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Daniel Goodman's empirical approach to Bayesian statistics

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ABSTRACT

Bayesian statistics, in contrast to classical statistics, uses probability to represent uncertainty about the state of knowledge. Bayesian statistics has often been associated with the idea that knowledge is subjective and that a probability distribution represents a personal degree of belief. Dr. Daniel Goodman considered this viewpoint problematic for issues of public policy. He sought to ground his Bayesian approach in data, and advocated the construction of a prior as an empirical histogram of "similar" cases. In this way, the posterior distribution that results from a Bayesian analysis combined comparable previous data with case-specific current data, using Bayes' formula. Goodman championed such a data-based approach, but he acknowledged that it was difficult in practice. If based on a true representation of our knowledge and uncertainty, Goodman argued that risk assessment and decision-making could be an exact science, despite the uncertainties. In his view, Bayesian statistics is a critical component of this science because a Bayesian analysis produces the probabilities of future outcomes. Indeed, Goodman maintained that the Bayesian machinery, following the rules of conditional probability, offered the best legitimate inference from available data. We give an example of an informative prior in a recent study of Steller sea lion spatial use patterns in Alaska.

KEYWORDS

Bayesian inference, hierarchical Bayes, informative priors, uncertainty, natural resource management, structured decision making

1. THE BAYESIAN PARADIGM

The two most widely used statistical paradigms in natural resource management are classical methods, which include null hypothesis significance tests and maximum likelihood estimates, and Bayesian methods. The pros and cons of Bayesian and classical methods have been debated for decades, both in ecology and other fields (Efron 1986, Reckhow 1990, Berger and Mortera 1991, Dennis 1996, Ellison 1996, Boyce 2002, Bayarri and Berger 2004). Other methods that don't fit into these two categories include machine-learning methods (Hastie et al. 2001) such as maximum entropy modeling (Phillips et al. 2006) and Random Forests (Breiman 2001). In this essay we focus on how Bayesian methods differ from classical methods, why Dr. Daniel Goodman thought the Bayesian approach was important, and what Goodman contributed to the subject.

At the time Goodman started using Bayesian ideas in the 1990s, the Bayesian paradigm was not widely accepted among ecologists. However, the use of Bayesian methods in ecological analyses has been rapidly increasing in recent years (Figure 1), due partly to modern computing power, partly to the development of more accessible software, and partly to the growing appreciation of the advantages of the Bayesian approach. There are even articles (Strogatz 2010, Carey 2011, Flam 2014) and books (McGrayne 2011, Silver 2012) for the general public about Bayesian ideas. Some analysts use Bayesian methods pragmatically. They might use Bayesian methods for complex hierarchical models, which are often easier to fit with Bayesian methods, but use non-Bayesian methods at other times. Goodman was committed to the Bayesian approach for philosophical reasons, and he believed firmly that it offered the best inference from available data.

The basic concept of the Bayesian approach is simple: after collecting data, we combine the new data with what is already known to arrive at an updated state of knowledge. This process largely reflects the scientific method, in which hypotheses are formed, evaluated with data, refined, then evaluated again with more data. Classical statistical methods (also called frequentist statistics) allow a null hypothesis to be rejected on the basis of observed data. However, rejection of the null hypothesis does not provide support for an alternative, nor does failure to reject the null hypothesis mean that it is true. In contrast, the Bayesian framework allows a hypothesis to be supported (or not), and the degree of support to be sequentially updated, through Bayes' formula, as more data become available. Bayes' formula is based on the axioms of conditional probability, and is mathematically uncontroversial. Simply put, Bayesian inference just follows the rules of conditional probability. Its adherents see that as its main strength; its detractors, while admitting its mathematical coherence, disagree with its use as a system of inference.

Maximum-likelihood and Bayesian methods both use likelihood functions, which describe the relationship between the data (x) and the parameter(s) that generated the data, and possibly a hypothesis (H) that depends on the parameters. Both methods condition on the observed data. The difference is that the maximum-likelihood approach answers the question "How *likely* (in a relative sense) are the parameter values given the data?", while the Bayesian approach answers the question "How *probable* (in an absolute sense) are the parameter values, given the data *and previous knowledge of the parameter values*?" The Bayesian answer, the posterior probability distribution $P(H|x)$, is computed via Bayes' formula,

$$P(H|x) \propto P(H)\, P(x|H),$$

where P(H) is the prior distribution, representing what is known about the hypothesis (parameters) prior to the data, and P(x|H) is the likelihood. The change between P(H) and P(H|x) can be viewed as the information content of the data (Gelman et al. 2013), and it can also be viewed as a measure of how much one "learns" by doing an experiment or collecting data (Ellison 1996).

The form of the likelihood function is often determined by the data. For example, the heights or lengths of members of a population may follow a normal or Gaussian distribution with parameters μ and σ^2 that describe the mean and variance of the distribution. The sexes of individuals in a group could be described with a binomial distribution with parameter θ that describes the probability that an individual is male. The ages or weights of members of a population may have a positively skewed distribution, which could be represented by a lognormal or gamma distribution, again described with mean and variance parameters.

