

EvoPER – An R package for applying evolutionary computation methods in the parameter estimation of individual-based models implemented in Repast

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ABSTRACT

Individual-based models are complex and they normally have an elevated number of input parameters which must be tuned in order to reproduce the experimental or observed data as accurately as possible. Hence one of the weakest points of such kind of models is the fact that rarely the modeler has the enough information about the correct values or even the acceptable range for the input parameters. Therefore, several parameter combinations must be checked to find an acceptable set of input factors minimizing the deviations of simulated and observed data. In practice, most of times, is computationally unfeasible to traverse the complete search space to check all parameter combination in order to find the best of them. That is precisely the kind of combinatorial problem suitable for evolutionary computation techniques. In this work we present the EvoPER, an R package for simplifying the parameter estimation using evolutionary computation techniques. The current version of EvoPER includes implementations of PSO, SA and ACO algorithms for parameter estimation of Repast models.

Keywords: Individual-Based Modeling, Parameter Estimation, Evolutionary Computation, Systems Biology

INTRODUCTION

The Individual-based modeling and simulation is a powerful methodology which is having more and more adoptions between researchers and practitioners of distinct branches of ecological modeling and microbial consortia. Certainly one of the main reasons for the success of this approach is the extreme simplicity for capturing micro-level properties, stochasticity and spatially complex phenomena without the requirement of a high level of mathematical background (Grimm and Railsback (2005)). But the counterpart of that facility to build complex models, is the difficulty to make credible results which can be attributed in part to the fact that modelers are prone to circumvent a thoroughly analysis for simulation output.

There are several reasons for the situation previously mentioned. The first and perhaps most important is that modeling and simulation is a vast discipline with a broad and complex body of knowledge having a theoretical background under the surface (Minsky (1965); Zeigler et al. (2000); Boccara (2003)) which are not completely mastered from modelers coming from disperse domains like biology, ecology or even computer science. Of course, this should not be an obstacle for the development of good models by the practitioners. We believe that the availability of easy tools for the model tuning and analysis which efficiently encapsulate such complex subject can greatly help to improve the quality and significance of simulation results.

In the next sections we will describe the scope and the usage examples of the EvoPER R package which has been developed for facilitating the tasks of estimating the parameters of Individuals-based models.

41 BACKGROUND

42 The terms model calibration and parameter estimation, although informally are used interchangeably
43 and being functionally similar are semantically distinct entities having a different scope and objectives.
44 In order to provide a more formal definition of these terms let us briefly define the basic structure of a
45 mathematical model. A model is normally expressed as some form of the algebraic composition expressing
46 the relationship between of three element types, namely the independent variables, the dependent or the
47 state variables and finally the constants. Therefore, for the sake of simplicity, a model expressing some
48 linear relationship between variables is shown below

$$y = \alpha + \beta x$$

49 where x and y are independent and the state variable respectively and α and β are the model constants.
50 The model constants are referred as the model parameters which necessarily do not have to have any
51 correspondence to some element in the system being modeled (Beck and Arnold (1977)). The direct
52 problem is, being known the model structure and also knowing the independent variables and the
53 parameters, to estimate the value of state variable. Of course this oversimplified case is rarely seen when
54 modeling real systems, especially when dealing with biological systems. In addition, in the most cases
55 the constants and the independent variables are impossible to observe directly being also unknown the
56 right model structure for representing the system under study.

57 Usually the only value elucidated experimentally or backed by observations of some population data
58 is the state variable; therefore, the parameters which are the structural part of model must be estimated
59 having as the only reference, the measurements of dependent variable. Hence the term **calibration** can be
60 defined as the procedure to where the values of state variable "y" are compared to the known standard
61 values, let's say "Y", which in the context of biological research are those sampled from population true
62 values Zeigler et al. (2000).

63 On the other hand, the **parameter estimation** is the task of estimating the values of the constants of
64 a model and it can be seen somehow as an inverse problem, since we are using the reference values Y
65 in order to determine the suitable values for the model constants (Ashyraliyev et al. (2009); Beck and
66 Arnold (1977)). The parameter estimation procedure implicitly encompasses the calibration process as, in
67 order to discover the values for the constants the model outputs must be checked to the reference values.
68 Thus the problem can be also stated as an optimization problem, just because the process requires the
69 search for the minimum values of some function $f(y_i, Y_i)$ measuring the distance between y_i and Y_i which
70 are the simulated and the reference values respectively.

