furthermore, the mean and standard deviation of the power variables (watts per kilogram) P1, P2 and P3 were 6.6±1.78 Wkg-1, 40.5±2.97 Wkg-1 and 9.98±1.04 Wkg-1 respectively and the nonlinear parameters (1, 2, 3) representing the rate parameters of theanaerobic subsystems were 0.94±0.05 s-1, 0.31±0.015 s-1and 0.11±0.004 s-1 respectively. As shown in Figure 3.3, the endogenous ATP concentrations decreased rapidly at the start of the race and contributed to most energy during the first 2 to 3 seconds of this 100-m sprint race. Then, PCr utilisation process buffered the drop in ATP for another 5 to 8 seconds during which the PCrutilisation curve reached its maximum much before the oxygen independent glycolysis energy-curve reached its maximum at about 9.1 seconds.

Figure 4: Total power, anaerobic power and aerobic power for all sprinters (*n*=8) are plotted vs. finishing times excluding measured reaction times.

Figure 5: Normalised maximum rate of energy production of the first rank sprinter (Maurice Greene) for each subsystem of the anaerobic metabolism vs. time excluding reaction times. The arrows represent the x and y coordinates of the points of intersection of the anaerobic subsystem curves.

By extrapolating the mathematical results, the effect of increasing the percentage of energy released from the PCrutilisation anaerobic subsystem was investigated using the computed anaerobic subsystem powers and the rate parameters for the first rank sprinter. The mathematical model predicted that if the percentage of energy released from the PCrUtilisation was increased to 110%, the finishing time of the first rank sprinter would have been 9.27 s, and if the percentage of energy contribution from this particular anaerobic subsystem was increased further to 120%, the finishing time would have been 8.88 s and these results are shown in Figure 6.

Figure 6: The effect of increasing the percentage of energy released from the PCr utilisation anaerobic subsystem for the first rank sprinter. The arrows represent the effect of increasing the percentage of energy produced from the PCr anaerobic subsystem and the expected finishing times for Maurice Greene.

3.3 RMSE of mathematical model

The percentage RMSE was calculated to determine the error between the exact calculated total anaerobic power at each discrete time from the sum of the simulated individual anaerobic subsystem powers at each discrete time (Figure 7). This total anaerobic power is the difference between the total aerobic power and power lost due to mechanical work and heat. The minimum percentageRMSE was 0.0022 and the maximum percentage RMSE was 0.018. The variability of the percentage errors were caused by the distinct kinematics as well as the distinct weights, heights and reaction times of the elite sprinters in finding convergent solutions to the variables.

Figure 7: Percentage RMSEin estimating the totalanaerobic power for all athletes (*n* = 8). The average value of RMSE was 0.009W.

4 Discussions and Conclusion

4.1 Model validation

The average value of the RMSE for this developed mathematical model in calculating the total anaerobic powers for all athletes was 0.009W. This indicates that is a good model (Ward-Smith and Radford, 2000; Wargon et al., 2009) in defining the total anaerobic power for this data, under these physical conditions and environmental as well. However, muscle biopsy studies conducted by Gaitanos et al. (1993) and Bogdanis (1996) found in maximal 6 seconds and 10 secondscycling sprints that power output was advocated by energy acquired mainly from PCr degradation (the concentration of that reduced by 57%). Also,there was a causal relationship between the percentage of PCr and speed which influenced sprint performance.

4.2 Aerobic and anaerobic metabolisms

The percentage of chemical energy derived from the anaerobic process was 95%, comparing with the literature where they found mathematically that 92% of chemical energy during the 100m sprint running was produced from anaerobic sources (Peronnet and Thibault, 1989; Ward-Smith, 1985). Thus, the calculated percentage of energy production from the anaerobic process as compared with the literature it offers that mathematical modelling may be a reliable tool in assessing the anaerobic and aerobic energy system pathways. The difference between the literature the mathematical in the percentage of the energy which derived from the anaerobic processmay be related to reducing finishing times of the 100-m sprint running over the last decades (IAAF, 2008). Furthermore, the values that are gained by both mathematical modelling and previous related studies (Peronnet and Thibault, 1989; Laurent and Locatelli, 2002) signals that it is possible for modelling physiological systems accurately by mathematical models.