The most straightforward example of the concept of updating occurs with time series models. Many populations are monitored through time (annual counts, for instance), and typical quantities of interest are population size or a trend in population size. Naturally, the model may be run after all data are collected, in which case all data are included in the likelihood function. Alternatively, the model may be run sequentially: when the population count is made at time t, the likelihood can be evaluated at this single data point, with priors specified as the posterior distributions from the model up to time t-1 (Wright et al. 2002, Goodman 2009). The result of analyzing the data sequentially agrees with analyzing the data all at once only in a system which follows the rules of conditional probability. Such coherence was one of the reasons that Goodman felt that using Bayesian methods offered the best legitimate inference from available data.

2. CONCEPTS OF PROBABILITY

Probability has been defined as a limiting frequency, which can be called an objectivist or frequentist point of view, but it has also been defined as a degree of belief, which is considered a belief-based or subjectivist point of view (Bernardo and Smith 2000). Historically, the objectivist view has been associated with classical statistical methods, and the subjectivist view with Bayesian methods. Goodman espoused a view which combined the objective rigor of a frequency-based concept of probability with the inferential coherence of the Bayesian approach (Goodman 2002b)

The concept of probability as limiting frequency is easily exemplified by flipping a fair coin. On a single flip, the proportion of heads can only be 0 or 1, not 0.5. In a series of ten flips, the proportion can still be far from 0.5. However, in the limit, that is, as the number of flips becomes larger and larger, the proportion of flips that comes up heads becomes closer and closer to 0.5, leading to the conclusion that the probability of heads is one half. Thus the frequency-based definition of probability is framed in terms of a very large number of replications under constant conditions. In practice this is a problem because, unlike simple coin-flipping, we cannot increase our sample size or replicate our experiments until we see a series of numbers converging on a true probability.

On the other hand, the view that probability represents a degree of belief which varies from person to person is incompatible with the scientific method. In particular, a personal belief-based view of knowledge is unacceptable for assessing risk and making decisions on issues of public policy. As Goodman put it "Subjective probability can lead to internally consistent systems relating belief and action for a single individual; but severe difficulties emerge in trying

to extend this model to justify public decisions. Objective probability represents probability as a literal frequency that can be communicated as a matter of fact and that can be verified by independent observers confronting the same information" (Goodman 2004).

However, the belief-based concept of probability also maintains that the state of our knowledge should be reflected by probabilities (De Finetti 1970a, b). De Finetti (2008, p. 212) said that it is "senseless to speak of the probability of an event unless we do so in relation to the body of knowledge [already] possessed". Representing state of knowledge by a probability distribution does not mean that knowledge (or "belief") has to be personal (or "subjective"). It can be objective and empirical, and this is what Goodman tried to implement. Probability as the state of knowledge may be most easily imagined as the assignment of values that account for all relevant information (or lack thereof) before placing a bet. For example, an intuitively appealing expression of complete ignorance is to assign equal probabilities to all possible outcomes. In the case of flipping a fair coin, assigning equal probabilities to heads or tails is one way of expressing complete ignorance, and it would, in fact, give us the best information with which to assess a bet based on the toss of a fair coin. However, the task of assigning a probability to all outcomes, or even defining all possible outcomes, quickly becomes complicated. Suppose we suspect that the coin flipper may be a charlatan, and that his jar of coins contains some unknown proportion of fair coins, weighted coins (in which case there can be an infinite number of weightings) or trick coins (same face on both sides). Now our assessment of possible outcomes must include some assessment of what may be in the jar, which, in turn, depends on our assessment of how likely the coin flipper is to be a con artist.

The Bayesian characterization of state of knowledge as a probability is a belief-based tenet, yet Bayesian inference does contain elements of frequency-based probabilities, provided the likelihood function contains a sample of data which were generated by an inherently random process. In contrast to the classical statistician, the Bayesian considers the parameter itself to be a random variable. A Bayesian is interested in realizations of that parameter, as opposed to multiple realizations of the data. The different questions posed by Bayesian and classical methods reflect the Bayesian interest in answering questions within the parameter space (and resulting willingness to represent knowledge, or its converse, uncertainty, as a probability) and the classical preference for a strict frequentist definition of probability (and resulting choice of inference to the data space). The conceptual difference can be illustrated succinctly by considering a single flip of a fair coin. After the coin is flipped, the coin is covered so the result can't be seen. The question is: "What is the probability that the result is heads?" A Bayesian says 0.5, because that probability represents our knowledge of the result. A classical statistician should refuse to answer the question, because it is an improper use of "probability." In the classical view, there is no uncertainty: the result is either heads or tails with probability one.

