# DPWSS: Differentially private working set selection for training support vector machines

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Support vector machine (SVM) is a robust machine learning method and is widely used in classification. However, the traditional SVM training methods may reveal personal privacy when the training data contains sensitive information. In the training process of SVMs, working set selection is a vital step for the sequential minimal optimization-type decomposition methods. To avoid complex sensitivity analysis and the influence of high-dimensional data on the noise of the existing SVM classifiers with privacy protection, we propose a new differentially private working set selection algorithm (DPWSS) in this paper, which utilizes the exponential mechanism to privately select working sets. We theoretically prove that the proposed algorithm satisfies differential privacy. The extended experiments show that the DPWSS algorithm achieves classification capability almost the same as the original non-privacy SVM under different parameters. The errors of optimized objective value between the two algorithms are nearly less than two, meanwhile, the DPWSS algorithm has a higher execution efficiency than the original non-privacy SVM by comparing iterations on different datasets. To the best of our knowledge, DPWSS is the first private working set selection algorithm based on differential privacy.

## 2 DPWSS: Differentially Private Working Set Selection for Training Support
## 3 Vector Machines

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9 **Abstract** Support vector machine (SVM) is a robust machine learning method and is widely used in classification.
10 However, the traditional SVM training methods may reveal personal privacy when the training data contains
11 sensitive information. In the training process of SVMs, working set selection is a vital step for the sequential
12 minimal optimization-type decomposition methods. To avoid complex sensitivity analysis and the influence of high-
13 dimensional data on the noise of the existing SVM classifiers with privacy protection, we propose a new
14 differentially private working set selection algorithm (DPWSS) in this paper, which utilizes the exponential
15 mechanism to privately select working sets. We theoretically prove that the proposed algorithm satisfies differential
16 privacy. The extended experiments show that the DPWSS algorithm achieves classification capability almost the
17 same as the original non-privacy SVM under different parameters. The errors of optimized objective value between
18 the two algorithms are nearly less than two, meanwhile, the DPWSS algorithm has a higher execution efficiency
19 than the original non-privacy SVM by comparing iterations on different datasets. To the best of our knowledge,
20 DPWSS is the first private working set selection algorithm based on differential privacy.

21 **Keywords** Differential privacy, exponential mechanism, sequential minimal optimization, support vector machines,
22 working set selection.

## 23 1. Introduction

24    In recent years, with the rapid development of artificial intelligence, cloud computing, and big data technologies,
25 data sharing and analysis are becoming easier and more practical. A large amount of individual information is stored
26 in electronic databases, such as economic records, medical records, web search records, and social network data,
27 which poses a great threat to personal privacy. Support vector machine (SVM) is one of the most widely used and
28 robust machine learning methods for classification. Boser et al.[1] proposed the earliest SVM classification idea by
29 maximizing the margin between the training patterns and the decision boundary. Cortes et al.[2] solved the
30 classification problem of non separable training data though non linearly mapping them to a very high dimension
31 feature space. Vapnik[3] considered three different kernels to construct learning machines with different types of
32 nonlinear decision surfaces in the input space. Bureges et al[4] gave an overview on linear SVMs and kernel SVMs
33 with numerous examples for pattern recognition. Chang and Lin[5] developed a popular library LIBSVM for SVMs
34 and presented all the implementation details. A SVM trains a classification model by solving an optimization
35 problem and requires only as few as a dozen examples for training. However, when the training data sets contain
36 sensitive information, directly releasing the SVM classification model may reveal personal privacy.

37    Generally speaking, training SVMs is to solve a large optimization problem of quadratic programming (QP).
38 Sequential minimal optimization (SMO)[6] is currently a commonly used decomposition method for training SVMs by
39 solving the smallest QP optimization problem, and only needs two elements in every iteration. Keerthi et al.[7]
40 employed two threshold parameters to derive modifications of SMO and it performed significantly faster than the
41 original SMO algorithm. In all kinds of SMO-type decomposition methods, working set selection (WSS) is an
42 important step. Different WSS algorithms determine the convergence efficiency of the SVM training process. Zuo et
43 al.[8] proposed an improved WSS and a simplified minimization step for the SMO-type decomposition method.

