# Recognition of emotions using Kinects

Shun Li<sup>1,2</sup>, Liqing Cui<sup>1,2</sup>, Changye Zhu<sup>3</sup>, Nan Zhao<sup>1</sup>, Baobin Li<sup>3</sup>, Tingshao Zhu<sup>1,4</sup>

<sup>1</sup> Institute of Psychology, Chinese Academy of Sciences, <sup>2</sup> The 6th Research Institute of China Electronics Corporation, <sup>3</sup> University of Chinese Academy of Sciences, <sup>4</sup> Institute of Computing Technology, Chinese Academy of Sciences

#### Abstract

Emotion recognition can improve the quality of patient care, product development and human-machine interaction. Psychological studies indicate that emotional state can be expressed in the way people walk, and the human gait can be used to reveal a person's emotional state. This paper proposes a novel method to do emotion recognition by using Microsoft Kinect to record gait patterns and train machine learning algorithms for emotion recognition.

59 subjects are recruited, and their gait patterns are recorded by two Kinect cameras. Joint selection, coordinate system transformation, sliding window gauss filtering, differential operation, and data segmentation are used for data preprocessing. We run Fourier transformation to extract features from the gait patterns and utilize Principal Component Analysis(PCA) for feature selection. By using NaiveBayes, RandomForests, LibSVM and SMO classifiers, the accuracy of recognition between natural and angry emotions can reach 80%, and the accuracy of recognition between natural and happy emotions can reach above 70%. The result indicates that Kinect can be used in the recognition of emotions with fairly well performance.

Keywords: emotion recognition, Microsoft Kinect, gait patterns

October 5, 2015

#### 1. Introduction

Emotion is a state that comprehensively represents human feeling, thought and behavior, thus takes an important role in inter-personal human communication. Emotion recognition aims to automatically discriminate different emotional states by using physiological and non-physiological signals acquired from people. In patient care, emotion recognition can find patients' different emotional states ,then help take corresponding nursing intervention. In product development, emotion recognition can find users' different emotional states when they use different products, then help improve the products. Emotion recognition can also improve the performance of human-machine interaction and to increase intuitive communication[1], make human-machine interaction more friendly and naturally.

Emotion identification is generally based on facial expressions, gestures, linguistic as well as acoustic feature in speech[2] etc. Body motion is regarded as additional modality to identify emotion states. Affective body movement provides important visual cues to distinguish emotions[3]. Since walking is a natural day to day motion, human gait is an ideal way to reveal a persons emotional state. In this paper, we propose to recognize emotions based on gait patterns.

Psychological studies found that emotional states can somehow be expressed in the way of walking. In recent years, much research has been done to analyze general gait patterns, which is very challenging, since gait is as unique as fingerprint[4]. Furthermore, gait can be influenced by many factors such as age, weight, and possible gait disorders. How such factors or their combinations affect gait is still unknown. So recognizing emotion by gait patterns' analysis is pretty difficult.

In this paper, we propose a novel method to recognize emotions by using Microsoft Kinect to record gait patterns and train machine learning algorithms for emotion recognition. The Microsoft Kinect is camera-based sensor primarily used to directly control computer games through body movement. The track of the position of limb and body without the need for handheld controllers or force platforms. Kinect uses a depth sensor to capture three-dimensional movement patterns. The system's software enables feature extraction to recognize body joints. Differing from conventional motion capture systems, such as the VICON system, Kinect is low-cost[5], portable, no-marker[6], and easy to deploy[7].

We recruit participants and collect the timing sequence data of people's gait patterns by using Kinect system, then use machine learning method for feature extraction and training classifiers.

The remainder of this paper is organized as follow. Section 2 introduces the related work of recognition of emotions in walking and the Kinect application. Section 3 describes the method of our work, including the experiment, the database, and the data processing. Classification and results are presented in section 4 and discussed in section 5. The paper ends with a conclusion in section 6.