71 The function measuring how close are the observed and the reference values is the goodness of fit
72 metric for assessing how well the model is able to reproduce the reference data. In other words, the
73 metric gives a numerical hint about how close are the output of model to the reference data. There are
74 fundamentally three approaches to define the goodness of fit for a model (Thiele et al. (2014)). The first
75 approach is based on using acceptable ranges for the model outputs being the most straightforward one.
76 That approach is also known as categorical calibration and works defining intervals for the model output
77 values and when the output falls inside the interval it is considered as having a good fit. One of the main
78 drawback of this approach is the fact that it is not possible to determine how close are the model and the
79 reference data. The second metric relies on measuring the differences between simulated and observed
80 values, being the least squares the most commonly used method for computing the quality of fit (Beck and
81 Arnold (1977)). Finally, that last approach requires the use of likelihood functions. It is hard to implement
82 and requires that the underlying distribution must be known.

83 In order to explore the search space, the calibration process requires many model executions as well
84 as many evaluations of goodness of fit function over the output data in order to find the best estimation for
85 the model parameters. This is a computationally expensive task, especially in the case of Individual-based
86 models, as the problem bounds increases with model complexity and the number of input parameters
87 which must be tested. Roughly speaking there are basically two different approaches for generating
88 the sample points required for estimating parameters. The first of them is based on the definition of
89 sampling schemes such as Monte Carlo, Factorial designs or the Latin Hypercube sampling that works by
90 generating an a priori set of samples in the search space, that is to say, a set of parameter combinations for
91 running model with all of these sampling points (Thiele et al. (2014); Viana (2013)). On the other hand,
92 in the case of optimization methods, we have to generate an initial set of points sampled from the input

93 space and modify them dynamically to search for neighboring solutions which could approximate better
94 to the minima. The exact method depends on the evolutionary algorithm chosen for parameter estimation.

95 DESCRIPTION

96 In order to facilitate the parameter estimation task of Individual-based models we introduce the GNU R
97 (R Core Team (2015)) package **EvoPER** - Evolutionary Parameter Estimation for Repast, an open source
98 project intended to facilitate de adoption and application of evolutionary optimization methods and algo-
99 rithms to the parameter estimation of IBMs developed using the Repast Symphony framework North et al.
100 (2013). The EvoPER package is released under the MIT license being the binaries available for download
101 from CRAN (<https://cran.r-project.org/web/packages/evoper/>) and the complete
102 source code for the project can be found on GitHub ([https://github.com/antonio-pgarcia/](https://github.com/antonio-pgarcia/evoper)
103 [evoper](https://github.com/antonio-pgarcia/evoper)).

104 The package EvoPER provides implementations of common evolutionary algorithms specially crafted
105 for search the optimum values for Individual-based models developed in Repast Symphony. Current
106 version of EvoPER package supports the Particle Swarm Optimization (PSO) (Kennedy and Eberhart
107 (1995)), the Simulated Annealing (SA) (Kirkpatrick et al. (1983)) and the Ant Colony Optimization (ACO)
108 (Dorigo et al. (2006)) algorithms for parameter estimation. We also plan to support more algorithms
109 in future versions. All of these algorithms use some kind of natural or physical system analogy having
110 each of them subtleties making them suitable for different types of problems. Nonetheless, despite of
111 the differences in the natural metaphor chosen all algorithms share an important aspect which is that the
112 search space is traversed downhill but allowing uphill moves in order to avoid to get trapped in a local
113 optimum far from the global one.

114 The basic PSO algorithm uses the idea of particles moving in a multidimensional search space being
115 the direction controlled by the *velocity*. The velocity has two components, one towards to the direction
116 of best value of particle p_i and other towards to the best value found in the neighborhood of particle p_i
117 (Kennedy and Eberhart (1995)). The behavior and convergence of the algorithm is controlled by the
118 particle population size and by the ϕ_1 , ϕ_2 parameters which respectively controls the particle acceleration
119 towards the local and the neighbor best. The algorithm implementation and the default values for the
120 algorithm parameters follows the guidelines and standard values provided by (Clerc (2012)).