4.3 PCr utilisation anaerobic subsystem

All the athletes speed started to reduce at around 5 to 7 seconds during this sprint race and also in the previous studies (Hirvonen et al., 1987). Figure 5 shows this reduce in speed synchronised with the highest rate of decay of the PCr utilisation energy curve. Over the total sprint, in PCr system the energy contribution was 12.8% that was higher than the energy contribution acquired from oxygen independent glycolysis. Bogdanis et al. (1996), had tested the contribution of PCr during repeated bouts of cycle ergometer sprints (10 to 30-s), demonstrated that there was a high correlation (*r*) between the percentage of PCr and the percentage of restoration of mean power output (MPO) and also the speed during the initial 10 seconds of the sprints ($r = +0.84$ and $r = +0.91$).Moreover, Bogdanis et al. (1996) did not detect any correlation between power output recovery and concentration of any other metabolites (lactate, hydrogen and dihydrogen phosphate ions). Nevertheless, Bogdanis' observations find that there was no correlation between the percentage of PCr and MPO during the last 20-s of the sprint (Bogdanis et al., 1996). In addition, the observations ofBogdanis that an independent study by Hirvonen et al. (1987) established, in a series of maximal cycling sprints (40 – 100m), that skeletal PCr stores were severely drained after 5 to 7-s. Interestingly, the elite sprinters used more of their available PCr stores over the first 5 to 7-s than sprinters of slightly less ability. In addition, in this mathematical modelling, it was found that the rate constant for the PCr anaerobic metabolic energy process ($0.31s^{-1}$) as determined in this research was greater than that of the oxygen-independent glycolysis metabolic process ($0.11s^{-1}$). This observed metabolic behaviour could be explained due to more time taken for ATP to be produced from the oxygen independent glycolysis process than that from the PCrutilisation energy process (Baechle and Earle, 2000; Wilmore and Costill, 2005).

Conclusion

In this research, the developed mathematical model for calculating the total anaerobic powers for all the athletes was very good as the root mean square error was very low. It was found that 95% of the chemical energy during 100 metres sprint running was derived from the anaerobic processes. PCr utilisation sub anaerobic system plays a very important part in the sprint performance of the 100m track and field elite athletes as the reduce in speed synchronised with the highest rate of decay of the PCr Utlisation energy curve.

References

[1] Shulman, R.G. & Rothman, D.L, 'The glycogen shunt in exercising muscle:a role for glycogen in muscle energetics and fatigue', Proc. Natl. Acad. Sci., USA 98,(2001) , pp. 457-461.

[2] de Koning JJ, Foster C, Bakkum A, Kloppenburg S, Thiel C, et al, Regulation of Pacing Strategy during Athletic Competition', (2011), PLoS NE 6(1):e15863. doi:10.1371/journal.pone.0015863.

[3] Hill A .& Lupton, H, 'Muscular exercise, lactic acid, and the supply andutilization of oxygen', Q. J. Med. 16, (1923), pp. 135-171.

[4] Lambert EV, St Clair Gibson A, Noakes TD, 'Complex systems model offatigue: integrative homeostatic control of peripheral physiological systems duringexercise in humans', B J Sports Med, 39, (2004) ,pp. 52-62.

[5] Bogdanis G.C., Nevill M.E., Boobis L.H., and Lakomy H.K.A, 'Contribution of phosphocreatine and aerobic metabolism to energy supply during repeatedsprint exercise', J. Appl. Physiol., 80, (1996), pp. 876-884.

[6] Gaitanos GC, Williams C, Boobis LH, Brooks S, 'Human muscle metabolismduring intermittent maximal exercise', J App. Physiol, 75 (2), (1993), pp.712-719.

[7] Edwards, R.H.T, 'Biochemical bases for fatigue in exercise performance:catastrophe theory in muscular fatigue', In:knuttgen HG, Vogel JA, Poortsman J,eds, (1983).

[8] Noakes, T. D, 'Physiological models to understand exercise fatigue and theadaptations that predict or enhance athletic performance', Scand. J. Med Sci.Sports, 10, (2000) ,pp. 123-145.

[9] Ulmer H-, ‘Concept of an extracellular regulation of muscular metabolic rate during heavy exercise in humans by psychophysiological feedback’, Experimentia, 52, (1996) ,pp. 416-20.

[10] Fox, E.L. et al, ‘The Physiological Basis for Exercise and Sport’, 5th ed.Madison: Brown & Benchmark, (1993).

[11] Matthew R, ‘Physiological methods and measurements in driving simulation’, Proc. of the human factors and Ergonomics Soc. Annual Meeting, vol.,48 (19), (2004), pp.2330-2334.

[12] Di Prampero P.E., Capelli C., Pagliaro P., Antonutto G., Giradis M., Zamparo P.,Soule, RG, ‘Energetics of best performances of middle distance running’.J. of Appl. Physiol. 74, (1993), pp. 2318-2324.

[13] Laurent MA, Locatelli, E, ‘Modeling the energetic of 100-m running byusing speed curves of world champions’, J Appl. Physiol., 92, (2002), pp. 1781-1788.

[14] Lloyd, BB, ‘World running records as maximal performances’, Circulation Research Suppl.I, (1967) ,pp. 218-226.

[15] Peronnet F, Thibault G, ‘Mathematical analysis of running performance and world running records’, J. Appl. Physiol., 67, (1989), pp. 453-465.