44    Differential privacy (DP)[9] was proposed by a series of work of Dwork et al. from 2006, which has been becoming
45 an accepted standard for privacy protection in sensitive data analysis. DP ensures that adding or removing a single item
46 does not affect the analysis outcome too much, and the privacy level is quantified by a privacy budget $\varepsilon$. DP is realized
47 by introducing randomness or uncertainty. According to the difference of data types, it mainly includes Laplace
48 mechanism[10], Gaussian mechanism, and exponential mechanism[11]. Among them, the Laplace mechanism and

49 Gaussian mechanism are mostly used for numerical data, while the exponential mechanism is used for non-numerical
50 data.

51  In this paper, we studied the privacy leakage problem of the traditional SVM training methods. There are some
52 shortcomings in the existing SVM classifiers with privacy protection, such as the low classification accuracy, the
53 requirements on the differentiability of the objective function, the complex sensitivity analysis, and the influence of
54 high-dimensional data on noise. We give a solution by introducing randomness in the training process of SVMs to
55 privately release the classification model. The main contributions in this paper conclude as follows:

56  ● We propose an improved WSS method for training SVMs and design a simple scoring function for the
57   exponential mechanism, in which the sensitivity is easy to analyze.

58  ● We propose a new differentially private working set selection algorithm (DPWSS) based on the exponential
59   mechanism, which is achieved by privately selecting the working set in every iteration.

60  ● To improve the utilization of the privacy budget, every violating pair is selected only once during the entire
61   training process.

62  ● We analyze theoretically that the DPWSS algorithm satisfies the requirement of DP, and evaluate the
63   classification accuracy, algorithm stability, and execution efficiency of the DPWSS algorithm versus the original
64   non-privacy SVM algorithm through extended experiments.

65  The rest of this paper is organized as follows. Section 2 discusses related work. Section 3 introduces the background
66 knowledge of SVMs, WSS, and DP. Section 4 proposes a novel DPWSS algorithm. Section 5 gives the experimental
67 evaluation of the performance of DPWSS. Lastly, Section 6 concludes the research work.

68 **2. Related work**

69  In this section, we briefly review some work related to privacy-preserving SVMs. Mangasarian et al.[12] considered
70 the classification problem of sharing private data by separating agents and proposed using random kernels for vertically
71 partitioned data. Lin et al.[13] pointed out an inherent privacy violation problem of support vectors and proposed a
72 privacy-preserving SVM classifier, PPSVC, which replaces the Gaussian kernel with an approximate decision function.
73 In these two methods, the degree of privacy protection cannot be proved as the private SVMs based on DP.

74  As DP is becoming an accepted standard for private data analysis, some SVM classification models based on DP
75 have produced in the recent two decades. Chaudhuri et al. proposed two popular perturbation-based techniques output
76 perturbation and objective perturbation[14][15]. Output perturbation introduces randomness into the weight vector $w$ after
77 the optimization process, and the randomness scale is determined by the sensitivity of $w$. On the contrary, objective
78 perturbation introduces randomness into the objective function before the optimization, and the randomness scale is
79 independent of the sensitivity of $w$. However, the sensitivity of the two perturbation-based techniques is difficult to
80 analyze[16] and the objective perturbation requires the loss function satisfying certain convexity and differentiability
81 criteria. Rubinstein et al.[17] proposed a private kernel SVM algorithm PrivateSVM for convex loss functions with
82 Fourier transformation and output perturbation to release the private SVM classification model. However, the
83 classification model is valid only for the translation-invariant kernels. To alleviate too much noise in the final outputs,
84 Li et al.[18] developed a hybrid private SVM model that uses a small portion of public data to calculate the Fourier
85 transformation. However, public data is hard to obtain in the modern private world. Zhang et al.[19] constructed a novel
86 private SVM classifier by dual variable perturbation, which adds Laplace noise to the corresponding dual variables
87 according to the ratio of errors.

88  Different from those kinds of perturbation-based techniques mentioned above, which introduce randomness into the
89 output result or objective function, the DPWSS algorithm introduces randomness during the process of WSS. Therefore,
90 it avoids complex sensitivity analysis and the influence of high-dimensional data on noise, meanwhile improves the
91 performance of the classification model to some extent.

92 **3. Preliminaries**

93 In this section, we introduce some background knowledge of SVM, WSS, and DP. Table 1 summarizes the notations
94 in the following sections.