Goodman's resolution of objectivist and subjectivist concepts of probability sought to combine the strengths of both approaches. He argued that a Bayesian analysis could fulfill both the belief-based tenet of representing the uncertainty associated with making a decision (*i.e.*, correctly assessing the risk and placing a rational bet) and a frequency-based definition of probability, because the frequency of interest was not based on hypothetically unlimited replication over the data space but rather on the frequency of estimated parameters in an indefinitely large sample from the prior parameter space (Goodman 2002b, 2004). In other words, if the prior parameter space were sampled enough by previous research, then a prior distribution based on these comparative parameter estimates should converge to the unknown parameter's true distribution. Bayes' formula then updates the frequency-based prior distribution

with the case-specific data of interest, producing a fully frequency-based posterior, which is the distribution of the parameter, given that the data were exactly as observed (Goodman 2004). Thus, Goodman emphasized "taking the prior seriously" by using an informative prior driven by an empirical frequency of actual prior cases. In Section 5 we discuss ways of doing this, including the situation where case-specific data are limited, and in Section 6, we give an example of using informative, data-driven priors to strengthen an analysis.

3. BAYESIAN STATISTICS AND DECISIONS

Decision making in natural resources involves uncertainty, risk, and updating knowledge with new data. As Goodman (2002b) noted, "each management decision is equivalent to the placing of a bet, and the measure of success isn't so much winning every bet, which is impossible, but rather, success lies in evaluating the odds well enough to place our bets so that in the long run our gains exceed our losses." Thus, the key element science can contribute is an improved understanding of the odds. These odds may generally be interpreted as the effects of management actions (including the status quo) in setting harvest levels, setting recovery targets, or deciding whether species should be given protected status. If uncertainty is properly represented, Goodman maintained that the decision process could, in its own way, be an exact science despite the uncertainty involved in these management decisions (Goodman 2002b). Similarly, effective policy development can be viewed as a scientific research enterprise (Goodman 2005).

A scientific approach to management decisions weights value statements (utility functions) about potential outcomes by the probabilities that they will occur (Berger 1985). Thus, the proper estimation of uncertainty, of both current and future states of nature, is central to the science of decision-making. In nearly all ecological studies, measurement uncertainty is large because of observation or sampling errors. For example, many species exist in complex food webs, and our understanding of these linkages, as well as the role of environmental variation, is poor. Further, because natural processes are inherently stochastic, prediction of future states is uncertain even if the current state and the processes leading to future states are known. Bayesian methods allow future uncertainty to be partitioned between the component due to inherent stochastic processes and the component due to parameter uncertainty (Goodman 2002a). Thus, risk can be reduced by the collection of more data (Goodman 2002b).

Estimation of the state of nature depends on data and statistical models. More (and better) data allow increasingly complex statistical models to be developed. In the simplest form, annual population counts can be analyzed in a linear regression framework (Goodman 2009), and at their most complex, advanced fisheries stock assessment models can be built, incorporating multiple types of data and complex population processes, such as movement and variation in recruitment (Hilborn and Walters 1992). Regardless of model complexity, all population models are capable of estimating the current population size, as well as forecasting future population sizes (*e.g.*, PVAs for extinction risk assessment).

The key difference between Bayesian and other methods is how uncertainty is treated in the models. Goodman talked about the *correct* calculation of uncertainty (Goodman 2002b, 2004). Just as we cannot win every bet, we cannot always correctly estimate the true state of nature. What we can do on each occasion, however, is to account correctly for what we know and what we do not know. Goodman felt strongly that Bayesian methods were the way to do this, and he did not hesitate to use words like "correct," "honest," and "legitimate" to describe the Bayesian accounting of uncertainty via the rules of probability.

While classical statistical methods (*e.g.*, likelihood ratio tests, p-values) have been used to make decisions about natural resource management, there are critical problems in using these methods, often with arbitrary significance levels, to make decisions. Small significance levels (*e.g*,. $\alpha = 0.05$) have been shown to over-penalize certain decisions, leading to management costs that are upwardly biased relative to optimal decisions (Field et al. 2004). Statistically, because these classical methods do not generate probabilities of states of nature, they cannot be used to express the expected utility of management actions. Because of this limitation, Bayesian methods provide the only statistical framework that is compatible with decision analysis – both in the natural resources and other fields (Goodman 1999).

Translating statistical output from a Bayesian analysis to decisions about natural resources requires several key elements (Punt and Hilborn 1997, Goodman 2002b). First, there must be uncertainty about the true state of nature (*e.g.*, population abundance). Second, a natural resource manager must specify a range of discrete management actions. For harvested fish stocks, these may involve alternative harvest regimes, and for species of conservation concern, this may involve various restoration actions or protections. Third, an expected utility function must be specified. The utility function represents the benefit of each action, and must be expressed in a common currency, such as dollars, biomass, expected catch, species diversity or relative ranking (Possingham et al. 2002, Burgman 2005), for each management action. Equivalently, a loss or cost function may be specified as a penalty for each decision (Dorazio and Johnson 2003). The fourth component of a decision analysis is the probability distribution of alternative states of nature (*e.g.*, population abundance, stock biomass). Given the expected utility (or cost) of each management action and the posterior probabilities of states of nature, the utility (or cost) of each management action can be calculated (Hilborn et al. 1994, Punt and Hilborn 1997). The majority of Bayesian decision analyses to date have been in a single species framework, but ecosystem-based management applications may require more complex multi-criteria utility functions (Mendoza and Martins 2006).