[16] Ward-Smith, AJ, Radford, PF, ‘Investigation of the kinetics of anaerobicmetabolism by analysis of the performance of elite sprinters’, J. of Biomechanics,33, (2000), pp. 997-1004.

[17] Ward-Smith, A.J., Mobey, A, ‘Determination of physiological data from a mathematical analysis of the running performance of elite female athletes’, J.Sports Sciences, 13, (1995), pp. 321-328.

[18] Ward-Smith, AJ, ‘A mathematical theory of running, based on the first lawof thermodynamics, and its application to the performance of world-class athletes’,J. of Biomechanics, 18, (1985) , pp. 337-349.

[19] Ferro A, Riviera A, Pagola I, Ferreruela M, Martin A, Rocandio V,‘Biomechanical analysis of the World Championships in Athletics Sevilla’99: 100m, 200 m, 400 m sprint events’, New studies in Athletics, 16, (2001).

[20] IAAF, ‘Top List - 100m’. [Retrieved on 2008-09-02].

[21] Lehninger, A, ‘Bioenergetics-theMolecular Basis of Biological EnergyTransformations’, 2nd . Ed. London: The Benjamin/Cummings PublishingCompany, (1971).

[22] Mitra, S.J, ‘Digital Signal Processing: A Computer-Based Approach’, 3rd ed., McGraw-Hill, (2006).

[23] Kamen, G, ‘Foundations of exercise: Special topics in biomechanics’, Lippincott Williams & Wilkins, (2002), p179.

[24] Grimshaw, P., Burden A, ‘Instant Notes in Sport and Exercise Biomechanics’, BI S Scientific Publ. (2004), pp. 248-249.

[25] Hoffman, K, ‘Stature, leg length and stride frequency’, Track Technique, 46:1463-69, (1971).

[26] Van IngenSchenau GJ, Jacobs R, and de Koning JJ, ‘Can cycle powerpredict sprint running performance?’,Eur J Appl. Physiol., 63, pp. 255–260.

[27]Ward-Smith, AJ, Radford, PF. (2000), ‘Investigation of the kinetics of anaerobicmetabolism by analysis of the performance of elite sprinters’, J. of Biomechanics,33, (1991), pp. 997-1004.

[28] Hogg, R and A. T. Craig ,‘Introduction to Mathematical Statistics’, 4th ed. New York: Macmillan, (1978).

[29] Wlodarczyk, J. and Kierdaszuk, B, ‘A new approach to interpretation ofheterogeneity of fluorescence decay: Effect of induced tautomeric shift andenzyme → ligand fluorescence resonance energy transfer’, J. BiophysicalChemistry, 123, (2006), pp. 146-153.

[30] Gu M, Kalaba RE, Taylor GA,‘ btaining initial parameter estimates for chaotic dynamical systems using linear associative memories’, Appl. Math. Comput,76,(1996), pp. 143–59.

[31] Chen, CH., and Fassois, SD, ‘Maximum likelihood idenfication of stochastic Wiener-Hammerstein-type nonlinear systems. Mech. Syst. Signal Process., 6(2), (1992), pp. 135-153.

[32] Kariya T and KurataH, ‘Generalized Least Squares’, Wiley, (2004).

[33]Wolberg J, ‘Data Analysis Using the Method of Least Squares: Extractingthe Most Information from Experiments’, Springer, (2005).

[34] Campbell SL and Meyer CD, Generalized Inverses of Linear Transformations. Dover, (1991)..

[35] Zheng, B and Bapat, R.B, ‘Generalized inverse A(2)T,S and a rank equation’, Applied Mathematics and Computation, 155, (2004), pp. 407–415.

[36] Wargon, M, Guidet, B, Hoang, T D, Hejblum, G, ‘A systematic review ofmodels for forecasting the number of emergency department visits’, Emerg. Med.J. 26, (2009), pp. 395-399.

[37] Peronnet F, Thibault G, 'Mathematical analysis of running performance and world running records', J. Appl. Physiol., 67, 1989, pp. 453-465.

[38] Hirvonen J, Rehunen S, Rusko H, Harkonen M, 'Breakdown of high energy phosphate-compounds and lactate accumulation during short supra-maximalexercise', Eur J Appl. Physiol., 56, (1987), pp. 253-259.

[39] Baechle TR and Earle RW, 'Essentials of Strength Training and Conditioning', 2nd ed. Champaign, IL: Human Kinetics, (2000).

[40] Wilmore JH and Costill D, 'Physiology of Sport and Exercise', 3rd Ed.Champaign, IL: Human Kinetics, (2005).

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Table 1 (on next page)

Table 1

10-m split data intervals for the 100m sprint race in Sevilla Spain 1999. (Rivera, A.; Pagola, I.; Ferreruela, M.; Martín, A.; Rocandio, "Biomechanical Analysis of the World Championships in Athletics Sevilla'99: 100, 200, 400m sprint events". New Studies in Athletics, 16 1\2 (2001)).