95 **Table 1.**

96 **3.1. Support vector machines**

97 The SVM is an efficient classification method in machine learning that originates from structural risk minimization[3].
98 It finds an optimal separating hyperplane with the maximal margin to train a classification model. Given training
99 instances $x_i \in R^n$ and labels $y_i \in \{1,-1\}$, the main task for training a SVM is to solve the QP optimization problem as
100 follows[20]:

101
$$\min_{\alpha} f(\alpha) = \frac{1}{2}\alpha^T Q\alpha - e^T\alpha \qquad (1)$$

102
$$\text{Subject to} \quad 0 \le \alpha_i \le C, i = 1,...,l \,,$$

103
$$y^T\alpha = 0 \,,$$

104

105 where $Q$ is a symmetric matrix with $Q_{ij}=y_iy_jK(x_i,x_j)$, and $K$ is the kernel function, $e$ is a vector with all 1's, $C$ is the
106 upper bound of vector $\alpha$.

107 **3.2. Working set selection**

108 Generally, the QP problem is hard to solve in the training process of the SVMs. When the optimization methods
109 handle the large matrix $Q$, the whole vector $\alpha$ will be updated repeatedly in the iterative process. Nevertheless, the
110 decomposition methods only update a subset of vector $\alpha$ in every iteration to solve the challenge and change from one
111 iteration to another. The subset is called the working set. The method for determining the working set is called WSS,
112 which originally derives from the optimality conditions of Karush-Kuhn-Tucker (KKT). Furthermore, SMO-type
113 decomposition methods restrict the working set to only two elements[6]. A pair of elements that violate the KKT
114 optimality conditions are called "violating pair"[7].

115 **Definition 1 (Violating pair[7][20]).** Under the following restrictions:

116
$$I_{up}(\alpha) \equiv \{t \mid \alpha_t < C, y_t = 1 \ or \ \alpha_t > 0, y_t = -1\} \,, \qquad (2)$$

117
$$I_{low}(\alpha) \equiv \{t \mid \alpha_t < C, y_t = -1 \ or \ \alpha_t > 0, y_t = 1\} \,. \qquad (3)$$

118 For the $k^{th}$ iteration, if $i \in I_{up}(\alpha^k)$, $j \in I_{low}(\alpha^k)$, and $-y_i\nabla f(\alpha^k)_i > -y_j\nabla f(\alpha^k)_j$, then $\{i,j\}$ is a "violating pair".

119 Violating pairs are important in WSS. If working set $B$ is a violating pair, the function value in SMO-type
120 decomposition methods strictly decreases[21]. Under the definition of violating pair, a natural choice of the working set
121 $B$ is the "maximal violating pair", which most violates the KKT optimality condition.

122 **WSS 1 (WSS via the "maximal violating pair"[7][20][24]).** Under the same restrictions (2) and (3) in Definition 1,

123 1. Select

124
$$i \in \arg\max_t\{-y_t\nabla f(\alpha^k)_t \mid t \in I_{up}(\alpha^k)\} \,, \qquad (4)$$

125
$$j \in \arg\min_t\{-y_t\nabla f(\alpha^k)_t \mid t \in I_{low}(\alpha^k)\} \,, \qquad (5)$$

126
$$\text{or } j \in \arg\max_t \{y_t \nabla f(\alpha^k)_t \mid t \in I_{low}(\alpha^k)\}. \tag{6}$$

127    2. Return $B = \{i, j\}$.

128    Keerthi et al.[7] first proposed the maximal violating pair, which has become a popular way in WSS. Fan et al.[20]
129    pointed out that it was concerned with the first order approximation of $f(\alpha)$ in (1) and gave a detailed explanation.
130    Meanwhile, they proposed a new WSS algorithm by using more accurate second order information.

131    **WSS 2 (WSS using second order information[20][24]).**
132
        1. Define $a_{it}$ and $b_{it}$,

133
$$a_{it} \equiv K_{ii} + K_{tt} - 2K_{it}, \tag{7}$$

134
$$b_{it} \equiv -y_i \nabla f(\alpha^k)_i + y_t \nabla f(\alpha^k)_t > 0, \tag{8}$$

135
$$\bar{a}_{it} \equiv \begin{cases} a_{it} & if\ a_{it} > 0, \\ \tau & otherwise. \end{cases} \tag{9}$$

136
        2. Select

137
$$i \in \arg\max_t \{-y_t \nabla f(\alpha^k)_t \mid t \in I_{up}(\alpha^k)\}, \tag{10}$$