4. GOODMAN'S EMPIRICAL APPROACH

Goodman sought an approach that combined the mathematical rigor of the Bayesian process with empirical observations. A central issue is the prior. Most Bayesian analyses use "weakly informative" or "vague" priors. (The term "noninformative" is sometimes used, but all priors contain information.) The idea behind vague priors is to minimize the influence of the prior, so that results depend only on the data for the case at hand. In a sense, trying to minimize the influence of the prior does not fully embrace the Bayesian paradigm. Goodman, who did embrace the paradigm, advocated using all available data, and that included before-the-fact comparative data. This is called an "informative" prior.

With a large amount of case-specific data, the likelihood term in Bayes' formula can overwhelm the prior, and the influence of the prior, even an informative prior, is small. With sparse data, however, an informative prior can strongly influence the posterior. To avoid "running aground on the rocks of subjective probability" (Goodman 2002a), Goodman advocated that an informative prior should be based, whenever possible, on "an empirical histogram of comparative information obtained from actual experience" (Goodman 2002b). Such an empirical prior should be based on a large number of "similar" cases (Goodman 2004, 2009). Goodman acknowledged, however, that the construction of such a data-based prior was difficult. In the next two sections we discuss construction of data-based priors and give an example based on Steller sea lion spatial use patterns.

A simple example may help make these abstract ideas concrete. Suppose we have to make a decision which depends on the number of dolphins in an area. There are some data about the density of animals in the area, but the data are sparse. Consequently our knowledge of the number of dolphins is quite uncertain. What should we do? One solution is to make the best estimate we can with the available data from the area, together with an honest reporting of the large uncertainty. But another possibility, and one Goodman would have advocated, would be to bring in comparative data, via the prior, about dolphin density in other areas. The comparative data, combined with the local data using Bayes' formula, should produce an improved estimate. It is important to note, however, that "comparative" is an imprecise term, and care should be taken in choosing the data that are used to construct a prior (Goodman 2004)

Bayes' formula is not the only way to combine previous and current data. We could combine a previous estimate of population size and a new estimate by simply taking the average of the two, for example. Or we could combine them in some way that weights one estimate more than the other, say by using the inverse of the variances of the estimates. Which way of combining the estimates is best? Bayes' formula tells us how to do it in a recognizably optimal way which is consistent with the laws of probability. The idea of updating the state of knowledge with new information is so natural and intuitive that the concept has been repeatedly re-discovered in various business, military and scientific situations, sometimes without realizing that it was Bayesian (McGrayne 2011).

5. METHODS FOR INFORMING PRIORS: EXPERT OPINION, EMPIRICAL AND HIERARCHICAL BAYES

Much of Goodman's work involved examples with limited data. When case-specific data are limited, Bayesian analyses can be used to guide decision-makers by incorporating comparative data to inform prior distributions. Comparative data may come from multiple data cases within the same dataset or from a meta-analysis of multiple data sets(Myers et al. 2002). As the complexity of population models has increased, particularly in fields such as fisheries, some model parameters may not be estimable for data-limited stocks. In these cases, meta-analyses have been developed for specific model parameters (Liermann and Hilborn 1997, Helser and Lai 2004, Thorson et al. 2013), and posterior distributions from these meta-analyses can be turned into priors at different focal levels (*e.g.*, population, species, genus) for the case of interest.

"Empirical Bayes" methods encompass a wide collection of techniques, all of which estimate the parameters of the prior distribution (hyperparameters) using non-Bayesian procedures (Casella 1992). A simple example of parameterizing an informative prior distribution through empirical Bayes methods is to equate the mean and variance of the prior distribution to the sample mean and variance of the data. While it is possible to use empirical Bayes methods to construct an informative prior from strictly comparative, not case-specific, data (Goodman 2004, Gelman et al. 2013), the term "empirical Bayes" often refers to analyses in which the case-specific data are used both in the likelihood function, and again with the comparative data, when estimating the hyperparameters (Carlin and Louis 2000). The word "empirical" in the phrase "empirical Bayes" describes a method and should not be confused with our description of Goodman's "empirical approach". Goodman's use of the term meant that prior information should be based on data rather than a belief system.

The fully Bayesian method of informing prior distributions is termed "hierarchical Bayes" (Gelman et al. 2013). In a hierarchical Bayes analysis, each hyperparameter is itself

drawn from a distribution called the hyperprior (Figure 2). The hyperprior is parameterized by fixed values that are generally chosen to give broad coverage across the hyperparameter support. This allows the comparative data, rather than the fixed values chosen for the hyperprior, to contribute the most information to the prior distribution. Notably, a hierarchical Bayes analysis can be viewed either as an inference on a single case-specific parameter, with an informative prior based exclusively on comparative data, or as a joint inference on the hyperparameter, using all of the data cases together. Some consider empirical Bayes methods approximations to a hierarchical Bayes analysis (Gelman et al. 2013), to be implemented when computational requirements or the format of the available data prohibit a hierarchical analysis.

In agreement with hierarchical Bayes theory, Goodman thought that case-specific data should be used exactly once (Goodman 2004). Above all, however, he emphasized the need to ground the prior in data. For practical reasons, empirical Bayes techniques might have to be used, but in these instances the amount of comparative data should be sufficient so that double inclusion of case-specific data does not influence the results of the analysis (Goodman 2004).