Sprinter Ranking number	Distance covered/metres and the split times in seconds									
	10	20	30	40	50	60	70	80	90	100
1	1.86	2.89	3.81	4.69	5.55	6.39	7.24	8.09	8.94	9.80
2	1.88	2.88	3.79	4.68	5.53	6.38	7.24	8.10	8.96	9.84
3	1.87	2.89	3.81	4.71	5.57	6.41	7.29	8.18	9.06	9.97
4	1.91	2.93	3.85	4.76	5.63	6.50	7.36	8.24	9.12	10.00
5	1.87	2.89	3.81	4.71	5.60	6.47	7.33	8.22	9.11	10.02
6	1.91	2.95	3.88	4.77	5.65	6.52	7.39	8.28	9.16	10.04
7	1.91	2.93	3.85	4.74	5.62	6.51	7.40	8.28	9.17	10.07
8	1.97	2.99	3.93	4.83	5.72	6.61	7.50	8.38	9.31	10.24

Figure 1(on next page)

Figure 1

Mean anaerobic power of a particular elite athlete and how the maxima (m_1, m_2, and m_3) are being identified.

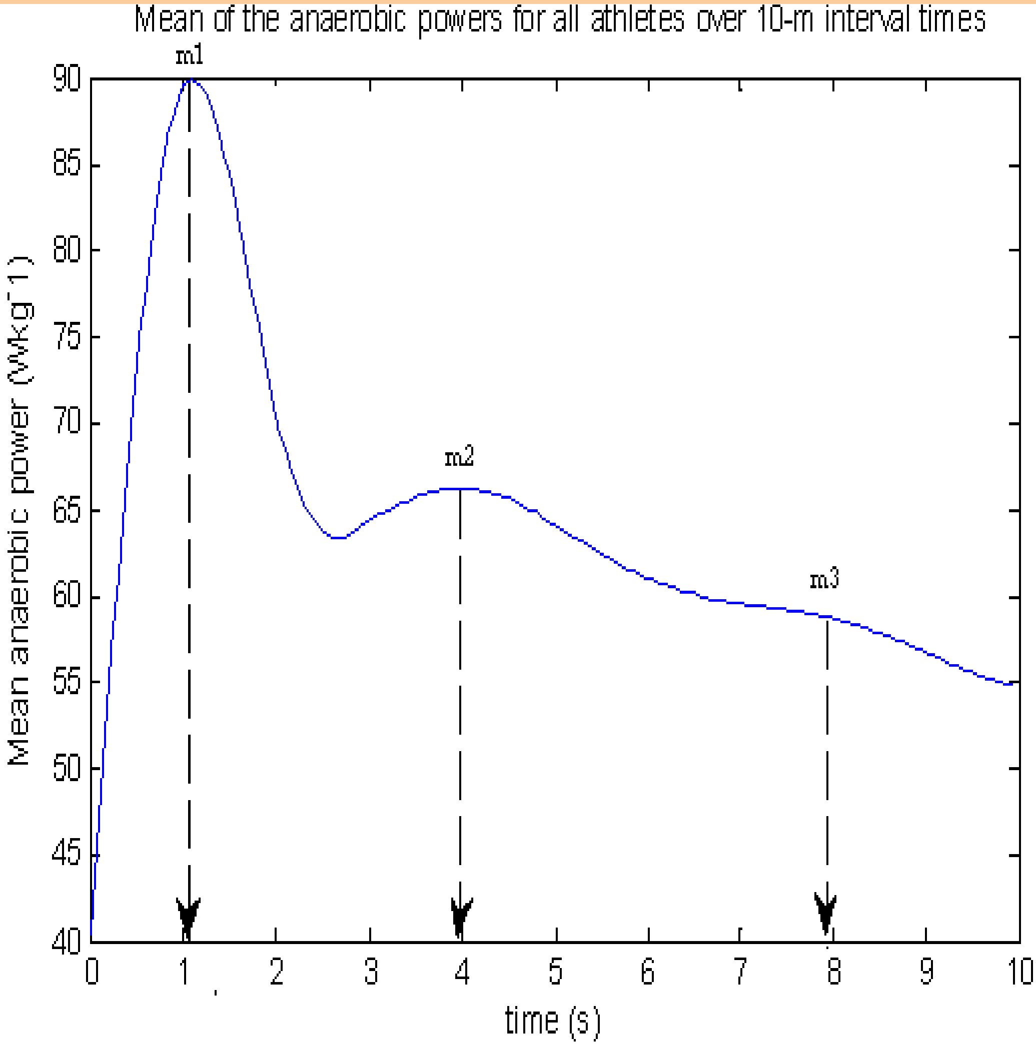

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Figure 2(on next page)

Figure 2

The flowchart diagrams summarise the mathematical model in simulating the various anaerobic energy subsystems (Laurent and Locatelli, 2002; Ward-Smith and Radford, 2000).