138
$$j \in \arg\min_t \left\{ -\frac{b_{it}^2}{\bar{a}_{it}} \mid t \in I_{low}(\alpha^k), -y_t \nabla f(\alpha^k)_t < -y_i \nabla f(\alpha^k)_i \right\}. \tag{11}$$

139    3. Return $B = \{i, j\}$.
140
141    WSS 2 uses second order information and checks only $O(l)$ possible working sets to select $j$ through using the same
142    $i$ as in WSS 1. The WSS 2 algorithm achieves faster convergence than existing selection methods using first order
143    information. It has been used in the software LIBSVM[5] (since version 2.8) and is valid for all symmetric kernel
        matrices $K$, including the non-positive definite kernel.

144
145    Lin[22][23] pointed out the maximal violating pair was important to SMO-type methods. When the working set $B$ is the
146    maximal violating pair, SMO-type methods converge to a stationary point. Otherwise, it is uncertain whether the
147    convergence will be established. Chen et al.[24] proposed a general WSS method via the "constant-factor violating pair".
148    Under a fixed constant-factor $\sigma$ specified by the user, the selected violating pair is linked to the maximal violating pair.
149    The "constant-factor violating pair" is considered to be a "sufficiently violated" pair. And they prove the convergence
        of the WSS method.

150
        **WSS 3 (WSS via the "constant-factor violating pair"[20][24]).**
151
        1. Given a fixed $0 < \sigma \leq 1$ in all iterations.
152
        2. Compute

153
$$m(\alpha^k) = \max_t \{-y_t \nabla f(\alpha^k)_t \mid t \in I_{up}(\alpha^k)\}, \tag{12}$$

154
$$M(\alpha^k) = \min_t \{-y_t \nabla f(\alpha^k)_t \mid t \in I_{low}(\alpha^k)\}. \tag{13}$$

155    3. Select $i, j$ satisfying

156
$$i \in I_{up}(\alpha^k) , \quad j \in I_{low}(\alpha^k) , \tag{14}$$

157
$$- y_i \nabla f(\alpha^k)_i + y_j \nabla f(\alpha^k)_j \geq \sigma(m(\alpha^k) - M(\alpha^k)) > 0 . \tag{15}$$

158    4. Return $B = \{i, j\}$.
159

160    Clearly (15) guarantees the quality of the working set $B$ if it is related to the maximal violating pair. Fan et al.[20]
explained that WSS 2 was a special case of WSS 3 under the special value of $\sigma$.

161

162    Furthermore, Zhao et al.[25] employed algorithm WSS 2 to test the datasets by LIBSVM. They find two interesting
163    phenomena. One is that some $\alpha$ are not updated in the entire training process. Another is that some $\alpha$ are updated again
164    and again. Therefore, they propose a new method WSS-WR and a certain $\alpha$ are selected only once to improve the
efficiency of WSS, especially the reduction of the training time.

165    **3.3. Differential privacy**

166    Recently, with the advent of the digital age, huge amounts of personal information have been collected by web
167    services and mobile devices. Although data sharing and mining large-scale personal information can help improve the
168    functionality of these services, it also raises privacy concerns for data contributors. DP provides a mathematically
169    rigorous definition of privacy and has become a new accepted standard for private data analysis. It ensures that any
170    possible outcome of an analysis is almost equal regardless of an individual's presence or absence in the dataset, and the
171    output difference is controlled by a relatively small privacy budget. The smaller the budget, the higher the privacy.
172    Therefore, the adversary cannot distinguish whether an individual's in the dataset[26]. Furthermore, DP is compatible
173    with various kinds of data sources, data mining algorithms, and data release models.

174    In dataset $D$, each row corresponds to one individual, and each column represents an attribute value. If two datasets
175    $D$ and $D'$ only differ on one element, they are defined as neighboring datasets. DP aims to mask the different results of
176    the query function $f$ in neighboring datasets. The maximal difference of the query results is defined as the sensitivity $\Delta f$.
177    DP is generally achieved by a randomized mechanism $M : D \rightarrow R^d$ , which returns a random vector from a probability
178    distribution. A mechanism $M$ satisfies DP if the affection of the outcome probability by adding or removing a single
179    element is controlled within a small multiplicative factor[27]. The formal definition is given as follows.

