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Tolerance-based interaction: A new model targeting opinion formation and diffusion in social networks

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ABSTRACT

One of the main motivations behind social network analysis is the quest for understanding opinion formation and diffusion. Previous models have limitations, as they typically assume opinion interaction mechanisms based on thresholds which are either fixed or evolve according to a random process that is external to the social agent. Indeed, our empirical analysis on large real-world datasets such as Twitter, Meme Tracker, and Yelp, uncovers previously unaccounted for dynamic phenomena at population-level, namely the existence of distinct *opinion formation phases* and *social balancing*. We also reveal that a phase transition from an erratic behavior to social balancing can be triggered by network topology and by the ratio of opinion sources. Consequently, in order to build a model that properly accounts for these phenomena, we propose a new (individual-level) opinion interaction model based on tolerance. As opposed to the existing opinion interaction models, the new tolerance model assumes that individual's inner willingness to accept new opinions evolves over time according to basic human traits. Finally, by employing discrete event simulation on diverse social network topologies, we validate our opinion interaction model and show that, although the network size and opinion source ratio are important, the phase transition to social balancing is mainly fostered by the democratic structure of the small-world topology.

Keywords: social networks, opinion diffusion, phase transition, discrete event simulation, tolerance

INTRODUCTION

Social networks analysis is crucial to better understand our society, as it can help us observe and evaluate various social behaviors at population level. In particular, understanding the social opinion dynamics and personal opinion fluctuation (Golbeck, 2013; Geven et al., 2013; Valente et al., 2013) play a major part in fields like social psychology, philosophy, politics, marketing, finances and even warfare (Easley and Kleinberg, 2010; Pastor-Satorras and Vespignani, 2001; Fonseca, 2011). Indeed, the dynamics of opinion fluctuation in a community can reflect the distribution of socially influential people across that community (Kempe et al., 2003; Hussain et al., 2013; Muchnik et al., 2013); this is because the social influence is the ability of individuals (agents) to influence others' opinion in either one-on-one or group settings(Maxwell, 1993; Wang and Chen, 2003; McDonald and Wilson, 2011). Without social influence, the society would have an erratic behavior which would be hard to predict.

Existing studies on opinion formation and evolution (Acemoglu et al., 2013; Yildiz et al., 2013; Valente et al., 2013; Hussain et al., 2013; Guille et al., 2013; Ruan et al., 2015) rely on the contagion principle of opinion propagation. However, such studies offer limited predictability and realism because they are generally based on opinion interaction models which use either fixed thresholds (Deffuant et al., 2000; Javarone and Squartini, 2014), or thresholds evolving according to simple probabilistic processes that are not driven by the internal state of the social agents (Fang et al., 2013; Deng et al., 2013). To mitigate these limitations, we reveal new dynamical features of opinion spreading that previous models fail to identify. The consistent and recurring real-world observations are then explained by introducing a new social interaction model which takes into account the evolution of individual's inner state. We finally validate the proposed model by analyzing empirical data from Twitter, MemeTracker and Yelp, and by using our opinion dynamics simulation framework - SocialSim (Topirceanu and Udrescu, 2014) - which includes multiple complex topological models, as well as customizable opinion interaction and influence models. Consequently, our main contributions are threefold:

1. **Identification of four distinct phases in opinion formation**; this aspect is *not* captured by existing models (Sznajd-Weron and Sznajd, 2000; Li et al., 2012a; Acemoglu et al., 2013; Chen et al., 2014; Guille et al., 2013; Fang et al., 2013) although previous research (Hołyst et al., 2000) has noticed that there exist a few stages in opinion evolution. We argue that the succession of opinion formation phases is critical to the *social balancing* phenomenon (i.e. the general opinion becomes stable despite constant local oscillations). We also identify a *phase transition* from an unstable opinion to social balancing which is related to the dynamics of opinion formation phases.

2. **Modeling opinion dynamics**: we propose a new graph and threshold based interaction model with stubborn agents (Acemoglu and Ozdaglar, 2011) which is able to reproduce the phenomena that we observe in real-world datasets. Inspired by social psychology, our new model assumes that individual's willingness to accept new opinions (i.e. tolerance) changes over time according to his/her inner state.

3. **Validation of the newly proposed tolerance model** via our discrete-event simulator SocialSim (Topirceanu and Udrescu, 2014). The analysis we provide reveals the deep connection between social balancing and the relevant parameters of social networks such as network size, topology, and opinion source ratio (i.e. stubborn agents distribution)(Acemoglu et al., 2013); this correlates well with our empirical observations on large social networks.

Taken together, these new contributions show that opinion dynamics in social networks exhibit specific patterns that depend on network size and ratio of stubborn agents (which we consider to be opinion sources), as well as underlying network topology. Consequently, our findings can be used to improve our understanding of opinion formation and diffusion in social networks, and predictability of social dynamics.

RESULTS

Opinion formation phases and social balancing

By analyzing data on opinion evolution using Twitter and MemeTracker hashtags, as well as user reviews and votes for local businesses from Yelp, we identify unique temporal patterns in all these datasets.

Figure 1 displays the popularity of six hashtags on MemeTracker and Twitter, expressed as posts/time evolution (posts are replies and tweets). Based on the observed fluctuations, we identify the following phases in opinion formation: an initiation phase (I) when new opinions are injected into the social network and the number of replies starts to increase rapidly; a fusion phase (F) when the opinion dynamics reaches a maximum and different opinions start to collide; a tolerance phase (T) which represents a fluctuating yet convergent behavior; and, occasionally, an intolerance phase ($\overline{T}$) when the fluctuations of opinion decrease and converge towards zero. Based on network topology and/or ratio of opinion sources, the diffusion process may reach the fourth phase of intolerance. Opinion sources, or stubborn agents (Acemoglu et al., 2011, 2013), are agents within the social network (i.e. Twitter or Yelp users) who try to instill a certain opinion by influencing their peers; they are represented by those people within the network who hold strong opinions that do not change over time. The concentration of opinion sources is expressed as their ratio relative to the entire population.

Additionally, the analysis of Twitter results in Figure 1b shows that tags 1-3 all exhibit a clear F phase (first spike), then they enter a balanced oscillation (T phase). This evidence supports the empirical observation of a phenomenon that we call *social balancing*, i.e., oscillations at microscopic scale of individuals opinion become stable and predictable at the macroscopic scale of the society. As such, social balancing is defined as the succession of $I-F-T$ phases, whereas social imbalance occurs if either the society does not reach T or, after reaching T, it decays into a $\overline{T}$ phase. For example, tag 4 (#Iran) in Figure 1b has a shorter, more abrupt oscillation. In this case, we consider that the number of opinion sources is not high enough (i.e. above a critical threshold) for social balancing to happen. Tag 5 (#Haiti) has a longer F phase because of the (probably) very high concentration of opinion sources. Indeed, the 2010 Haiti earthquake was breaking news so there were many outbreaks of opinion, scattered across the globe, resembling a random network topology of sources of opinion; nonetheless, for tag 5 the society reaches social balance. Tag 6 is an example of social imbalance with a decisive crystallization of just one opinion, as there is no T phase.

Phase transition

Apart from the quantitative measure of posts/time, we also consider the qualitative information from Yelp submitted by votes to local businesses (Figure 2a-c). With data from Yelp, we show the effects of a phase transition from social instability to social balancing which can occur when a critical concentration of opinion sources is reached in a social network. Figures 2a-c highlight the fact that opinion (i.e. the stars given by users to a particular business) stabilizes only after reaching a critical ratio of opinion sources (i.e. votes representing strong opinions). This can be viewed in Figure 2a at time point $OX = 35$, in Figure 2b at time point $OX = 32$, and in Figure 2c at time point $OX = 28$ and again at $OX = 58$, where the total number of reviews and votes rises dramatically (see the vertical red line). We interpret this phenomenon as a rise beyond a σ threshold for the concentration of opinion sources, which determines the *social balancing,* i.e. the average opinion stabilizes despite of opinion oscillations at local level. As such, in Figure 2b, we observe a stabilization of the average score given by users at time point $OX = 35$. The same type of stabilization occurs in Figure 2b at time point $OX = 32$. Moreover, in Figure 2b, we identify two stabilization points: $OX = 28$ and $OX = 58$.