## 2. Related work

In psychology, evidence exists that emotion can be expressed in walking and recognized by human observers. In 1987, Montepare et al. found that observers can identify emotion from variations in walking style[8]. In 2008, Janssen used the conventional camera to acquire kinetic and kinematic data and do recognition of emotions in gait[9]. Krag made person-dependent and inter-Individual recognition of emotions by marker-based gait analysis using motion capturing system[10, 11, 1]. Although conventional 3-dimension video-based motion analysis systems allow for comprehensive kinematic and kinetic analysis of gait, they require considerable expertise and are expensive. And the marker-based motion capture systems (i.e., the VICON system) require precise, tedious and time-consuming maker preparation, which may affect the subjects' emotional states and also expensive.

Low cost options could include inertial monitoring sensors such as accelerometers [12] and gyroscopes, however, these sensors possess sources of error such as signal drift and noise which impedes their accuracy[13].

Microsoft Kinect is a rapidly developing, inexpensive, portable and nomarker motion capture system. Previous research suggests that Kinect can identify pose[14] and simple stepping movements[15] in healthy adults. Recently, clinical researchers have reported interesting applications using Kinect. For example, an interactive game-based rehabilitation tool for balance training[16] and a 3-D body scanner[17]. And methods have been proposed to detect gait patterns in walking data obtained with Kinect: in 2014, Auvinet used Kinect to detect the gait cycles in treadmill[18]; Yeung evaluated the Kinect as a clinical assessment tool of body sway[19]; Galna used Kinect to measure the movement in people with Parkinson's disease[20].

Previous studies also have validated the Kinect as a motion capture system. Accuracy and sensitivity of kinematic measurements obtained from Kinect, such as reaching distance, joint angles, and spatial-temporal gait parameters, were comparable to a VICON system[21]. Evidence exists that it can accurately assess the gait patterns dynamics during walking[22].

Given the emotion can be indicated by the gait patterns, and Kinect is able to acquire the gait patterns without any interference, we propose to identify emotion by using gait patterns acquired by Kinect.

#### 3. Methods

#### 3.1. Experiment

59 healthy young subjects (32 females and 27 males) from University of Chinese Academy of Sciences (UCAS) participated in this study. They reported no injuries, illnesses or other condition influence their gait patterns. This study was approved by Institute of Psychology, Chinese Academy of Sciences (approved number: H15010), and all subjects provided informed consent.

There was a 6 meters long footpath in the experiment environment, and two Kinects were placed at the two sides of the footpath. The experiment environment is shown in Figure 1 and Figure 2.

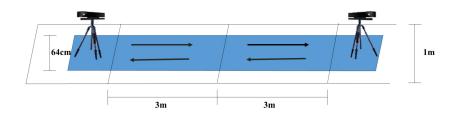


Figure 1: the description of the experiment environment

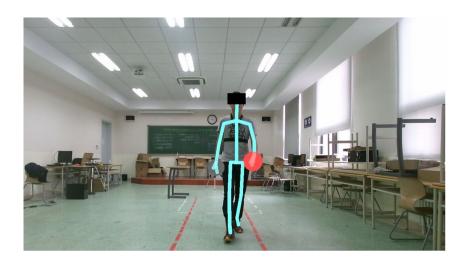


Figure 2: the scene of the experiment environment



In order to acquire subjects' gait patterns with different emotions, we conducted two-round experiments. Subjects took part in each round of experiment one by one. At the beginning of each round, subject was instructed to stand in front of the starting line which was located at one side of the footpath. Then the subject kept walking back and forth on the footpath for 2 minutes. At the same time, two Kinect cameras recorded the subject's gait patterns. When the subject finished walking, he/she was asked to report his/her current emotional score on a scale of 1 to 10. In the first-round experiment, subjects reported their scores of anger. In the second-round, happy scores were reported instead. Then the subject watched a film clip[23] for emotion priming, since previous studies have validated that the film can elicit people's emotion [24]. At the first-round experiment, the film clip attempted to arouse subjects' angry emotion. There was a happy emotional film clip instead in the second-round experiment. After watching the file clip, the subject continued walking back and forth on the footpath for another 1 minute. Each subject was asked to report his/her current emotional score on a scale of 1 to 10 when finished the second walking, and recall the emotional score just after watching the film clip. To ensure the emotion aroused by the film clip can last during subjects' walking, we didn't ask subject to report his/her emotional score immediately when the subject finished watching the film clip. To avoid the subject's emotional states in the first-round experiment influence his/her gait in the second-round experiment, each subject was allowed to do the second-round experiment at least 3 hours later. We acquired the subjects' gait patterns from two Kinects separately.