121 On the other hand, the Simulated Annealing uses the idea of cooling scheme to control how the
122 problem solutions are searched. The algorithm generates an initial solution and then iterates, searching for
123 neighbor solutions accepting new solutions when they are better than the current solution or with some
124 probability P which is function of current temperature and the cost of solutions. Important parameters are
125 the initial temperature T_0 , the final temperature and the cooling scheme Kirkpatrick et al. (1983). In our
126 implementation the default function for temperature update is $T = \alpha T$, being α the parameter controlling
127 how fast the temperature is decremented.

128 The package designed using an object-oriented approach being structured around the classes repre-
129 senting the objective function to be minimized. These classes are the basic input for the optimization
130 algorithms available on the EvoPER package. There is a parent class called *ObjectiveFunction* with
131 two subclasses, namely the *PlainFunction* and the *RepastFunction*. The purpose of the first subclass is
132 allow the user run the optimization algorithms to their own mathematical functions, the second subclass
133 encapsulates the Repast Model calls and perform the parameter estimation. A brief description of package
134 classes and the main methods is given in Table 1.

135

Table 1. The EvoPER classes for encapsulating the objective function for parameter estimation.

Class name	Methods	Description
ObjectiveFunction		The base class in hierarchy providing the skeleton for running the optimization algorithms.
	Parameter	Sets a model parameter with range between a minimum and a maximum values.
	GetParameter	Returns a previously defined parameter.
	Evaluate	Evaluate the objective function.
PlainFunction	Value	Returns the value of last objective function evaluation.
	initialize	Allows the optimization of plain functions implemented in R. Class constructor. Requires any R function as parameter. For instance $f < -function(x_1, x_2)\{(1 - x_1)^2 + 100(x_2 - x_1^2)^2\}$
RepastFunction	Evaluate	Override superclass method to the specific function call.
	initialize	Wrapper the Repast Model
	Evaluate	Requires the model directory, an aggregated data source, the simulation time and a user defined cost function. Override superclass method to the specific function call.

136

137 The object oriented approach allows the easy extension of the package for other types of Individual-
 138 based modeling tools or methods. As can be seen in Table 1 the only requirement to apply the methods
 139 contained in the EvoPER package is to extend the *ObjectiveFunction* class and override the Evaluate
 140 method to support the new parameter estimation target. One of the useful aspects of EvoPER implementa-
 141 tion is the possibility to specify constraints in the search space by individually setting lower and upper
 142 bounds for every parameter being analyzed using the *ObjectiveFunction\$Parameter(name, min, max)*
 143 method. That is an important point for limiting the parameter values only to the acceptable biological
 144 range.

145 The workflow for carry out the parameter estimation consists in a simple sequence of steps. First,
 146 an object instance of any *ObjectiveFunction* subclasses must be created and properly initialized. As
 147 mentioned previously, currently we have two options available for parameter estimation: one for simple
 148 functions which could be used for testing purposes (*PlainFunction*) and another for estimating parameters
 149 of Repast models (*RepastFunction*). Once the objective function has been initialized, the required
 150 parameters must be provided with the appropriate lower and upper bounds. Finally, the *extremize* function
 151 can be applied to the previously defined function. The required parameters are the optimization method
 152 and the objective function instance. The function has a third optional parameter for providing the custom
 153 options for the underlying optimization method.

154 The optimization functions and its accessory helper functions are shown in the Table 2 for providing an
 155 overview on the package contents, the package is in continuous improvement and development therefore
 156 the list could change over the time. The package manual will be the most updated source of information
 157 for the package contents.

158

Table 2. The partial list of EvoPER optimization functions for parameter estimation.

Function	Description
abm.pso	The function call for running the particle swarm optimization method. The parameters are the <i>ObjectiveFunction</i> and an instance of <i>Options</i> class with the suitable parameter set.
pso.neighborhood.K2	This neighborhood function returns two neighbors of particle x_i , where the neighbors are the particles x_{i-1} and x_{i+1} using a ring topology Zambrano-Bigiarini et al. (2013).
pso.neighborhood.K4	Returns four neighbors of particle x_i using a von Neumann neighborhood function.
pso.neighborhood.KN	Return the whole set of particles. The neighborhood is a complete graph.
pso.Velocity	Calculate the particle velocity Poli et al. (2007)
pso.chi	Calculate the constriction coefficient Poli et al. (2007).
initSolution	Creates a random initial population of size N for the model parameters.
enforceBounds	Verify the upper and lower limits of every parameter
abm.saa	The Simulated Annealing implementation. The parameters required are the <i>ObjectiveFunction</i> and an instance of <i>Options</i> class with the suitable parameter set.
saa.neighborhoodI	Generate a neighborhood solution for simulated annealing perturbing randomly one value from current best solution and using the distance parameter.
saa.neighborhoodH	Generate a neighborhood solution for simulated annealing perturbing randomly the half of values of current best solution.
saa.neighborhoodN	Generate a neighborhood solution for simulated annealing perturbing randomly all values of current best solution.
extremize	This is a wrapper encapsulating the calls for all parameter estimation methods. The parameters are the optimization method, the <i>ObjectiveFunction</i> and an instance of <i>Options</i> class with the suitable parameter set.