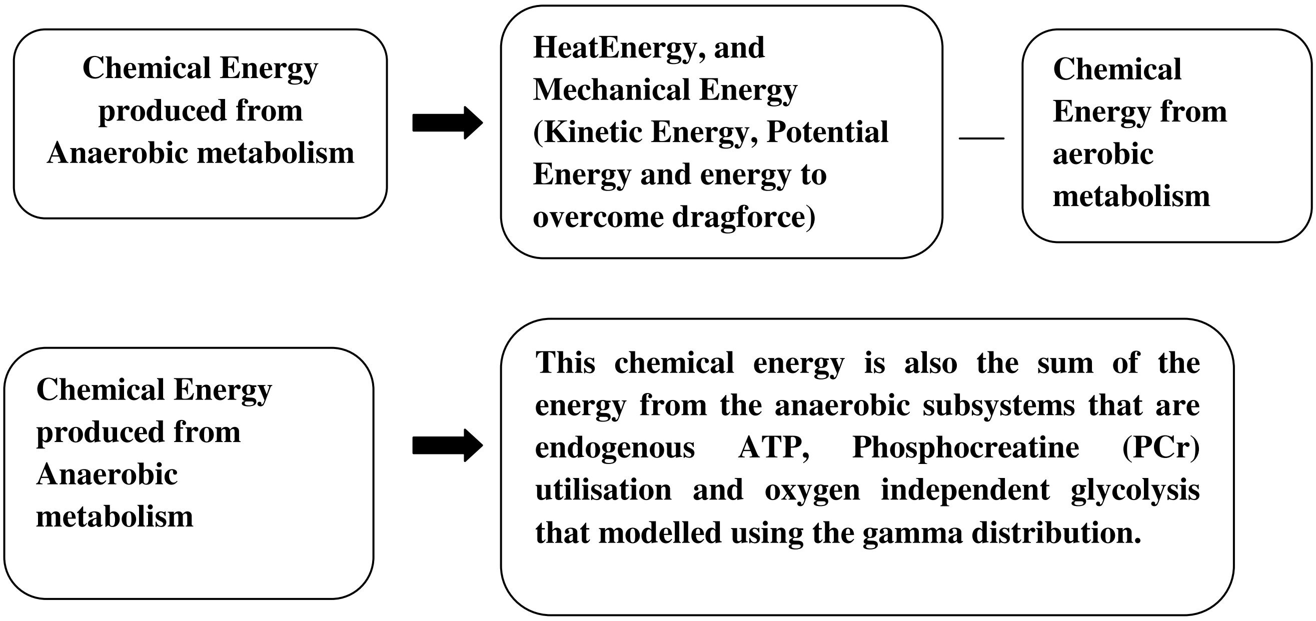

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Figure 3 (on next page)

Figure 4

Total power, anaerobic power and aerobic power for all sprinters (n=8) are plotted vs. finishing times excluding measured reaction times.