Corroborating all these empirical observations, we can state that Twitter and MemeTracker illustrate a *responsive* type of behavior, i.e. an immediate evolution towards the F phase, so a high opinion change is quickly reached for a relatively small ratio σ of opinion sources. This behavior, in turn, correlates well with another study which shows that Twitter online networks have a strong random and small-world component (Duma and Topirceanu, 2014).

In contrast, the Yelp dataset can be associated with a *saturated* type of behavior, as the ratio σ (relative to the maximum number of votes) needed to trigger the phase transition towards social balancing is high in all three cases. Balancing does not occur until a high concentration of opinion sources (we interpret them as similar to opinion-influencing ”stubborn agents” (Acemoglu et al., 2013) or ”blocked nodes” (Ruan et al., 2015)) are inserted into the social network.

New tolerance-based opinion model

This section analyzes the characteristics of a new opinion model that can reproduce this kind of real-world phenomena, i.e. the four opinion formation phases and phase transition towards social balancing.

In terms of network *structure*, our analysis includes the basic topologies such as mesh, random (Erdös and Rényi, 1960), small-world (Watts and Strogatz, 1998), and scale-free networks (Barabási and Albert, 1999). Also, based on the last decade of research on realistic social network topology generation which either adds the small-world property to scale-free models (Holme and Kim, 2002; Fu and Liao, 2006; Li et al., 2012b), or adds a power-law degree distribution to the small-worlds (Jian-Guo et al., 2006; Chen et al., 2007; Wang and Rong, 2008; Zaidi, 2013), we also consider the Watts-Strogatz with degree distribution (WSDD) (Chen et al., 2007).

In terms of *opinion dynamics*, we rely on a predictive opinion interaction model that can be classified as being graph- and threshold-based (Guille et al., 2013). Generally speaking, previous models use fixed thresholds (Javarone and Squartini, 2014; Biswas et al., 2011; Li et al., 2012a; Das et al., 2014; Li et al., 2013) or thresholds extracted from real-world examples (Galuba et al., 2010; Saito et al., 2011). However, there are a few models which use dynamic thresholds (Fang et al., 2013; Deng et al., 2013; Li et al., 2011), but their evolution is not driven by the internal states of the social agents. On the other hand, our empirical references (i.e Twitter, MemeTracker and Yelp) indicate that opinion does not cease to oscillate and consensus is a rare case in real world. Therefore, we propose an opinion interaction model based on stubborn agents, because it assumes that the society does not reach consensus. Based on recent research on stubborn agents which use a discrete (Yildiz et al., 2013) or continuous (Acemoglu et al., 2013) representation of opinion, we integrate the following opinion models: one-to-one (simple contagion) versus one-to-many diffusion (complex contagion) (Centola and Macy, 2007), and discrete (0 or 1) versus continuous (0 to 1) opinion representation. By combining opinion representation and opinion diffusion, we obtain 4 distinct models; they are defined in Figure 3a and exemplified in Figures 3b

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and 3c. We build our tolerance-based opinion interaction model by using the SD (1) and SC (2) opinion representations as defined in Figure 3a.

Given a social network $G = \{V, E\}$ composed of agents $V = \{1, 2, ..., N\}$ and edges E, we define the neighborhood of agent $i \in V$ as $N_i = \{\, j \mid (i, j) \in E\}$. The disjoint sets of stubborn agents $V_0, V_1 \in V$ never change their opinion, while all other (regular) agents $V \setminus \{V_0 \cup V_1\}$ update their opinion based on the opinion of one or all of their direct neighbors.

We use $x_i(t)$ to represent the real-time opinion of agent i at time t. Normal (regular) agents can start with a predefined random opinion value $x_i(0) \in [0,1]$. The process of changing the opinion of regular agents is triggered according to a Poisson distribution and consists of either adopting the opinion of a randomly chosen direct neighbor, or an averaged opinion of all direct neighbors.

We represent with $s_i(t)$ the discrete opinion of an agent i at moment t having continuous opinion $x_i(t)$. In case of the discrete opinion representation SD (1) (Figure 3a), $x_i(t) = s_i(t)$; in case of the continuous opinion representation SC (2) (Figure 3a), $s_i(t)$ is given by equation 1.

$$s_i(t) = \begin{cases} 0 & if\ 0 \leq x_i(t) < 0.5 \\ 1 & if\ 0.5 \leq x_i(t) \leq 1 \end{cases} \tag{1}$$

Furthermore, $s(t)$ denotes the average state of the population at a certain time t by averaging the opinion of all individual agents $i \in V$.

$$s(t) = \frac{1}{|V|} \sum_{i \in V} s_i(t) \tag{2}$$

The previous social interaction models (Deffuant et al., 2000; Javarone and Squartini, 2014; Li et al., 2012a; Chau et al., 2014; Das et al., 2014; Fang et al., 2013; Li et al., 2011) do not assign nodes (i.e. individuals or social agents) the basic properties of humans, i.e. humans evolve, learn, react, and adapt in time. The reason for the simplicity behind the existing models is twofold: first, the state-of-the-art models are only suited for theoretical contexts so bringing additional complexity to the interaction model would significantly increase the difficulty of mathematical analysis; second, involving measures of human personality (e.g. quantifying an individuals trust, credibility, or emotional state) is a complicated endeavor, in general; this was not the main goal of previous work.

Individual tolerance: interpretation and formalism

In order to improve the existing opinion interaction model based on a fixed threshold, we consider the evolution of personal traits by taking inspiration from social psychology. As a new contribution to the state-of-the-art, we introduce the concept of *tolerance* which reflects the individual's inner state and personal beliefs regarding surrounding opinions. For instance, egocentrism, as it is called in psychology, is highly correlated with individual's emotional state (Elkind, 1967). We choose to extend this model because the egocentrism-emotional state correlation is a trait that has been shown to influence and evolve with individual opinion (Windschitl et al., 2008).

Corroborating literature on attitude certainty (Clarkson et al., 2013), consensus (Clarkson et al., 2013), confirmation bias (Nyhan and Reifler, 2010), social group influence (Roccas and Amit, 2011), and ingroup emotion (Moons et al., 2009), we extrapolate the mechanism that leads to the formation of opinion into a measurable parameter. As such, we define *tolerance* θ as a parameter that reflects the willingness of an agent to accept new opinions. Similar to real life, individuals with higher tolerance will accept the others opinion easier; thus, this parameter can be defined as a real number $0 \leq \theta \leq 1$. An agent with a tolerance value of 1 is called fully tolerant, whereas an agent with a tolerance of 0 is called fully intolerant (i.e. stubborn agent). Tolerance values which are greater than 0.5 describe a tolerance-inclined agent, while values smaller than 0.5 describe an intolerance-inclined agent.

Similar to the threshold-based continuous opinion fluctuation model described by Acemoglu et al. (Acemoglu et al., 2013), tolerance can be used as a trust factor for an agent relationship; however, as opposed to the trust factor, tolerance changes its value over time:

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$$x_i(t) = \begin{cases} 0 & if\ i \in V_0 \\ 1 & if\ i \in V_1 \qquad for\ t > 0 \\ \theta_i(t)x_j(t) + (1-\theta_i(t))x_i(t-1) & if\ j \in N_i \end{cases} \tag{3}$$

where the new opinion $x_i(t)$ is a weighted sum of the agent's prior opinion $x_i(t-1)$ and the current opinion $x_j(t)$ of one randomly selected direct neighbor. The weights for the two opinions are given by the current tolerance $\theta_i(t)$ of the agent, thus, the extent of how much it can be influenced depends on its internal state.