## 3.2. Database

The Kinect cameras are placed at the two sides of the footpath, 30Hz video data is acquired from each Kinect camera using the official Microsoft software development SDK Beta2 version and customized software (Microsoft Visual Studio 2012). One frame data contains the 3-dimensional position of 25 joints over time. The 25 joints include head, shoulders, elbows, wrists, hands, spine (shoulder, mid and base), hips, knees, ankles and feet as shown in Figure 3.

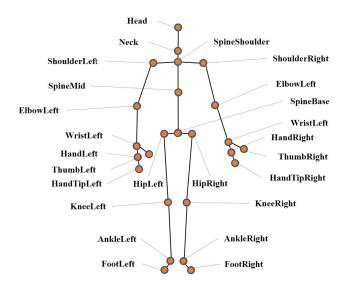


Figure 3: Stick figure and location of body joint centers estimated by Kinect

The 3D coordinate system of Kinect use the Kinect camera as the origin and the unit of 3 dimension is meter. The 3D coordinate system of Kinect is shown in Figure 4.

# 3.3. Data processing

#### 3.3.1. Preprocessing

Joint selection. According to sport anatomy theory, some joints' position doesn't change much while walking, so we choose 14 significant joints to analyze gait patterns, including spinebase, neck, shoulders, wrists ,elbows, hips, knees, and ankles. The spinebase joint is used to reflect people's position on the footpath relative to Kinect, and will be used in coordinate system transformation. After selecting the significant joints, one frame data contains the 3-dimmension position of 14 significant joints, which affords a 42 dimension vector:

$$j_t = [x_1, y_1, z_1, x_2, y_2, z_2, \dots, x_{14}, y_{14}, z_{14}]$$
(1)

We denote one walk as the way that one subject walks around the footpath

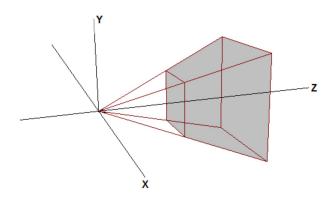


Figure 4: Kinect 3D coordinate

one time (with an emotional state), which consists of T frames. The data of one walk is described by the matrix:

$$J = [j_1, j_2, \dots, j_t, \dots, j_T]^T$$
 (2)

Coordinate system transformation. Given that different subjects may have different position relative to Kinect camera when they walk on the footpath, so using Kinect coordinate system has error in gait patterns' analysis. To eliminate the bias, we change the coordinate system by use the position of spinebase joint in each frame of data as the origin.

In the vector  $j_t$ , the first three columns are the 3-dimension coordinates of spinebase joint, so the coordinate transformation is given by:

$$x_{i}^{t} = x_{i}^{t} - x_{1}^{t}$$

$$y_{i}^{t} = y_{i}^{t} - y_{1}^{t}$$

$$z_{i}^{t} = z_{i}^{t} - z_{1}^{t}$$

$$(1 \le t \le T, 2 \le i \le 14)$$
(3)

Sliding window gauss filtering. The gait patterns' dataset acquired by Kinect system has noises and burrs. To smooth the dataset, we apply sliding window

gauss filtering to each column of J, the length of the window is 5 and the convolution kernel c = [1, 4, 6, 4, 1]/16, which is frequently-used low pass gauss filter[25].