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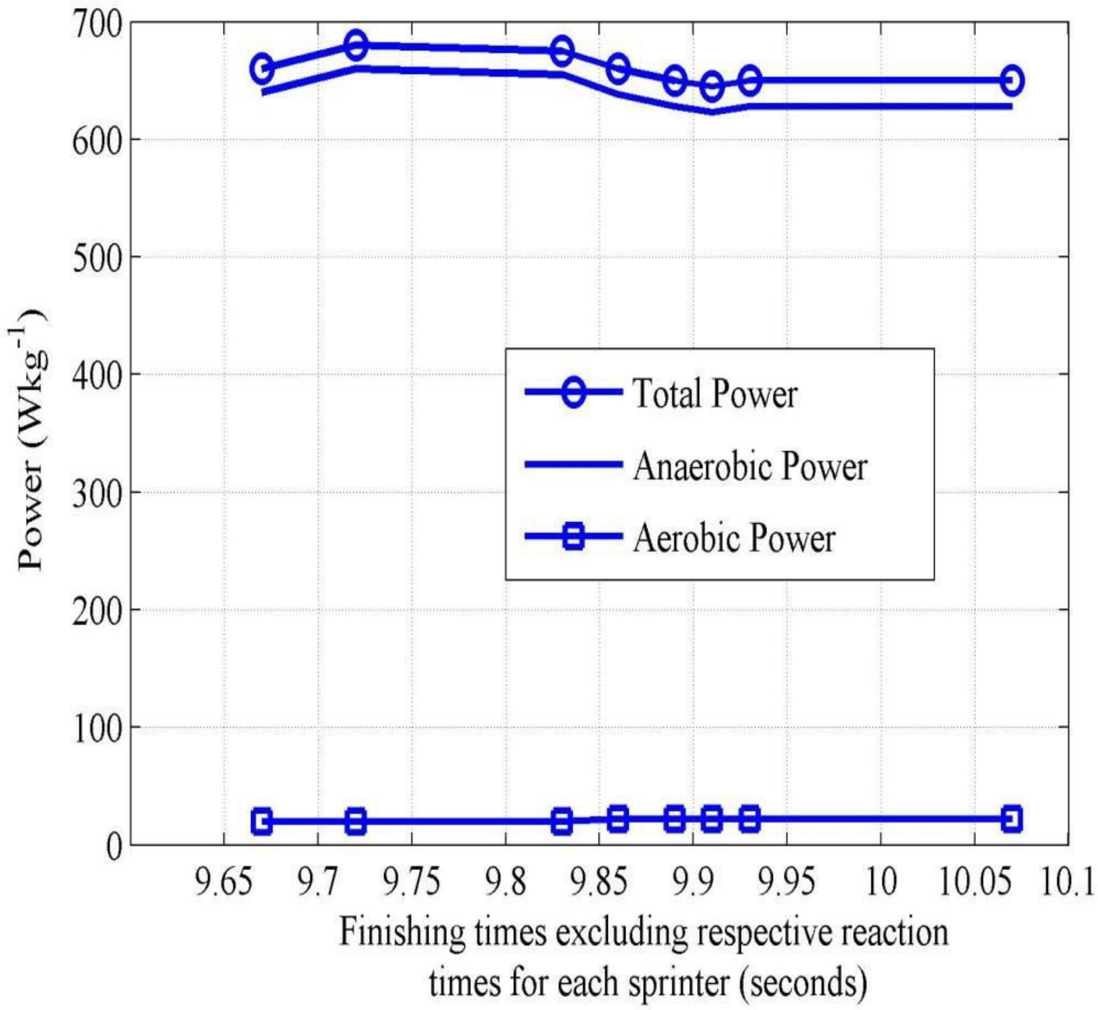

Power (Wkg^{-1})
700
600
500
400
300
200
100
0
Total Power
Anaerobic Power
Aerobic Power
9.65
9.7
9.75
9.8
9.85
9.9
9.95
10
10.05
10.1
Finishing times excluding respective reaction times for each sprinter (seconds)

Figure 4(on next page)

Figure 5

Normalised maximum rate of energy production of the first rank sprinter (Maurice Greene) for each subsystem of the anaerobic metabolism vs. time excluding reaction times. The arrows represent the x and y coordinates of the points of intersection of the anaerobic subsystem curves.

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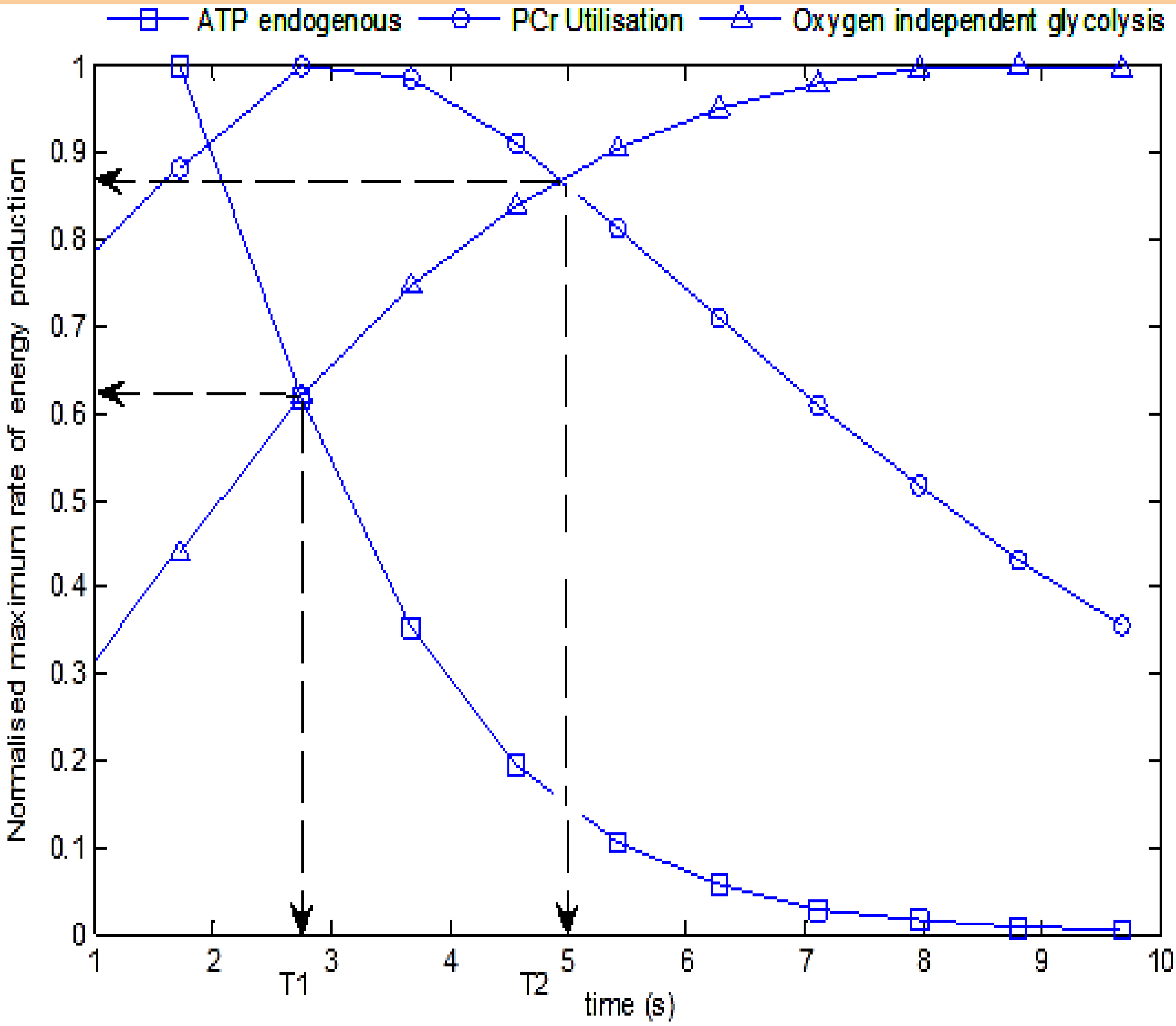
ATP endogenous
PCr Utilisation
Oxygen independent glycolysis
Normalised maximum rate of energy production
time (s)
T1
T2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1
2
3
4
5
6
7
8
9
10