As can be inferred from equation 3, the greater the tolerance of an agent, the easier it can accept external opinions from others. At the beginning of the opinion formation process ($t = 0$), all agents are considered as having a high tolerance ($\theta_i(0) = 1$), but, as the society evolves, agents become intolerant, therefore segregated in clusters which tend to have a more stable opinion. We further define the tolerance θ of the entire population as a normalized average of all individual tolerances:

$$\theta(t) = \frac{1}{|V|}\sum_{i \in V} \theta_i(t) \tag{4}$$

We also introduce the concept of *opinion change* ω as the ratio of agents which have changed their current state (discrete time step t) since the last observation (time $t-1$):

$$\omega(t) = \frac{1}{|V|}\sum_{i \in V} |s_i(t) - s_i(t-1)| \tag{5}$$

If an agent changes its state from one opinion to another, then the absolute difference$|s_i(t) - s_i(t-1)|$ will be 1; conversely, it will be 0 if the agent state does not change. This change, averaged over all agents at the interaction (discrete) moment t, defines the opinion change of the population $\omega(t)$. This metric is used to draw insights regarding the current tolerance level across the entire society.

Progressive tolerance model

Our model for tolerance evolution stems from the idea that the evolution towards both tolerance and intolerance varies exponentially (Hegselmann and Krause, 2002; Weidlich, 2002), e.g. a person under constant influence becomes convinced at an increased rate over time. If that person faces an opposing opinion, it will eventually start to progressively build confidence in that other opinion. Thus, our proposed progressive model represents the tolerance fluctuation as a non-linear function, unlike other models in literature. For the first time, we integrate these socio-psychological characteristics in the dynamical opinion interaction model; as such, the new tolerance state is obtained as:

$$\theta_i(t) = \begin{cases} max\ (\theta_i(t-1) - \alpha_0\varepsilon_0, 0) & if\ s_i(t-1) = s_j(t) \\ min\ (\theta_i(t-1) + \alpha_1\varepsilon_1, 1) & otherwise \end{cases} \tag{6}$$

In equation 6, tolerance decreases by a factor of $\alpha_0\varepsilon_0$ if the state of the agent before interaction, $s_i(t-1)$, is the same as the state of the interacting neighbor (randomly chosen from all direct neighbors) $s_j(t)$. If the states are not identical, i.e. the agent comes in contact with an opposite opinion, then the tolerance will increase by a factor of $\alpha_1\varepsilon_1$. Variable t represents the time step where an opinion update is triggered; these moments are considered as being randomly distributed. The two scaling factors, α_0 and α_1, both initially set as 1, act as weights (i.e. counters) which are increased to account for every event in which the initiating agent keeps its old opinion (i.e. tolerance decreasing), or changes its old opinion (i.e. tolerance increasing). Therefore, we have:

$$\alpha_0 = \begin{cases} \alpha_0 + 1 & if\ s_i(t-1) = s_i(t) \\ 1 & otherwise \end{cases} \tag{7}$$

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$$\alpha_1 = \begin{cases} 1 & if\ s_i(t-1) = s_i(t) \\ \alpha_1 + 1 & otherwise \end{cases} \tag{8}$$

On even terms with the observation of the *majority illusion* (Lerman et al., 2015), which explains that globally rare opinions and bias may be strongly present in local neighborhoods as a result of the topology of social networks, we dynamically model bias using the two scaling factors α_0 and α_1. Whenever an event occurs, the counter corresponding to the other type of event is reset. These factors are used to increase the magnitude of the two tolerance modification ratios ε_0 (intolerance modifier weight) and ε_1(tolerance modifier weight). The two ratios are chosen with the fixed values of ε_0 = 0.002 and ε_1 = 0.01. To determine these values, we have tried various $\varepsilon_0 : \varepsilon_1$ ratios as follows: if ε_0 is increased such that $\varepsilon_0 : \varepsilon_1 = 1:1$, most nodes will quickly become intolerant, as opinion will cease to diffuse; conversely, if ε_0 is decreased closer to a 1:10 ratio, then the society will become tolerance-inclined, with random opinion fluctuations. The used $\varepsilon_0 : \varepsilon_1$ ratio of 1:5 was chosen through consistent experimentation in order to provide a good balance between the deviations towards tolerance and intolerance, respectively.

As an illustration of the 1:5 ratio for $\varepsilon_0 : \varepsilon_1$, Figure 4 represents the non-linear tolerance function as implemented in equation 6. The displayed examples show that a total of 10 consecutive steps are required to maximize the tolerance if an agent starts with $\theta_i(0) = 0.5$, because the cumulative sum of $\theta_i(0) + \varepsilon_0 \sum_j \alpha_0$ reaches 1 after 10 iterations. Similarly, in Figure 4b, the sum $\theta_i(0) - \varepsilon_1 \sum_j \alpha_1$requires $t = 45$ iterations to reach intolerance ($\theta_i(t) = 0$), having started from $\theta_i(0) = 1$.

MODEL VALIDATION

Our dynamical opinion model adds significant complexity to the opinion interaction model. Therefore, we use discrete event simulation (SocialSim (Topirceanu and Udrescu, 2014)) over complex social network topologies, in order to validate our model's capability to reproduce real-world phenomena like the opinion formation phases and the phase transition towards social balancing.

Simulation on basic topologies

Regular networks

The first simulation setup is based on regular topologies, i.e. lattice and mesh. The results show that a homogeneous cluster of stubborn agents divides the overall society opinion (i.e. green (1) vs. red (0)) with a ratio that is directly proportional with their initial distribution. Figure 5 shows how a mesh network of 100,000 agents evolves under the influence of 64 stubborn agents – 32 of each opinion evenly distributed among the population. This way, we observe the same opinion formation phases as identified by our empirical observations: initiation I (Figure 5a), fusion F (Figure 5b), tolerance T (Figure 5c), and intolerance $\overline{T}$ (Figure 5d). The situation in Figure 5c may lead to one of two scenarios: a perpetual (proportional) balance of the two opinions, introduced by us as *social balancing* (the society remains in the T phase, and $\overline{T}$ is never reached), or a constant decrease in opinion dynamics which ultimately leads to a stop in opinion change (the society reaches the $\overline{T}$ phase), as depicted in Figure 5d.

Figure 6a illustrates a society which tends towards the tolerance phase T and social balance, by providing the evolution of the overall society state $s(t)$ (as defined in equation 2), tolerance $\theta(t)$ (see equation 4), and opinion change $\omega(t)$ (equation 5). For the society described in Figure 6a, the initiation phase I is revealed by the early increase of $\omega(t)$, as the number of individuals with opinion increases. The climax of $\omega(t)$ represents the fusion phase F. At this stage, there is a maximum number of bordering agents with distinct opinions (a situation that is also depicted in Figure 5b) and $s(t)$ evens out. In the tolerance phase T, the agents tend to stabilize their opinion, i.e. $\theta(t)$ stabilizes and $s(t)$ converges towards the ratio of stubborn agents (which was chosen as 1:1).

Another observation is that opinion fluctuation is determined by the stubborn agents density (see Figures 6b, c and d). Because of the regular topology, the fewer stubborn agents (regardless of their opinions) there exist in the society, the more the opinion fluctuates. This is explained by the fact that having few stubborn agents means few points of opinion control and stabilization in the local mesh structure; conversely, many stubborn agents make possible the control of more regular agents. Because of this, $s(t)$ may drastically get biased in someone's favor until the entire society stabilizes (Figure 6b). Also, due to the small influencing power of a few agents, the opinion will not necessarily stabilize with

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the same distribution ratio. As expected, the opinion distribution of a society with a high opinion source concentration will tend towards the ratio of the two stubborn agent populations (Figure 6c).