6. AN APPLICATION TO STELLER SEA LION SPATIAL USE

While the fully Bayesian analysis advocated by Goodman is theoretically sound, implementation remains challenging due both to limitations of computational power and to the difficulty in acquiring data or appropriate relationships for priors. Raw data may not be available to perform a hierarchical analysis, or it may take a considerable amount of time to gather such data, even when found in publications. Appropriate priors can sometimes be found from only one previous analysis, but those results may be reported without uncertainty. Oftentimes a data-informed prior simply does not exist. Expert opinion based on well-thought-out expert solicitation can aid in building priors (Martin et al. 2005, Choy et al. 2009), but the process can be time-consuming. Partly because of such challenges, the practical application of Goodman's approach to Bayesian analyses has only just begun, at least in ecology. Indeed, Goodman himself had yet to implement fully a hierarchical Bayes model with priors based on data from similar cases.

Bayesian analyses that obtain multiple data cases from within the same dataset are a potentially more tractable way to inform priors with data, and their use is increasing (Cressie et al. 2009, Ogle 2009). Although this requires large datasets, priors for one case are derived from other cases within the dataset. The following empirical Bayes example, inspired by Goodman's teachings, exemplifies how a Bayesian analysis using multiple cases from within the same data set provides us with answers to important ecological questions. In this example, data were available across a broad spatial domain, but inference was conducted at a much smaller local scale. As a result, point estimates of hyperparameters were calculated from the entire dataset, and the data-driven priors were then combined with the local case-specific data to derive posterior distributions. Given the large sample size (> 18,000 cases), a hierarchical analysis was not tractable. Details of this analysis follow.

In April 1990, National Marine Fisheries Service (NMFS) listed the Alaska Steller sea lion (*Eumetopias jubatus*) as Endangered due to a rapid and dramatic decline in abundance in the western portion of its range (75% decline over 14 years). NMFS decided in 1997 to split the species into two distinct population segments (Western and Eastern DPS, divided at 144°W) based on further declines in the western region, as well as information about genetic and morphological differences between the populations. Increasing numbers in a large portion of the range led NMFS to delist the Eastern DPS in October 2013, whereas in some areas the Western

DPS continues to decline (NOAA 2013). The Steller sea lion was a species on which Goodman attempted to put his ideas into practice (Himes Boor and Wade 2015).

Understanding the distribution of the Western DPS is of paramount importance when determining critical habitat and potentially limiting fisheries activities in response to the needs of the sea lions. Until recently, critical habitat had been mostly determined via satellite telemetry data. While the telemetry data were comparatively rich (n = 302), those data did not represent all age classes or sexes equally nor cover the extensive range of the Steller sea lion (Himes Boor and Small 2012). Using an empirical Bayes approach, Himes Boor and Small (2012) were able to quantify Steller sea lion relative spatial density using platform of opportunity (POP) data that covered extensive ranges of the Pacific Ocean and was not selective with respect to age or sex.

The POP data analyzed in Himes Boor and Small (2012) were assembled by NMFS and spanned years 1958 – 2000. The data represented standardized records of marine mammal at-sea sightings from many types of survey platforms (vessels, aircraft, shore) and by observers with varying degrees of training. While data were well vetted for quality control, two analytical challenges presented themselves. First, effort was never recorded, so data were not available for cases where an observer was looking in an area, but no animals were seen. Second, surveys were not conducted in a systematic format, with some areas receiving frequent visits by opportunistic platforms, and other areas few or none.

To address both issues, Himes Boor and Small (2012) divided the North Pacific Ocean into 15 x 15 km cells and limited their analysis to cells with any marine mammal sighting records (not just Steller sea lions). A sighting within a cell indicated some form of effort occurred in that cell (N = 18,321). Each cell could have sightings from multiple platforms and/or multiple days, called platform-days. From these data, the posterior distribution of Steller sea lion encounter rate (number of animals seen per day, y) was estimated for each cell j with i platform-days. The likelihood is a negative binomial distribution on data y_j with unknown parameters m_j and k_j.

$$P(y_j|m_j,k_j) = \prod_{i=1}^{n_j} \frac{(y_{ij}+k_j-1)!}{y_{ij}!\,(k_j-1)!}\left(\frac{k_j}{m_j+k_j}\right)^{k_j}\left(\frac{m_j}{m_j+k_j}\right)^{y_{ij}}$$

where n is the number of platform-days. The number of platform-days varied by cell, between 1 and 236. To build an informative prior for m_j, which represents the mean of the negative binomial distribution, Himes Boor and Small (2012) calculated the mean encounter rate for each cell

$$\bar{y}_j = \frac{\sum_{i=1}^{n_i} y_{ij}}{n_i},$$

and fit a gamma distribution to the histogram of the mean encounter rates across all cells. The parameter values from the fitted gamma distribution were used as the hyperparameters of the gamma prior. To create a prior for the negative binomial dispersion parameter k_j, the authors calculated the method of moments estimate (MME) across all cells ($\hat{k}$), and used it as the mode of a gamma distribution prior