Most of the aspects implemented in the optimization code are standard and, perhaps the only points which are specific to the EvoPER package, are the neighborhood function for *pso.neighborhood.K4* and *saa.neighborhood*. The von Neumann neighborhood for particle swarm optimization is generated using a topology created converting the linear collections of particles to a matrix using the R code

```
m <- matrix(seq(1, N), nrow=(ceiling(sqrt(N))))
```

where N is the swarm size.

In the case of neighborhood solution for Simulated Annealing we have used the following logic for generating new solutions: first we pick randomly the parameters to be perturbed¹ and update them using the expression $S' = S + S * U(-1, 1) * distance$ where S' , S , U and $distance$ are respectively the new neighbor solution, the current solution, a uniform random number between $[-1, 1]$ and the desired distance from current solution.

The package provides acceptable default values for most of parameters related to the optimization method in use. In spite of the fact that the parameter estimation functions can be called directly, the users should use the function *extremize(m, f, o)* which is the standard entry point for the optimization methods. As has been mentioned previously, the function has three parameters, which are respectively the method (m), the objective function (f) and the options (o). Only the first two are required and the third is optional. When the options parameter is not provided the default values are used. If setting different from the default values are required, the user must pass an instance of the corresponding option class. For example, if more iterations are required for PSO method an instance of *OptionsPSO* must be created and the method *setValue("iterations", value)* with the appropriate value. Many other parameters can be customized in order to fit the specific needs for the model being analyzed such as the neighborhood functions or the temperature update for the simulated annealing.

¹Our implemented neighborhood functions allows to choose from 1, 1/2 n or n, being n the number of parameters

181 **EXAMPLES**

182 In this section we will show some small and illustrative examples about how to use the EvoPER package
 183 for estimating the model parameters. It is worth mentioning that although the package is oriented to the
 184 application of evolutionary optimization methods to the parameter estimation of models developed using
 185 Repast Symphony it can also be used to minimize basic mathematical functions. In the following example
 186 shown in Figure 1 we demonstrate the package usage applying it to the two variables Rosenbrock's
 187 function.

```

1 # Step 0
2 rm(list=ls())
3 set.seed(161803398)
4 library(evoper)
5
6 # Step 1
7 rosenbrock2<- function(x1, x2) { (1 - x1)^2 + 100 * (x2 - x1^2)^2 }
8
9 # Step 2
10 objective<- PlainFunction$new(rosenbrock2)
11
12 # Step 3
13 objective$Parameter(name="x1",min=-100,max=100)
14 objective$Parameter(name="x2",min=-100,max=100)
15
16 # Step 4
17 results<- extremize("pso", objective)

```

Figure 1. A simple example for minimizing the Rosenbrock's function using the EvoPER package.

188 As can be seen in Figure 1 the **step 1** shows the definition of a simple function to be minimized; the
 189 **step 2** demonstrate how to create an instance of *PlainFunction* class; in the **step 3** the parameter ranges
 190 for each function's parameter is provided and finally in the **step 4** the EvoPER *extremize* function is used
 191 to minimize the objective function. The results of running the example are shown in Figure 2 where can
 192 be seen the estimated parameters, the value of fitness function, the execution time and the number of
 193 times the function has been evaluated.

```

1 > system.time(results<- extremize("pso", f))
2   user  system elapsed
3  1.50    0.00    1.53
4 > results
5      x1      x2 pset      fitness
6 1 1.000762 1.001341  4 3.948505e-06
7 > f$stats()
8   total_evals converged
9 [1,]         1616      1

```

Figure 2. The R console output session showing the results of running the previous example.