If the ratio of the two stubborn agent populations is not 1:1, then the opinion fluctuation will be around that ratio only during the initiation phase I. Afterwards, the overall opinion will get more biased towards the opinion of the larger stubborn agent population. In Figure 6d the ratio is 1:4 between green and red stubborn agents, therefore the fluctuation starts around 20% green opinions, but eventually stabilizes at 8%.

The scenarios presented above hold true for lattices. Consequently, these conclusions are more of theoretical interest, as real social networks are typically not organized as such regular topologies. Next, we consider more realistic network topologies.

Small-world networks

By constructing a Watts-Strogatz small-world network of 100,000 nodes, (Watts and Strogatz, 1998; Strogatz, 2001; Wang and Chen, 2003; Tsvetovat and Carley, 2005; Chen et al., 2007; Bandyopadhyay et al., 2011) we show experimentally that a different type of behavior can emerge. For instance, Figures 7a and b present the society as having a mixed opinions distribution with no noticeable clusters. As opposed to the representation in Figure 5, this topology does not allow multiple agents to cluster around the stubborn agents and converge towards their opinion. Consequently, this model not only increases the dynamics of opinion fluctuation, but also keeps the society in *social balance*. The fourth and final phase of opinion evolution - the intolerance phase - does not occur, and opinion change $\omega(t)$ is maintained at a (high) constant level. Moreover, the state of the society $s(t)$ is stable.

The society depicted in Figure 7a is homogeneously mixed from an opinion standpoint. Clusters do not form because many agents have long range links to other distant agents whose opinion can be different from the local one. This leads to a perpetual fluctuation which remains in balance. The noticeable effect on a small-world network is that the opinion stabilizes very fast and always at the ratio of the two stubborn agent populations (i.e. 1:1 in our case). In a mesh network, having few stubborn agents leads to an imbalance of opinion, but in the case of small-world topologies, opinion across the entire population always stabilizes. Opinion change $\omega(t)$ is also much higher compared to the mesh (i.e. 42% versus 10% under the same conditions) due to the long range links.

Scale-free networks

We apply the same methodology by constructing a 100,000 node Barabasi-Albert scale-free network and highlight the unique behavior it enacts.(Barabási and Albert, 1999; Pastor-Satorras and Vespignani, 2001; Albert and Barabási, 2002; Wang and Chen, 2003; Song et al., 2005; Chen et al., 2007) As Figure 7c shows, the society does not reach a balance at the expected value ($32 : 32 \Rightarrow 50\%$); instead, it gets biased towards one opinion or another. The reason behind this behavior is related to the power-law degree distribution (Wang and Chen, 2003). As such, scale-free networks behave more like a tree-structure with hubs rather than as a uniform graph. Indeed, as opinion flows from one agent to another, the higher impact of the hub nodes on the opinion formation at the society level becomes clear. If, for example, a green stubborn agent is placed as the root of a sub-tree filled with red stubborn agents, that sub-tree will never propagate red opinion as it cannot pass through the root and connect with other nodes. Experimentally, this is illustrated in Figure 7c. The green agents have been placed over nodes with higher degrees, and this can be seen in the evolution of the opinion. There is some initial fluctuation in the society and although the stubborn agent distribution is even, the fluctuation rapidly imbalances as the overall tolerance $\theta(t)$ plummets and all agents become sort of "indoctrinated" by the green opinion. The rapid drop in tolerance coincides with the drop in opinion change $\omega(t)$ and the stabilization of the state $s(t)$ at over 90%. Simulations were also run on the WSDD topology (Chen et al., 2007), which has a strong scale-free component, and yield similar results which lead to the same set of observations.

Phase transition in opinion dynamics

This section aims at analyzing the impact of topology, network size, interaction model, stubborn agent placement, ratio and concentration on the opinion change (ω), and on convergence towards intolerance (θ).

Simulations show that, in a society with a fixed stubborn agent distribution, the topology τ determines if:

- the society enters the intolerance phase I: $\theta \to 0$ (with $\theta < 0.1$), which also results in $\omega \to 0$;

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- the society *balances* and never enters the intolerance phase I: $\theta \rightarrow \theta_{limit}$, where $\theta_{limit} > 0.1$ and maintains a high ω;
- the society continues to oscillate for $0.1 < \theta < 1$, but the tolerance level does not stabilize.

In case of the Yelp dataset, we notice that for a given topology τ, and a network of size N, when the concentration of stubborn agents is bigger than a critical ratio σ, the society never becomes intolerant. In such cases, the society becomes balanced, with slight oscillation in tolerance or opinion change. The goal is therefore to find the tuples (τ, N, σ) at which this phenomenon occurs.

To obtain our results we have used five topologies τ (mesh, random, small-world, scale-free and WSDD), network sizes N of 400 up to 100,000 nodes, our new tolerance interaction model, a ratio of 1:1 between green (1) and red (0) stubborn agents, and an increasing concentration of stubborn agents ranging from 1% to 36%.

Impact of topology

The tolerance and opinion change with respect to the number of stubborn agents, as depicted in Figurs 8a and b, highlight a clear difference between the five topologies, namely mesh, random, small-world, scale-free, and WSDD. There is a total of three clearly distinguishable behaviors: a *responsive* behavior (present in small-worlds and random graphs), a *linear* behavior (for mesh networks), and a *saturated* behavior (corresponding to scale-free and WSDD networks).

The tolerance increases *linearly* for the mesh, as the population of stubborn agents increases. Consequently, there is no critical σ for which a phase transition occurs due to the high regularity of the network, but there is a visible saturation point (when the blue graph begins to drop in Figure 8a). This happens because the society is physically filled with more stubborn agents than regular ones and because all stubborn agents have $\theta = 0$, the overall tolerance begins to drop.

The *responsive* behavior exhibited by the random network and small-world networks suggests that these two topologies behave similarly in the context of opinion source saturation. The two topologies are almost identical under the conditions defined here, as they behave almost as the opposite of mesh networks: once the critical point σ is reached, their tolerance rises to the maximum value. Then, as the stubborn agents population increases, the tolerance and opinion change values decrease proportionally. The random and small-world topologies are equivalent with the mesh topology as the society becomes saturated with stubborn agents (i.e. see Figures 8a and b in terms of tolerance θ and opinion change ω, respectively).

Finally, the *saturated* behavior groups together the scale-free and WSDD topologies, both of which have the feature of degree-driven preferential attachment. The two topologies show smaller responsiveness to social balancing. As depicted in Figures 8a and b, the critical point of stubborn agents concentration for scale-free is by far the greatest one (i.e. $\sigma = 16\%$) and the maximum tolerance θ reached is the smallest among the simulations aiming at the impact of topology (20%). The WSDD topology shows a better response, at a much lower critical stubborn agents concentration point ($\sigma = 4\%$) and reaches social balance at $\theta = 30\%$.

Impact of network size

When analyzing the opinion change at society level, the same observations and classification are valid for all other network sizes. The larger the size N is, the more accurate the delimitation shown in Figures 8a and b becomes.

The impact of size offers a comparison of different tolerance stabilization on the same topology. The results in Figures 8c, d, e, and f show how well the social balancing effect scales with increasing sizes of the network.

The behavior of meshes, presented in Figure 8c, shows a linearly proportional increase of the critical stubborn agents concentration σ (around 20-25%) in accordance with the network size N. A similar evolution is visible in Figure 8f, on networks with preferential attachment, where the required σ is also proportionally bigger on larger networks. In Figures 8d and 8e, the random and small-world networks exhibit similar behavioral patterns: they achieve the critical point σ with maximal opinion change, and then evolve towards intolerance at a pace that is corroborated with N (i.e. a slower drop in tolerance for larger networks occurs).

All simulations presented in this section confirm our main observations (Twitter, MemeTracker, Yelp) on opinion formation phases and phase transition towards social balancing.