$$\hat{k} = \frac{\bar{Y}^2}{S^2-\bar{Y}},$$

where $\bar{Y} = \frac{1}{c}\sum_{j=1}^{c}\bar{y}_j$, $S^2 = \frac{1}{c}\sum_{j=1}^{c}\left(\bar{y}_j-\bar{Y}\right)^2$, and c is the total number of cells. The variance of the gamma distribution was set to 100 to allow for the wide range of values for k_j calculated in the individual cells j for which the MME could be obtained. This led to prior distributions of the negative binomial parameters $m_j \sim Gamma(\alpha=0.0216, \beta=0.0396)$ and $k_j \sim Gamma(\alpha=1.001558,$

β=0.1), and the parameters of interest were m_j. In cells with few platform-days, *i.e.*, few data, posterior distributions of encounter rate estimates were similar to the prior (Figure 3). In such cases the use of a well-informed prior was particularly advantageous.

Results indicated that while current delineations of critical habitat do encompass areas of high sea lion encounter rates, many high encounter rate areas are not defined as critical habitat (Figure 4). This analysis provides an excellent example of using empirical Bayes methods to approximate a hierarchical Bayes analysis when computational requirements made the latter intractable. Notably, the large sample size (>18,000 data points) that complicated the use of a hierarchical analysis also made results of the empirical analysis robust to using both the case-specific data and the comparative data in estimating the gamma hyperparameters. In other words, the contribution of the encounter rate observed in any particular cell to the prior distribution would be negligible given that the prior was determined by >18,000 values. In addition, a hierarchical Bayes analysis would not have changed the important result that potential areas of high sea lion use were outside the boundaries of defined critical habitat. The informative prior was specifically useful in identifying potential critical habitat in areas where sampling was not as frequent. Without the informative prior, posterior uncertainty in encounter rates in these regions would have been higher, providing less information for the decision-making process.

CONCLUSIONS

In this document we, as Dr. Goodman's students, have tried to summarize his thoughts about how Bayesian statistics should be used for risk assessments and for making decisions for natural resource management. For all of us, Goodman influenced the way we think about and use Bayesian methods for the analysis of ecological data. We hope that this article may inspire some readers to consider Bayesian statistics as a useful and viable approach in risk assessment, ecology and other fields. In Goodman's view, Bayesian statistics was the only legitimate way to deal with uncertainty, correctly and honestly.

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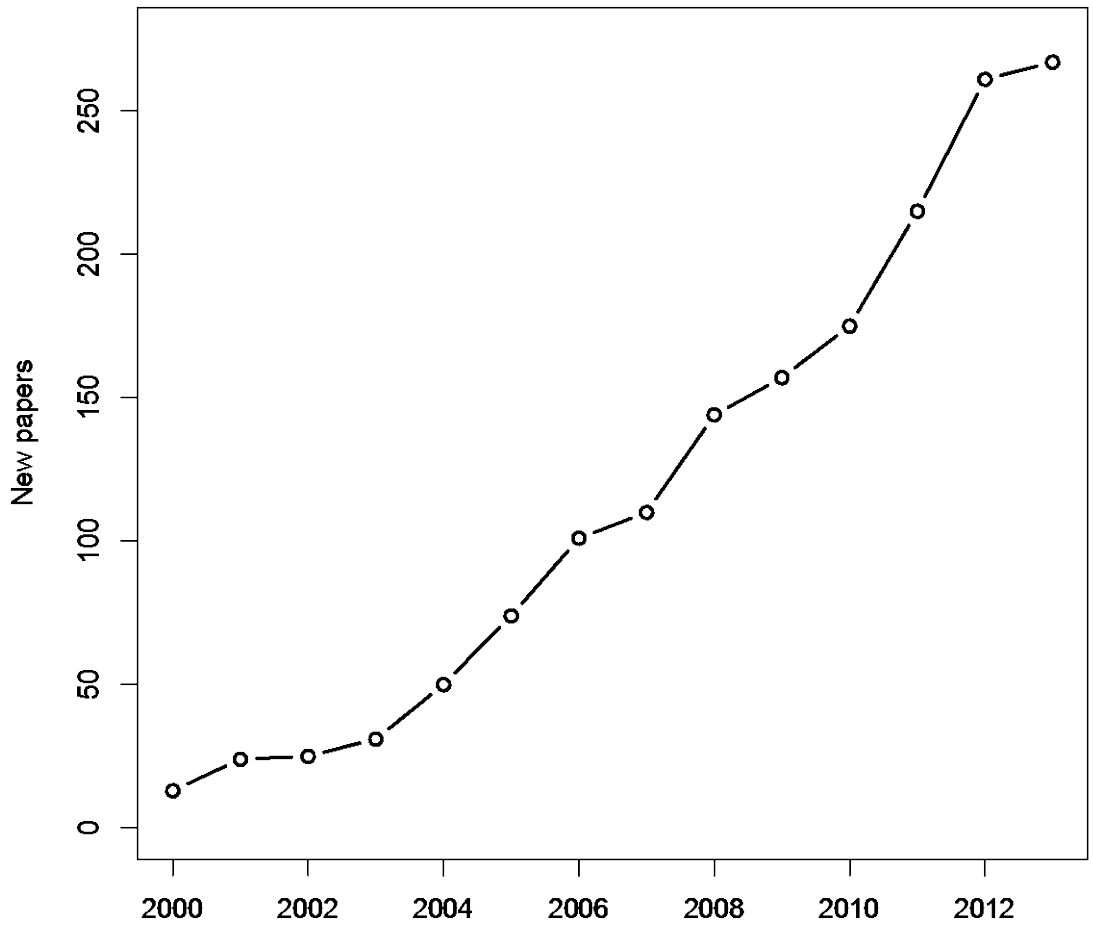