The procedure of filtering is presented as follows:

$$\begin{aligned} x_i^t &= [x_i^t, x_i^{t+1}, x_i^{t+2}, x_i^{t+3}, x_i^{t+4}] \cdot c \\ y_i^t &= [y_i^t, y_i^{t+1}, y_i^{t+2}, y_i^{t+3}, y_i^{t+4}] \cdot c \\ z_i^t &= [z_i^t, z_i^{t+1}, z_i^{t+2}, z_i^{t+3}, z_i^{t+4}] \cdot c \\ (1 \leq t \leq T - 4, 1 \leq i \leq 14) \end{aligned} \tag{4}$$

Differential operation. Since the change of joints' position between each frame can reflect the people's gait patterns more than the joints' position itself, we apply the differential operation on J, then the change of 3-dimension position of 14 joints between each frame is stored in J.

The differential operation is given by:

$$j_{t-1} = j_t - j_{t-1} (2 \le t \le T) \tag{5}$$

Data segmentation. Since there are several straight walk segments of one walk, and the joints' coordinate acquired by Kinect is not accurate when the participant turns around. So we divide one walk into several straight walk segments. The segments record the gait patterns when subjects face to the Kinect, called front segments, and the other segments are back segments. To ensure each segment covers at least one stride, we only choose the segment which contains at least 40 frames.

Suppose one walk J contains n front segments and m back segments, which is described by a series of matrices:

$$\begin{cases}
Front_i, 1 \le i \le n \\
Back_j, 1 \le j \le m
\end{cases}$$
(6)

#### 3.3.2. Feature Extraction

The gait patterns record by Kinect between the front segments and the back segments are pretty different, so we extract features from front segments and back segments separately.

From what has been presented above, the process of one subject walk around the footpath one time (with an emotion state), called one walk, described by a matrix J, contains n front segments and m back segments.

First, we extract features from front segments. Given human walking is periodic and each segment covers at least one stride, we run Fourier transformation to acquire the behavior of the each front segment  $Front_i$ , we apply Fourier transformation on each column of  $Front_i$ , the main frequency  $f_1^i, f_2^i, \ldots, f_{42}^i$  and the corresponding phase  $\varphi_1^i, \varphi_2^i, \ldots, \varphi_{42}^i$ , are extracted. Since one walk contains n front segments, and different processes of walk has different number of segments. We select mean features of all front segments, and finally we extract 84 features, denoted as  $Feature_{front}$ :

$$Feature_{front} = \frac{1}{n} \sum_{i=1}^{n} [f_1^i, f_2^i, \dots, f_{42}^i, \varphi_1^i, \varphi_2^i, \dots, \varphi_{42}^i]$$
 (7)

Second, extract features from back segments in same way, get another 84 features, denoted as  $Feature_{back}$ :

$$Feature_{back} = \frac{1}{m} \sum_{i}^{n} [f_1^i, f_2^i, \dots, f_{42}^i, \varphi_1^i, \varphi_2^i, \dots, \varphi_{42}^i]$$
 (8)

Combine  $Feature_{front}$  and  $Feature_{back}$  to get 168 features, as Feature:

$$Feature = [Feature_{front}, Feature_{back}] \tag{9}$$

These 168 features describe the one subject's gait patterns with an emotional state (natural, angry or happy). Since the value of these features may vary dramatically, in case some important features with small values are ignored while training the model, which may seriously affect the recognition results, we run Z-score normalization on these features.

In general, training data with high dimension not only increases computational complexity, but also brings much redundant information. For efficient dimension reduction, we utilize Principal Component Analysis (PCA) to do fea-

ture selection, since previous study found that PCA can perform much better than other techniques on small size of train sets[26].

#### 4. Results

For recognition, several standard classifiers are compared. NaiveBayes, Random Forests, LibSVM and SMO classifiers are used for classification. The recognition rate is calculated using 10-fold cross validation.