DISCUSSION

The results for the proposed tolerance-based opinion interaction model show that, if individual traits are considered for modeling social agents, then we can realistically reproduce real-world dynamical features of opinion formation such as opinion formation phases, as well as their evolution towards social balancing. At the same time, we demonstrate that the dynamics of opinion formation is influenced by topology, network size and stubborn agent (opinion source) distribution across the entire population. Overall, the topology seems to have the strongest influence on opinion formation and spread; this can be summarized by the following different tendencies:

- **Responsive behavior**: Tolerance stabilization is attained right after reaching a relatively low critical ratio of stubborn agents. Inserting additional stubborn agents entail a drop in autonomy and opinion flow. Such a behavior is achieved by random and small-world topologies, and it can be motivated by the uniform degree distribution and the existence of both local and long-range links, which foster opinion diversity and *social balancing*; this can be representative for a decentralized and democratic society.

- **Linear behavior**: The critical threshold at which tolerance becomes stable for mesh topologies increases linearly with the stubborn agents concentration. The mesh topology corresponds to a limited, almost ”autistic” social interaction behavior (where each agent only interacts with close proximity neighbors); therefore, the probability of opinion diversity only increases with the proportional addition of stubborn agents. For meshes, *social balancing* is attained only if a substantial number of stubborn agents is inserted.

- **Saturated behavior**: Tolerance converges slowly around a specific low value. This behavior is achieved in scale-free and WSDD networks. Due to the nature of these topologies, even though long-range links exist, nodes tend to be preferentially attached to the same hub nodes, meaning the same opinion sources. The amount of stubborn agents required to reach *social balance* is much higher and the resulting balance saturates quickly. It is thus a conservative, stratified and oligarchic type of society which reacts later and slower to new stimuli. Most individuals within this type of society remain intolerant and opinion change is treated as suspicious and non-credible.

Besides these original contributions, the results obtained with our model confirm prior studies which show how individuals converge towards the state of their ingroup (Moons et al., 2009; Van Der Schalk et al., 2011). This is especially noticeable on networks with high modularity, like the WSDD network in which every member in a community converges towards the community’s dominant opinion, yet every community converges towards a different state.

An important real-world aspect of our new tolerance model (which assumes that the level of acceptance of neighboring opinions evolves over time) is that the tolerance level of an agent $\theta_i(t)$ is proportional to the degree of the node. In other words, the more neighbors a node has, the more likely it is to receive different influences which can guarantee a higher tolerance level. This observation is backed up by a recent study which proves that individuals with a higher (in)degree are less likely to be influenced, and the influence of friends is not significantly moderated by their friends’ indegree and friendship reciprocity (Geven et al., 2013).

The results rendered with our tolerance model also fall in line with a research direction started by Gross et al. (Gross and Blasius, 2008) where the authors show that there is a self-organization in all adaptive networks, including multi-agent opinion networks. Our real-world observations and opinion simulation results show a similar topological self-organization based on stubborn agent topological properties.

Finally, the study of opinion dynamics through our proposed concept of *social balancing* shows key features that may be used in practical applications, like marketing or conflict resolution. Under the requirement of keeping the social state stable, while never reaching intolerance, we provide a classification of network topologies based on the social balancing property. Networks with the democratic small-world structure promote balancing; the phenomenon is also exhibited if there is a high concentration of stubborn agents to stabilize opinion in mesh networks. If there are significantly fewer stubborn agents in the network, balancing will only be achieved if one side is using a placement strategy to counter its rivals (Gionis et al., 2013). A small-world network will not offer an advantage to any of the opinions due the link layout and uniform degree distribution. On the other hand, the oligarchic scale-free topology shows

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a clear importance of strategically placed agents in hub nodes which intrinsically render the opposing nodes on lower levels of the tree virtually powerless. The balancing phenomenon does not occur in networks with scale-free properties. Clearly, the social balancing concept remains open for further debate, improvement, and real-world validation.

METHODS

We rely on the following datasets, which contain opinion fluctuation data with time information:

The Yelp dataset: contains graded (1-3 stars) user reviews of American businesses, each with a timestamp. One can obtain insights on the popularity of a business at a given time. The usable information is the number of reviews at a given moment in time (interpreted as network size of individuals with an opinion), the average grade in time (the average opinion over time), and the number of votes to each review (ratio of agents with strong or "stubborn" opinions, because when an agent votes, his opinion is already made up). The dataset contains 366,715 users, 61,814 businesses and 1,569,264 reviews.

MemeTracker and Twitter hashtags with time information from the Stanford Large Network Dataset Collection (SNAP); which contain the history (repost rate in time) of diverse, popular hashtags. We can use this data to analyze the evolution of a particular opinion in time. MemeTracker phrases are the 1,000 highest total volume phrases among 343 million phrases collected within 2008-2009. Twitter hashtags are the 1,000 highest total volume hashtags among 6 million hashtags from Jun-Dec 2009.

Discrete simulation methodology

Like any discrete event simulation, we define the salient properties of the experimental setup which was used to obtain the simulation results with our Java-based opinion dynamics simulator, SocialSim (Topirceanu and Udrescu, 2014).

Events are synchronized by the simulation clock; we call the period of this clock a simulation *day*. One day is a simulation period in which agents can interact with their neighbors. However, an agent does not interact daily, in fact each agent picks a random number of days to be inactive after each active day. In our simulation, we use a random timeout interval between 1 day and 50 days. Only after this time has elapsed, will an agent interact again with one random neighbor. After that interaction, the agent will again choose to be inactive for a random period of days.

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ADDITIONAL INFORMATION

Competing financial interests

The author(s) declare no competing financial interests.

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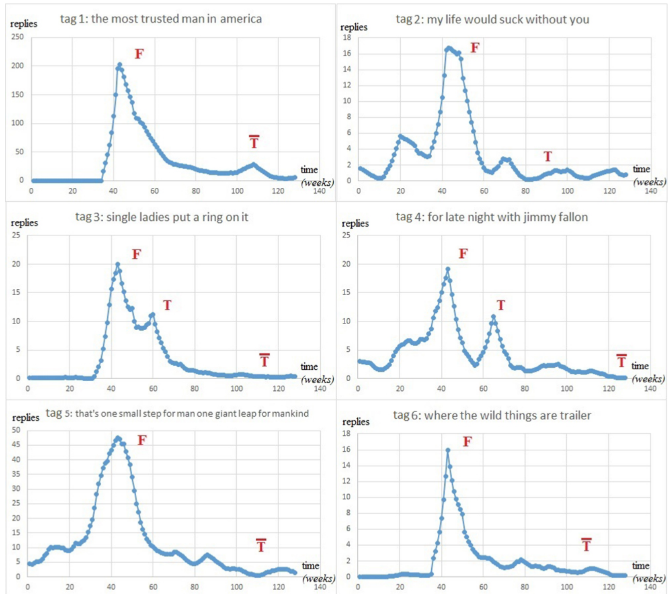

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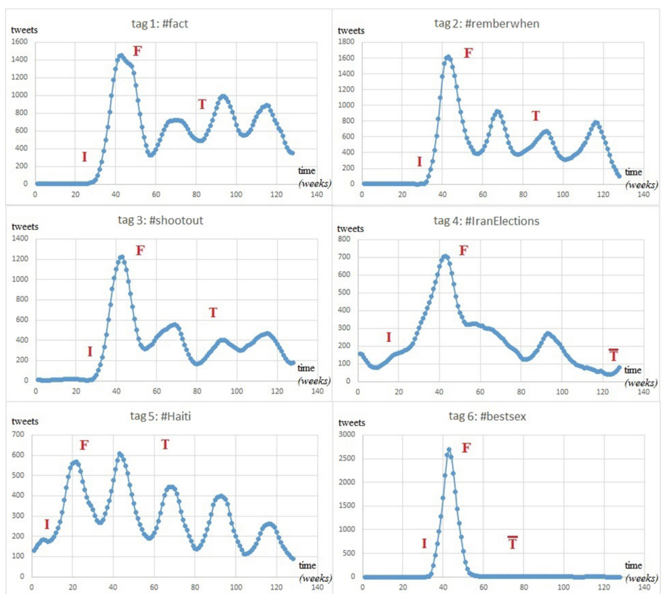
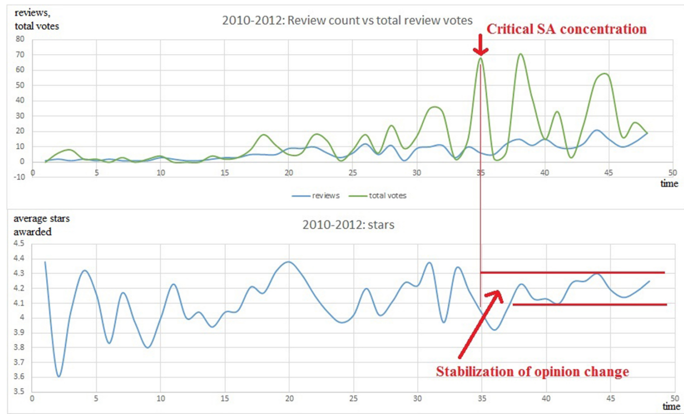