194 One of important aspects is that the syntax is simple and consistent independent of the function for
 195 which parameters are being estimated. In next example shown in Figure 3 we can observe the simplicity
 196 for running the optimization code for Repast parameter estimation. As can be seen the same steps
 197 are required: (1) create the function to minimize based on the model characteristics; (2) Create the
 198 *RepastFunction* instance with model data; (3) Initialize the model parameters with the acceptable ranges
 199 and (4) Run the optimization function. In this example we are basically trying to find the best combination
 200 of model parameters which minimize the differences between the observed and the simulated data for the
 201 variable *Rate* and the method used is the normalized root mean square deviation.

202 Finally, in the last example show in Figure 4 we want to show an example on how to craft the cost
 203 function for tuning the model parameters in order to accomplish a specific output. Specifically, a simple
 204 toy model representing the Lotka-Volterra, also known predator-prey is presented and we want to estimate
 205 the parameters required to make the output oscillate with an approximate period of twenty-four hours.
 206 This model, despite of being developed for modeling the predator and prey relationship, has a broad range
 207 of applications and can be used for representing a many types of ecological and biological interactions

```

1 # Step 0
2 rm(list=ls())
3 set.seed(161803398)
4 library(evoper)
5
6 # Step 1
7 my.cost<- function(params, results) {
8   Rate<- AoE.NRMSD(results$simulated, results$experimental)
9   criteria<- cbind(Rate)
10  return(criteria)
11 }
12
13 # Step 2
14 objective<- RepastFunction$new("/usr/models/BactoSim", "ds::Output", 300, my.cost)
15
16 # Step 3
17 objective$Parameter(name="cyclePoint", min=1, max=90)
18 objective$Parameter(name="conjugationCost", min=0, max=100)
19 objective$Parameter(name="pilusExpressionCost", min=0, max=100)
20 objective$Parameter(name="gamma0", min=1, max=10)
21
22 # Step 4
23 results<- extremize("saa", objective)

```

Figure 3. The minimum code required to accomplish the parameter estimation for a repast model.

208 (Shonkwiler (2008)). The parameters we are trying to estimate are c_1 , c_2 , c_3 and c_4 which represent
 209 respectively the growth rate of prey, the predation rate, the predation effect on predator growth rate and
 210 finally the death rate of predator. The session output is presented in Figure 5 where the values for the
 211 parameters required to produce oscillations with the desired period are shown. The Figure 6 shows
 212 graphically the results for the tuned parameters.

```

1 # Step 0
2 rm(list=ls())
3 set.seed(161803398)
4 library(evoper)
5
6 # Step 1
7 my.cost<- function(params, results) {
8   predators<- AoE.NRMSD(period(results$predators), 24)
9   criteria<- cbind(predators)
10  return(criteria)
11 }
12
13 # Step 2
14 objective<- RepastFunction$new("/usr/models/PredatorPrey", "ds::population", 180, my.cost)
15
16 # Step 3
17 f$Parameter(name="c1", min=0.5, max=8)
18 f$Parameter(name="c2", min=0.5, max=8)
19 f$Parameter(name="c3", min=0.5, max=8)
20 f$Parameter(name="c4", min=0.5, max=8)
21
22 # Step 4
23 results<- extremize("pso", objective)

```

Figure 4. Tuning the oscillation period of predator-prey model.

213 CONCLUSIONS

214 The systematic parameter estimation should be a fundamental part of individual-based modeling but
 215 it is normally omitted by modelers. One of the main reasons is the relative complexity of available
 216 methods and the lack of simple tools for the practitioners which usually come from different domains with
 217 different backgrounds. Individual-based models are complex and non-linear and the evaluation of model's
 218 input parameters is precisely the kind of combinatorial optimization problem for which evolutionary
 219 computation provides good results.

220 In this work we have introduced the set of features available on EvoPER package alongside with

```

1 > system.time(results<- extremize("pso", f))
2   user  system elapsed
3   91.2    0.00   91.29
4 > results
5   c1      c2      c3      c4 pset      fitness
6 1 0.6305862 0.8146169 1.192911 1.611731 4 5.01271e-03
7 > f$stats()
8   total_evals converged
9 [1,]          800         0

```

Figure 5. The R console output session showing the results of running predator-prey model in Figure 4.

221 some brief usage cases. The package is being developed bearing in mind the idea of minimizing the effort
 222 required to the application of sophisticated methods in the parameter estimation process of Individual-
 223 based models. This package will allow the modelers to try different alternatives without having to code ad
 224 hoc and complex integration code to the existent packages.