## 4.1. The recognition of natural emotion and angry emotion

Table 1 shows the accuracy of each classifier to recognize the natural and angry emotions, and the gait dataset is collected by KINECT1, in first-round experiment.

Table 1: The accuracy of recognition between natural and angry emotions on KINECT1

Classifier	NaiveBayes	RandomForests	${ m LibSVM}$	SMO
Accuracy(%)	80.5085	52.5424	72.0339	52.5424

Table 2 shows the accuracy of each classifier to recognize natural and angry emotions, the gait dataset is collected by KINECT2, in first-round experiment.

Table 2: The accuracy of recognition between natural and angry emotions on KINECT2

Classifier	NaiveBayes	RandomForests	${ m LibSVM}$	SMO
Accuracy(%)	75.4237	_	71.1864	_

#### 4.2. The recognition of natural and happy emotion

Table 3 and Table 4 present the accuracy of each classifier to recognize the natural and happy emotions, the gait datasets are collected by KINECT1 and KINECT2, in second-round experiment.

Table 3: The accuracy of recognition between natural and happy emotions on KINECT1

Classifier	NaiveBayes	RandomForests	${\rm LibSVM}$	SMO
Accuracy(%)	79.6610	51.6949	77.9661	_

Table 4: The accuracy of recognition between natural and happy emotions on KINECT2

Classifier	NaiveBayes	RandomForests	${ m LibSVM}$	SMO
Accuracy(%)	61.8644	51.6949	52.5414	_

#### 4.3. The recognition of angry and happy emotion

Table 5 and Table 6 present the accuracy of each classifier to identify angry and happy, the gait datasets are collected by KINECT1 and KINECT2, in first-round and second-round experiments.

Table 5: The accuracy of recognition between angry and happy emotions on KINECT1

Classifier	NaiveBayes	RandomForests	LibSVM	SMO
Accuracy(%)	52.5424	55.0847	-	51.6949

## 5. Discussion

Recognition of emotions based on gait patterns is a challenging data mining task. Although the expression of emotions may vary from person to person, by using machine learning technique and features extracted from gait patterns, it is still possible to acquire cues about peoples' emotion state.

Comparing with marker-based system, Kinect may be inaccurate since it estimates joint locations by computation, which might influence the performance of trained models.

From the result we can see that the recognition accuracy between natural and angry can reach 80%, and 70% for classifying natural vs. happy. From the self-report emotion score, we can see that the participants' emotion have been really elicited by the film clip. The participants also reported their emotion's change, which could be expressed by their gait to some extend. But the recognition

Table 6: The accuracy of recognition between angry and happy emotions on KINECT2

Classifier	NaiveBayes	RandomForests	LibSVM	SMO
Accuracy(%)	_	51.6949	-	50.8475

accuracy between different unneutral emotions (i.e., angry and happy) is not very good. As people's gait may be similar or less difference in joints' position's change no matter whether for angry or happy, which makes it is difficult to distinguish angry from happy using gait patterns acquired by Kinect.

After choosing relevant joints during data preprocessing, the result indicates that the accuracy of identification increases. Using the whole 25 joints position leads to lower recognition rates, which is less than 70%.

#### 6. Conclusion and Future Work

This paper introduces a novel method to do emotion recognition, by using Kinect to acquire gait data, training classifiers on features extracted. The emotions can be recognized fairly well based on gait patterns, with 80% accuracy for classifying natural and angry emotion as well as 70% accuracy for classifying natural and happy emotion. Due to limited sample size of participants, there still exists much spaces to improve the performance of trained models. In the future, we plan to do more gait patterns data acquisition by recruiting more subjects, extracting more new features, and training advanced classification algorithms.

#### 7. Acknowledgement

The authors gratefully acknowledge the generous support from National High-tech R&D Program of China (2013AA01A606), National Basic Research Program of China(2014CB744600), Key Research Program of Chinese Academy of Sciences(CAS)(KJZDEWL04), and CAS Strategic Priority Research Program (XDA06030800).