(b)

Figure 1. Opinion dynamics for six popular hashtags on: **a.** MemeTracker. Tags 1, 5, and 6 all exhibit the fusion phase (F) (opinion spike), then they slowly converge towards intolerance. Tags 2 and 4 have an initial spike before the F phase and more oscillations after F. The tolerance phase is depicted in tag 2 as the oscillation exists, but it is balanced. Tag 3 exhibits a second spike after the F phase, then enters the intolerance phase; as such, social balancing does not occur in tag 3. **b.** Twitter. Tags 1, 2, 3 and 5 exhibit the fusion phase F (first opinion spike), then they oscillate during the tolerance phase keeping social balance. Tags 4 and 6 show an example of convergence towards the intolerance phase, as social balancing does not occur.



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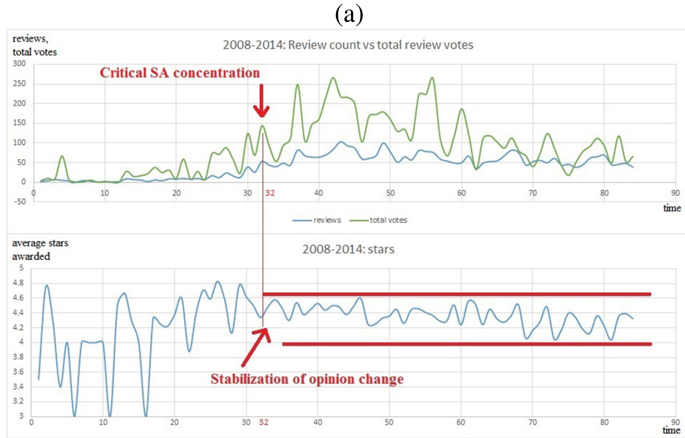

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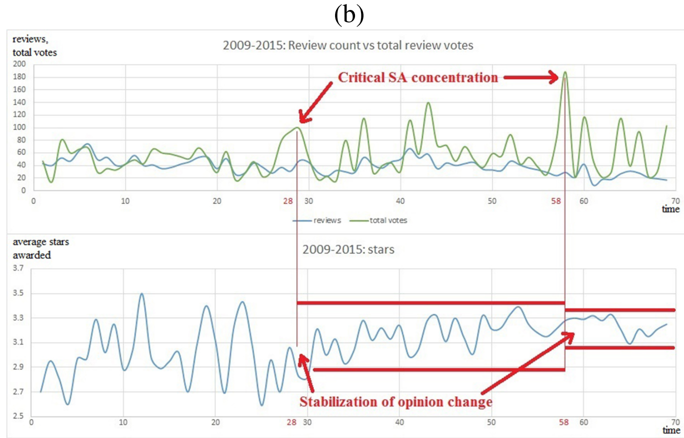

(c)

Figure 2. Evolution of reviews count and reviews votes for three popular businesses on Yelp over the period of 2008-2015. Accompanying each review trend, is the the average user defined popularity of the respective business over the same period of time. We highlight each critical opinion source concentration (i.e. ratio of reviews and votes) on the horizontal OX axis and corroborate it with a stabilization of the opinion change given as the evolution of average stars awarded.

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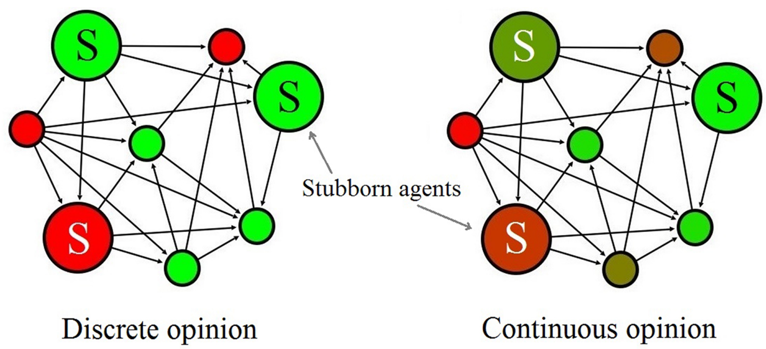
Table: interaction model taxonomy

		Opinion representation	
		Discrete	Continuous
Diffusion model	Simple	SD(1)	SC(2)
	Complex	CD(3)	CC(4)

(a)



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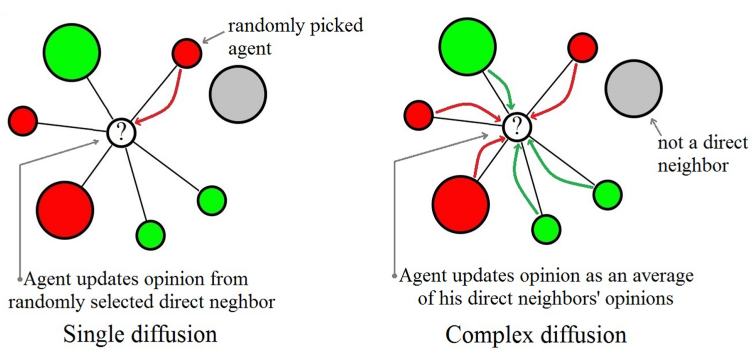

(c)

Figure 3. The interaction models, based on the two types of opinion representation and two types of diffusion. **a.** Taxonomy. **b.** Opinion representation types where the larger nodes (labeled with S) represent stubborn agents (or opinion sources) which can also have any value for opinion, with the property that their opinion value never changes. Discrete opinion (left): nodes have opinion 0 (red) or 1 (green) at any time (SD). Continuous opinion (right): nodes have any opinion between 0 and 1, highlighted by the color gradient transitioning from red to green (SC). **c.** A scenario highlighting the two opinion diffusion models for discrete representation. Single diffusion (left): the central white node picks one random neighbor and adopts his opinion (SD). Complex diffusion (right): the white node polls all neighbors for their opinion and then adopts an averaged opinion (CD). Note that only direct neighbors can influence opinion, not friends of friends etc. (e.g. the gray node).