Figure 1. Web of Science citations with “Bayesian” and either “ecology” or “fisheries” in the title, abstract or keywords, 2000-2013.

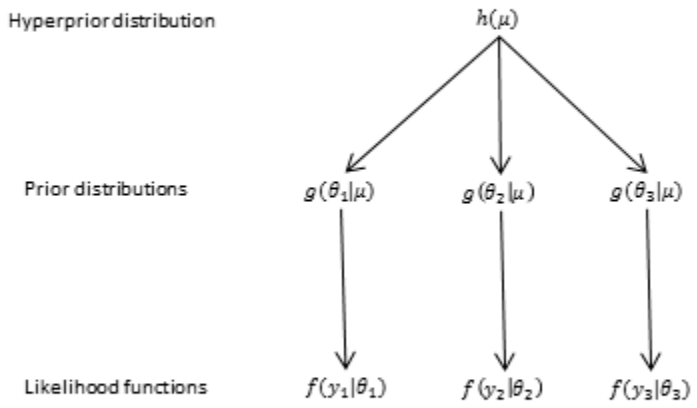

Figure 2. Hierarchical Bayes set-up. Each data set, y_i, is generated by a case-specific parameter, θ_i. Each parameter, θ_i, is drawn from a prior distribution governed by a hyperparameter, μ, and μ is drawn from a hyperprior distribution governed by fixed values.