#### References

- [1] M. Karg, K. Kuhnlenz, M. Buss, Recognition of affect based on gait patterns, Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on 40 (4) (2010) 1050–1061. doi:10.1109/TSMCB.2010.2044040.
- [2] C. P. R. Beale, Affect and emotion in human-computer interaction.
- [3] F. E. Pollick, H. M. Paterson, A. Bruderlin, A. J. Sanford, Perceiving affect from arm movement, Cognition 82 (2) (2001) B51–B61.
- [4] A. Kale, A. Sundaresan, A. Rajagopalan, N. Cuntoor, A. Roy-Chowdhury, V. Kruger, R. Chellappa, Identification of humans using gait, Image Processing, IEEE Transactions on 13 (9) (2004) 1163–1173. doi:10.1109/ TIP.2004.832865.
- [5] S. Gaukrodger, A. Peruzzi, G. Paolini, A. Cereatti, S. Cupit, J. Hausdorff,
   A. Mirelman, C. U. Della, M38, Gait & posture 37 (Supplement 1) (2013)
   S31. doi:10.1016/j.gaitpost.2012.12.062.
- [6] Pogrzeba, Loreen, Wacker, Markus, Jung, Bernhard, P22, Gait & Posture 38 (Supplement 1) (2013) S94-S95. doi:10.1016/j.gaitpost.2013.07. 195.
- [7] E. Stone, M. Skubic, M. Rantz, C. Abbott, S. Miller, Average inhome gait speed: Investigation of a new metric for mobility and fall risk assessment of elders, Gait & Posture 41 (1) (2015) 57 62. doi:http://dx.doi.org/10.1016/j.gaitpost.2014.08.019.
  URL http://www.sciencedirect.com/science/article/pii/S0966636214006833
- [8] J. Montepare, S. Goldstein, A. Clausen, The identification of emotions from gait information, Journal of Nonverbal Behavior 11 (1) (1987) 33–42. doi:10.1007/BF00999605.
  - URL http://dx.doi.org/10.1007/BF00999605

- [9] D. JanJanssen, W. I. Schöllhorn, J. Lubienetzki, K. Fölling, H. Kokenge, K. Davids, Recognition of emotions in gait patterns by means of artificial neural nets, Journal of Nonverbal Behavior 32 (2) (2008) 79–92. doi: 10.1007/s10919-007-0045-3.
  - URL http://dx.doi.org/10.1007/s10919-007-0045-3
- [10] M. Karg, R. Jenke, K. Kühnlenz, M. Buss, A two-fold pca-approach for inter-individual recognition of emotions in natural walking., in: MLDM Posters, 2009, pp. 51–61.
- [11] M. Karg, R. Jenke, W. Seiberl, K. Kuuhnlenz, A. Schwirtz, M. Buss, A comparison of pca, kpca and lda for feature extraction to recognize affect in gait kinematics, in: Affective Computing and Intelligent Interaction and Workshops, 2009. ACII 2009. 3rd International Conference on, 2009, pp. 1–6. doi:10.1109/ACII.2009.5349438.
- [12] J. Wang, R. Chen, X. Sun, M. F. She, Y. Wu, Recognizing human daily activities from accelerometer signal, Procedia Engineering 15 (2011) 1780 1786, {CEIS} 2011. doi:http://dx.doi.org/10.1016/j.proeng.2011.08.331.

  URL http://www.sciencedirect.com/science/article/pii/S1877705811018327
- [13] R. A. Clark, Y.-H. Pua, A. L. Bryant, M. A. Hunt, Validity of the microsoft kinect for providing lateral trunk lean feedback during gait retraining, Gait & Posture 38 (4) (2013) 1064 - 1066. doi:http://dx.doi.org/10.1016/j.gaitpost.2013.03.029. URL http://www.sciencedirect.com/science/article/pii/ S0966636213001884
- [14] F. Kondori, S. Yousefi, H. Li, S. Sonning, S. Sonning, 3d head pose estimation using the kinect, in: Wireless Communications and Signal Processing (WCSP), 2011 International Conference on, 2011, pp. 1–4. doi:10.1109/WCSP.2011.6096866.