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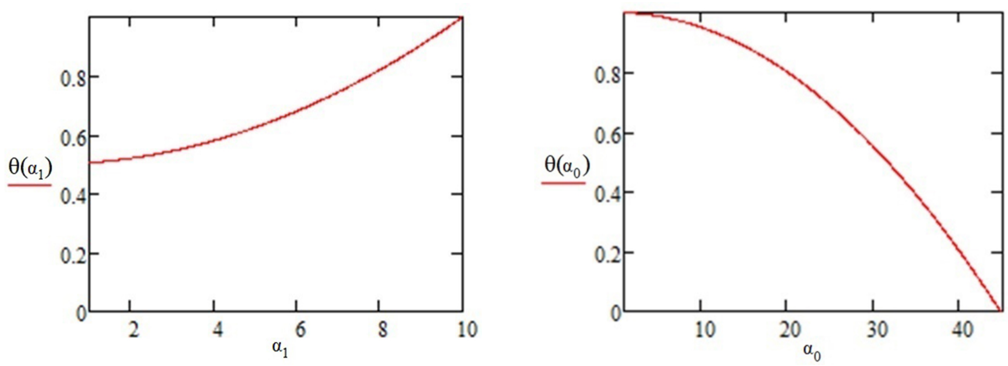
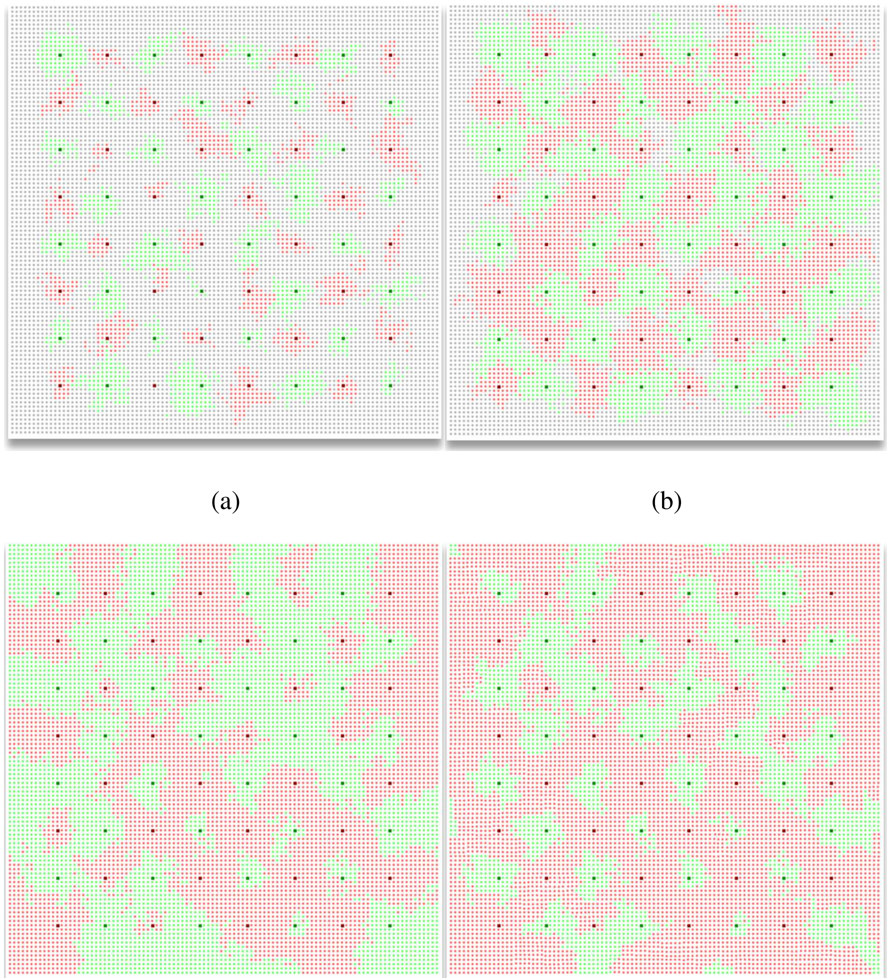


(a) (b)

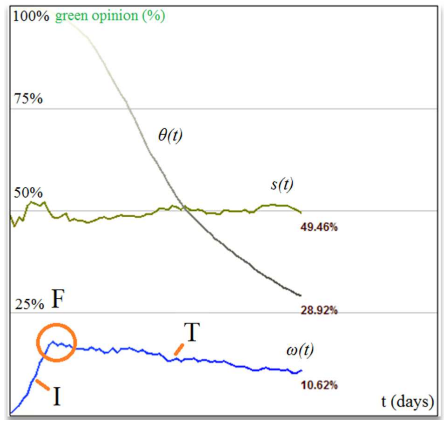
Figure 4. The tolerance function as defined by the progressive tolerance model. **a.** Tolerance scaling: shows how tolerance θ increases with the $\alpha_1\varepsilon_1$ scaling, as a result of continuous opinion change for an agent i. **b.** Intolerance scaling: shows how tolerance θ drops with the $\alpha_0\varepsilon_0$ scaling, from an initial tolerance $\theta_i(0) = 1$ to complete intolerance ($\theta_i(t) = 0$).



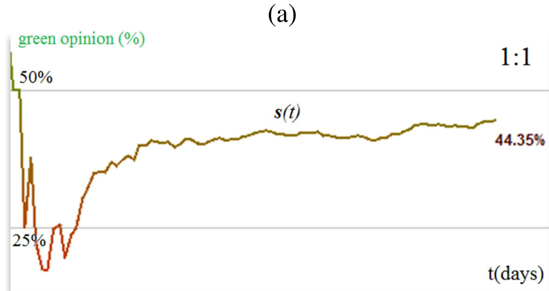
(a) (b)

(c) (d)

Figure 5. Green (1) vs. red (0) opinion evolution with homogeneous stubborn agent distribution in a 100,000 node social network. The network is initialized with 32 red and 32 green stubborn agents (represented as the darker nodes) which start influencing the neighboring regular agents. Initially, the regular agents have no opinion and are colored with grey. We distinguish between the following phases of opinion formation: **a.** The initiation phase I where the society has no opinion, i.e. the stubborn agents exercise their influence to the surrounding neighborhood without being affected by any other opinion. The opinion change $\omega(t)$ rises during this phase, whereas tolerance $\theta(t)$ remains high. **b.** The fusion phase F where the society is now mostly polarized (green or red) and different opinion clusters expand and collapse throughout the society. The opinion change $\omega(t)$ reaches a maximum, and tolerance $\theta(t)$ begins to slowly decrease. **c.** Tolerance phase T, where the cluster interaction stabilizes and new, larger, more stable clusters emerge. Most of the individuals within the society have been in contact with both opinions; each agent's opinion $s_i(t)$ begins to converge, and the tolerance $\theta(t)$ is steadily declining or becomes stable. **d.** Intolerance phase $\overline{T}$, where the overall tolerance of agents has decreased to a point where opinion fluctuation ceases and the red opinion becomes dominant ($\theta(t) < 0.1$). The society may or may not reach this phase.



(a)



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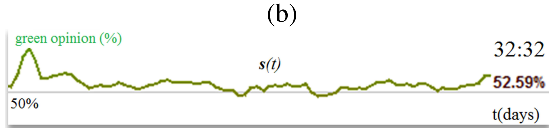

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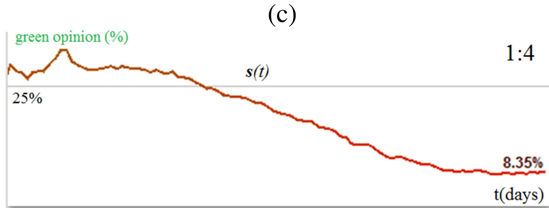

(d)

Figure 6. Simulation of a 100,000 mesh network with SocialSim (Topirceanu and Udrescu, 2014), displaying a representative example for the evolution of $s(t)$, $\theta(t)$, and $\omega(t)$, as well as the opinion evolution $s(t)$ with various stubborn agents distributions. **a.** Representative setup for for the mesh topology, where the lowest panel displays the opinion change (ω) evolution over three simulation phases: (I) initiation, (F) fusion, and (T) tolerance. The opinion state (s) and its tolerance (θ) are also displayed in the middle and upper panels. **b.** Opinion evolution $s(t)$ with few and evenly distributed SA (1:1 ratio: 1 green, 1 red). **c.** Opinion evolution with many and evenly distributed stubborn agents (1:1 ratio: 32 green, 32 red), **d.** Opinion evolution with few and unevenly distributed stubborn agents (1:4 ratio: 1 green, 4 red).