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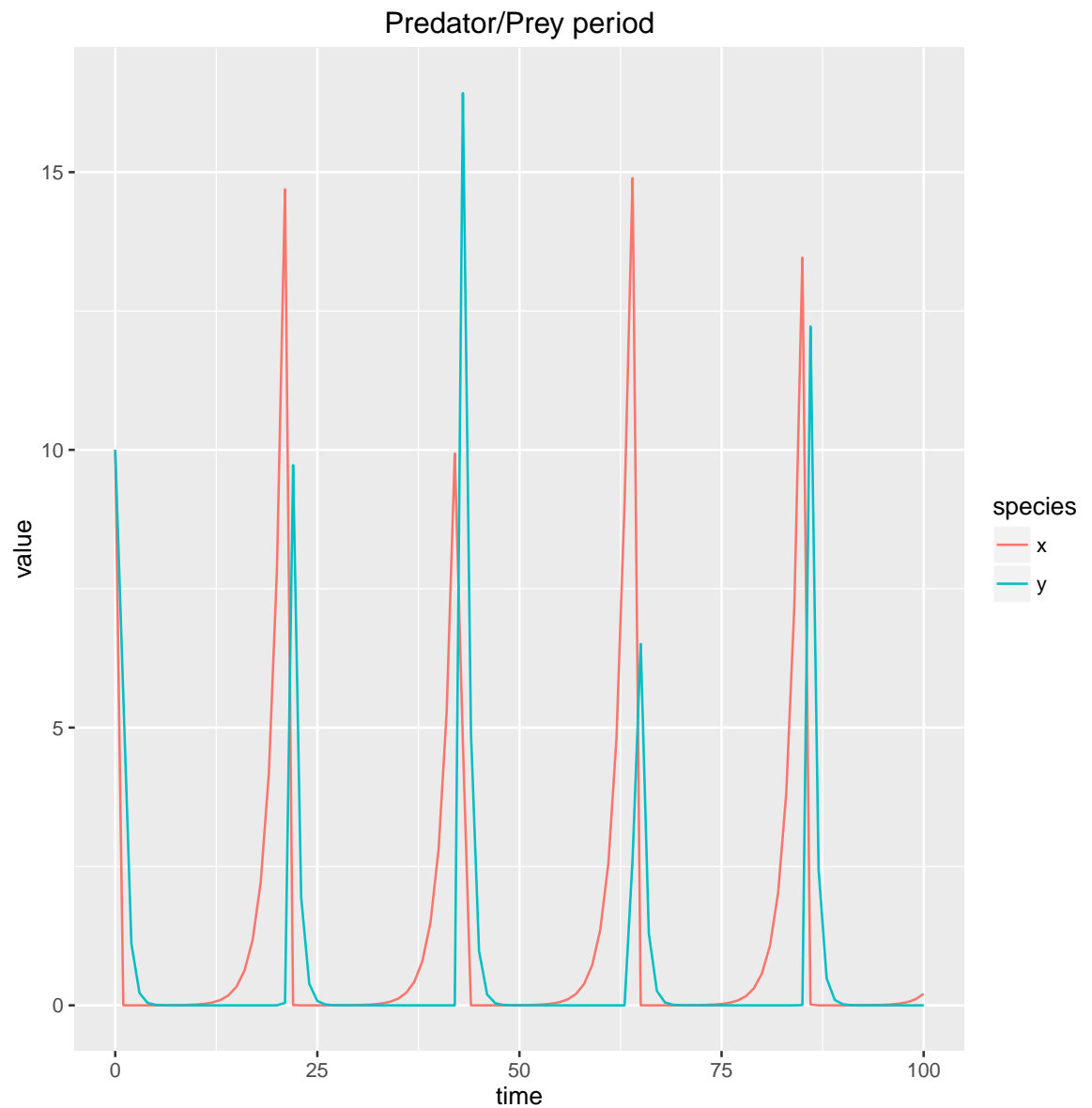


Figure 6. An example of tuning the oscillation period of predator-prey model. The objective function can be tweaked to reproduce any desired output behavior. The most common one is to assess the quality of fit between simulated and experimental data but it is no limited and can be used to find parameter combinations which generate some global behavior. In this figure we can observe how x and y species, respectively the prey and predator components oscillates with an approximated period of 24 hours using the parameter combination shown in Figure 5.