- [15] A. Fern'ndez-Baena, A. Susin, X. Lligadas, Biomechanical validation of upper-body and lower-body joint movements of kinect motion capture data for rehabilitation treatments, in: Intelligent Networking and Collaborative Systems (INCoS), 2012 4th International Conference on, 2012, pp. 656–661. doi:10.1109/iNCoS.2012.66.
- [16] B. Lange, C.-Y. Chang, E. Suma, B. Newman, A. Rizzo, M. Bolas, Development and evaluation of low cost game-based balance rehabilitation tool using the microsoft kinect sensor, in: Engineering in Medicine and Biology Society, EMBC, 2011 Annual International Conference of the IEEE, 2011, pp. 1831–1834. doi:10.1109/IEMBS.2011.6090521.
- [17] J. Tong, J. Zhou, L. Liu, Z. Pan, H. Yan, Scanning 3d full human bodies using kinects, Visualization and Computer Graphics, IEEE Transactions on 18 (4) (2012) 643–650. doi:10.1109/TVCG.2012.56.
- [18] E. Auvinet, F. Multon, C.-E. Aubin, J. Meunier, M. Raison, Detection of gait cycles in treadmill walking using a kinect, Gait & Posture 41 (2) (2015) 722 - 725. doi:http://dx.doi.org/10.1016/j.gaitpost.2014.08.006. URL http://www.sciencedirect.com/science/article/pii/ S0966636214006705
- [19] L. Yeung, K. C. Cheng, C. Fong, W. C. Lee, K.-Y. Tong, E-valuation of the microsoft kinect as a clinical assessment tool of body sway, Gait & Posture 40 (4) (2014) 532 538. doi:http://dx.doi.org/10.1016/j.gaitpost.2014.06.012.
  URL http://www.sciencedirect.com/science/article/pii/S0966636214006122
- [20] B. Galna, G. Barry, D. Jackson, D. Mhiripiri, P. Olivier, L. Rochester, Accuracy of the microsoft kinect sensor for measuring movement in people with parkinson's disease, Gait & Posture 39 (4) (2014) 1062 – 1068. doi:http://dx.doi.org/10.1016/j.gaitpost.2014.01.008.



- URL http://www.sciencedirect.com/science/article/pii/S0966636214000241
- [21] I. Weber, J. Koch, J. Meskemper, K. Friedl, K. Heinrich, U. Hartmann, Is the ms kinect suitable for motion analysis?, Biomedical Engineering/Biomedizinische Technik 57 (SI-1 Track-F).
- [22] E. Stone, M. Skubic, Evaluation of an inexpensive depth camera for inhome gait assessment, Journal of Ambient Intelligence and Smart Environments 3 (4) (2011) 349–361.
- [23] J. Song, G. Zhao, Y. Xiong, X. Sun, Y. Ge, Chinese emotional film clips database.
- [24] R. D. Ray, Emotion elicitation using films, Handbook of emotion elicitation and assessment (2007) 9–28.
- [25] P. Gwosdek, S. Grewenig, A. Bruhn, J. Weickert, Theoretical foundations of gaussian convolution by extended box filtering, in: A. Bruckstein, B. ter Haar Romeny, A. Bronstein, M. Bronstein (Eds.), Scale Space and Variational Methods in Computer Vision, Vol. 6667 of Lecture Notes in Computer Science, Springer Berlin Heidelberg, 2012, pp. 447–458. doi:10.1007/978-3-642-24785-9\_38.
  URL http://dx.doi.org/10.1007/978-3-642-24785-9\_38
- [26] A. M. Martinez, A. Kak, Pca versus lda, Pattern Analysis and Machine Intelligence, IEEE Transactions on 23 (2) (2001) 228–233. doi:10.1109/ 34.908974.