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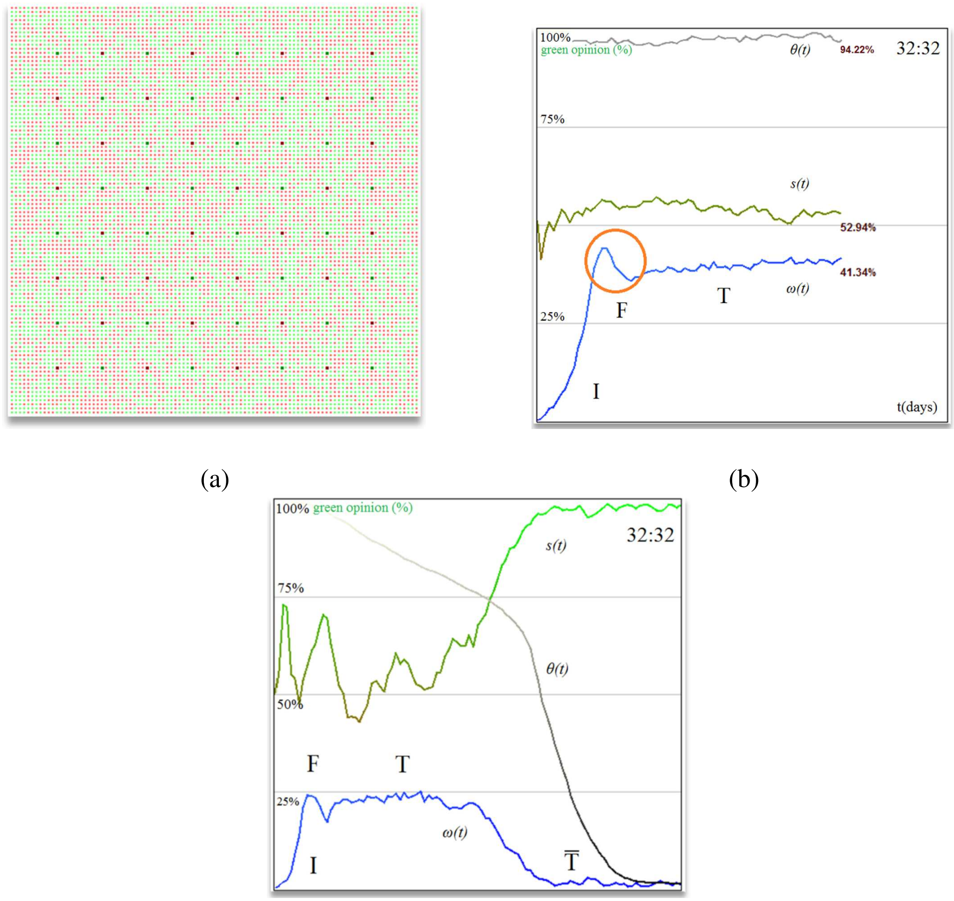

(a) (b)

(c)

Figure 7. Opinion evolution with homogeneous stubborn agent distribution (32:32) in small-world and scale-free networks. **a.** Tolerance phase where no visible clusters emerge for small-world networks. **b.** For small-world networks, social balancing is attained because tolerance remains extremely high ($\theta(t) > 90\%$), opinion change (ω) exhibits the three opinion evolution phases (initiation I, fusion F, and tolerance T), and never reaches intolerance. The state of the society $s(t)$ is stable. **c.** Social balancing is not achieved for scale-free networks: tolerance drops constantly and the society reaches the intolerance phase ($\overline{T}$). The state of the society $s(t)$ is unstable during the first three phases of opinion change, then stabilizes as tolerance (θ) and opinion change (ω) fall.

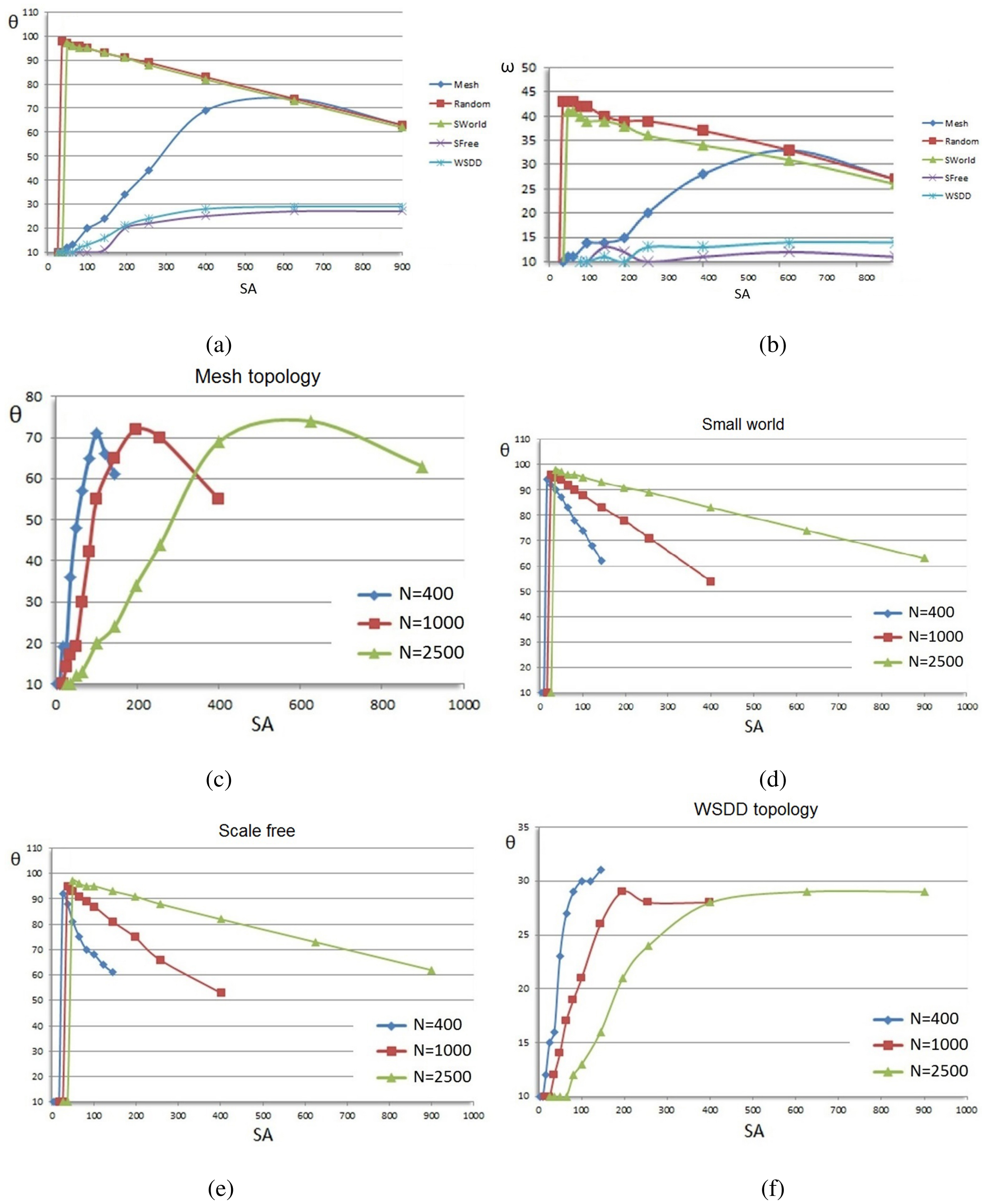

Figure 8. Tolerance (θ) and opinion change (ω) evolution with the increasing concentration of evenly distributed stubborn agents and increasing network sizes. values over the five topologies for an increasing concentration of evenly distributed stubborn agents. **a** and **b.** θ and ω respective values, over the five topologies when the size of the network is fixed as $N = 2500$, and the concentration of stubborn agents ranges from 4% to 36%. **c**, **d**, **e**, and **f.** Tolerance θ stabilization values at which social balancing occurs over increasing network sizes (N=400 to 2500 nodes) on mesh, small-world, scale-free, and WSDD networks, respectively.

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