Figure 5 (on next page)

Figure 6

The effect of increasing the percentage of energy released from the PCr utilisation anaerobic subsystem for the first rank sprinter. The arrows represent the effect of increasing the percentage of energy produced from the PCr anaerobic subsystem and the expected finishing times for Maurice Greene.

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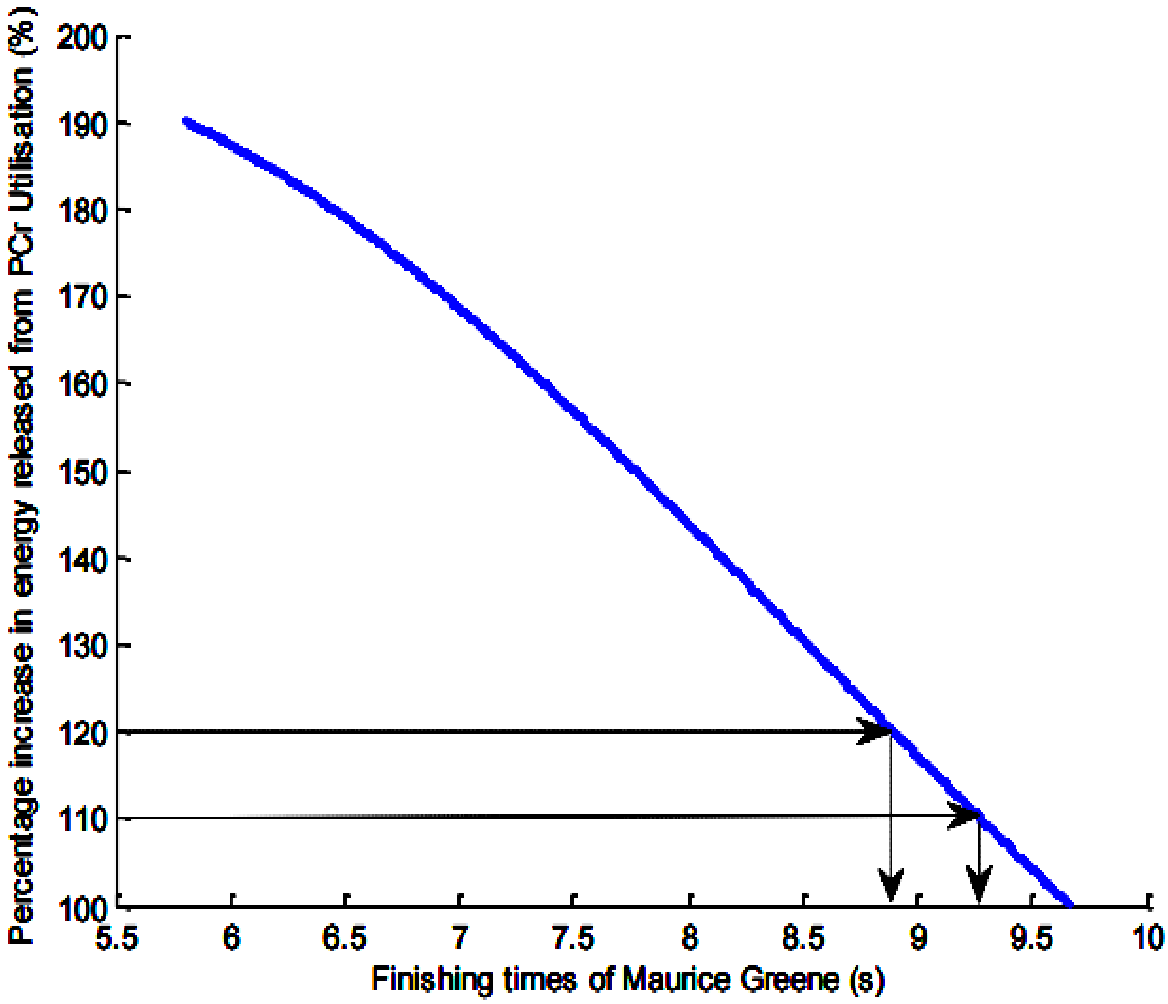
Percentage increase in energy released from PCr Utilisation (%)
200
190
180
170
160
150
140
130
120
110
100
5.5
6
6.5
7
7.5
8
8.5
9
9.5
10
Finishing times of Maurice Greene (s)