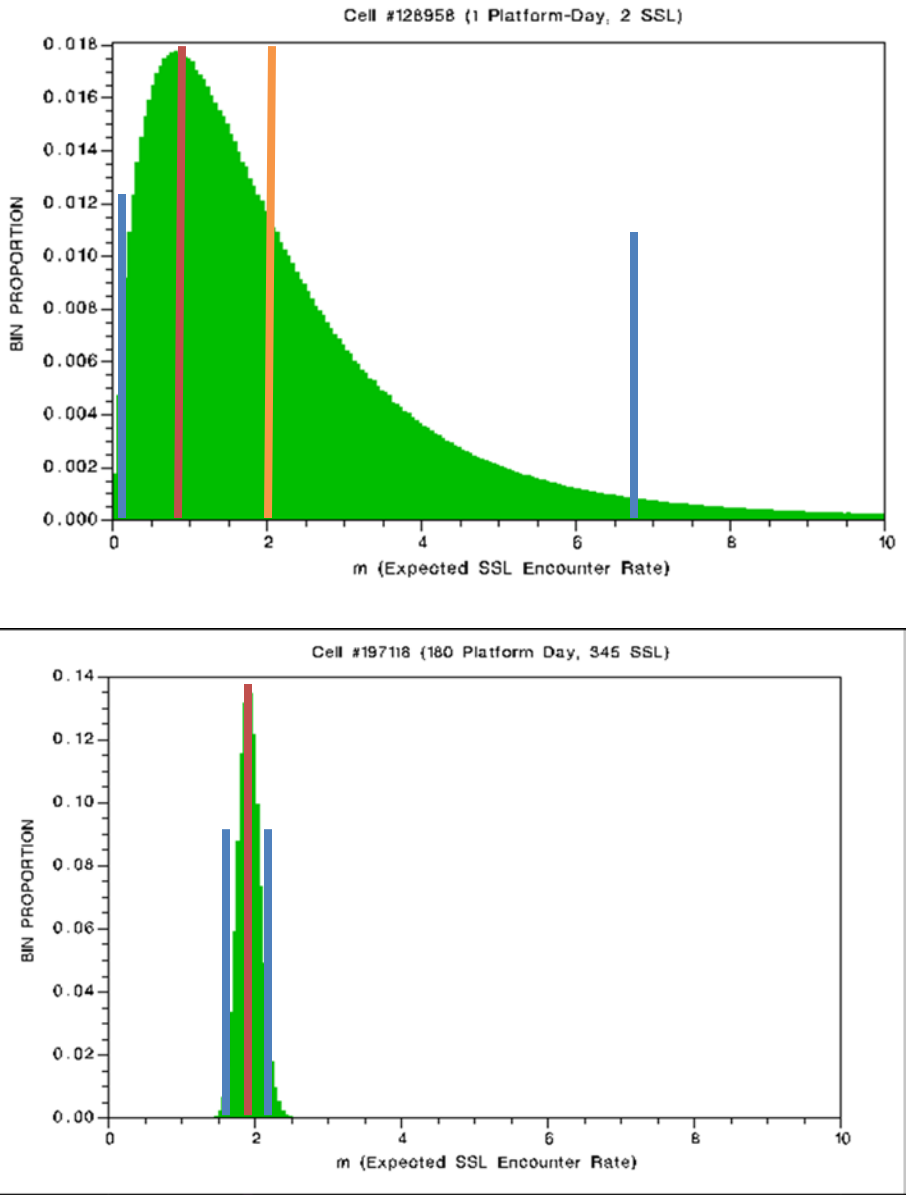

Figure 3. Posterior distribution of mean Steller sea lion encounter rate given different number of platform days: a) one platform day with two sea lion sightings, b) 180 platform days with 345 sea lion sightings. Orange line indicates the point estimate of encounter rate. Red lines are posterior modal values, and blue lines bound the 95% posterior interval.

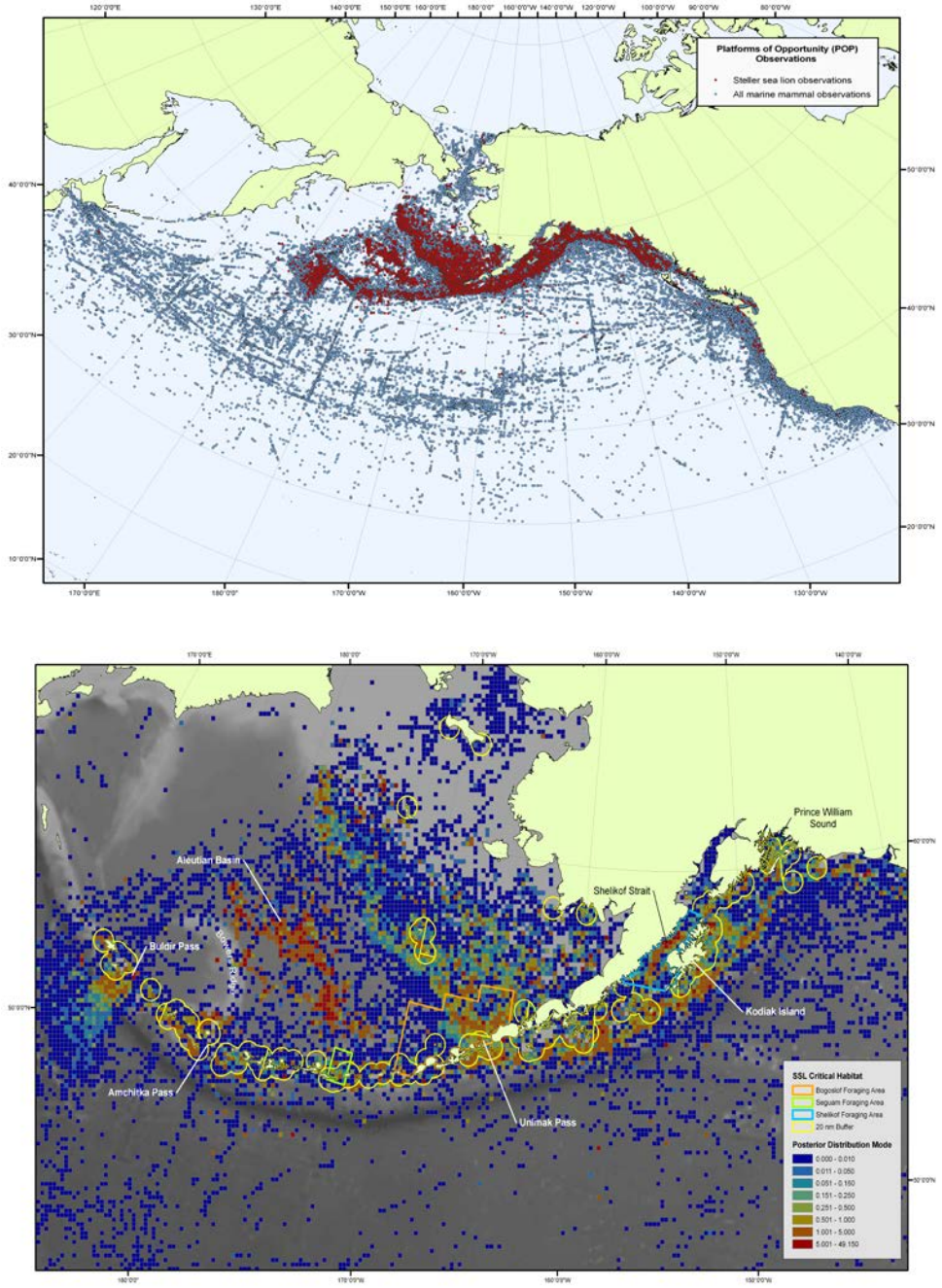

Figure 4. a) Sea lion sightings (red dots) and marine mammal sightings without sea lions (gray dots) from platform of opportunity data. b) posterior modes of sea lion encounter rate for each grid cell.

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