Figure 6 (on next page)

Figure 7

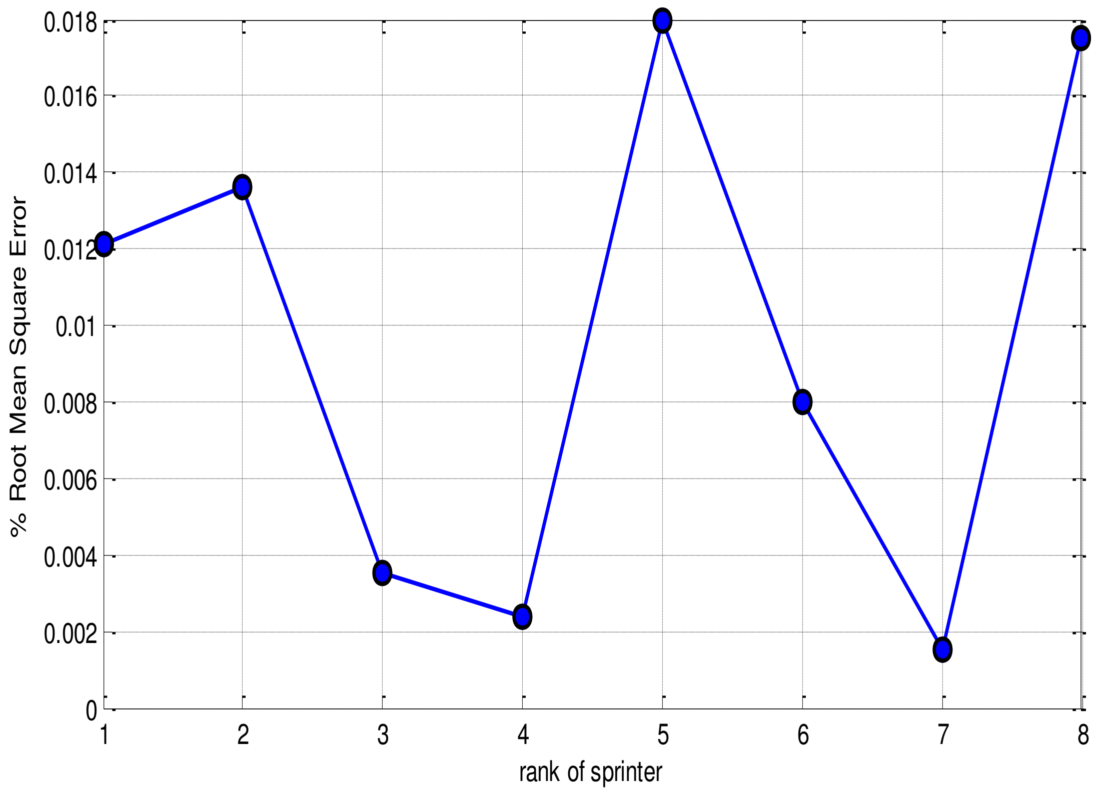
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
% Root Mean Square Error
1
2
3
4
5
6
7
8
rank of sprinter

Figure 7 (on next page)

Figure 3

(**A**) Velocity of all the 100m-dash elite sprinters (*n* = 8); **(B)** Velocity of all the 100m-dash elite sprinters (*n* = 8) to show the change in velocity for each sprinter between time = 3s to time = 11s.

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