180    **Definition 2 ($\varepsilon$-differential privacy**[9]**).** A randomized mechanism $M$ gives $\varepsilon$-DP if for all datasets $D$ and $D'$ differing
181    on at most one element, and for all subsets of possible outcomes $S \subseteq \mathrm{Range}(M)$,

182
$$\Pr[M(D) \in S] \leq \exp(\varepsilon) \times \Pr[M(D') \in S] . \tag{16}$$

183    Sensitivity is a vital concept in DP that represents the largest affection of the query function output made by a single
184    element. Meanwhile, sensitivity determines the requirements of how much perturbation by a particular query
185    function[28].

186    **Definition 3 (Sensitivity**[9]**).** For a given query function $f : D \rightarrow R^d$ , and neighboring datasets $D$ and $D'$, the
187    sensitivity of $f$ is defined as

188
$$\Delta f = \max_{D, D'} \left\| f(D) - f(D') \right\|_1 . \tag{17}$$

189    The sensitivity $\Delta f$ depends only on the query function $f$, and not on the instances in datasets.

190    Any mechanism that meets Definition 2 is considered as satisfying DP[27]. Currently, two principal mechanisms have
191    been used for realizing DP: the Laplace mechanism[11] and the exponential mechanism[10].

192    **Definition 4 (Laplace mechanism**[11]**).** For a numeric function $f : D \rightarrow R^d$ on a dataset $D$, the mechanism $M$ in Eq.
193    (18) provides $\varepsilon$-DP.

194
$$M(D) = f(D) + Lap(\frac{\Delta f}{\varepsilon})^d .$$
(18)

195     The Laplace mechanism gets the real results from the numerical query and then perturbs it by adding independent
196     random noise. Let $Lap(b)$ represent the random noise sampled from a Laplace distribution according to sensitivity. The
197     Laplace mechanism is usually used for numerical data, while for the non-numerical queries, DP uses the exponential
198     mechanism to randomize results.

199     **Definition 5 (Exponential mechanism[10]).** Let $q(D,r)$ be a score function on a dataset $D$ that measures the quality of
200     output $r \in R$, $\Delta q$ represents the sensitivity. The mechanism $M$ satisfies $\varepsilon$-DP if

201
$$M(D) = \left( return\ r \propto \exp\left( \frac{\varepsilon q(D,r)}{2\Delta q} \right) \right).$$
(19)

202     The exponential mechanism is useful to select a discrete output in a differentially private manner, which employs a
203     score function $q$ to evaluate the quality of an output $r$ with a nonzero probability.

204     **4. DPWSS algorithm**

205     In this paper, we study the problem of how to privately release the classification model of SVMs while satisfying DP.
206     To overcome the shortcomings of the privacy-preserving SVM classification methods, such as low accuracy or
207     complex sensitivity analysis of output perturbation and objective perturbation, we proposed the algorithm DPWSS for
208     training SVM in this section.   The DPWSS algorithm is achieved by privately selecting the working set with the
209     exponential mechanism in every iteration. As far as we know, DPWSS is the first private WSS algorithm based on DP.

210     **4.1. An improved WSS method**

211     In the process of training SVMs, WSS is an important step in SMO-type decomposition methods. Meanwhile, the
212     special properties of the selection process in WSS are perfectly combined with the exponential mechanism of DP. WSS
213     3 algorithm is a more general algorithm to select a working set by checking nearly $O(l^2)$ possible $B$'s to decide $j$,
214     although under the restricted condition of parameter $\sigma$. By using the same $i \in \arg m(a^k)$ as in WSS 2, which checks
215     only $O(l)$ possible $B$'s, we propose WSS 4 to select a working set based on WSS 3 as below. To make the algorithm
216     easy to understand, we replace $M(a^k)$ with $M'(a^k)$.
217

**WSS 4 (An improved WSS via the "constant-factor violating pair")**
218
    1. Given a fixed $0 < \sigma \le 1$ in all iterations.
219
    2. Compute

220
$$m(\alpha^k) = \max_t \{ -y_t \nabla f(\alpha^k)_t \mid t \in I_{up}(\alpha^k) \} ,$$
(20)

221
$$M'(\alpha^k) = \max_t \{ y_t \nabla f(\alpha^k)_t \mid t \in I_{low}(\alpha^k) \} .$$
(21)

222     3. Select $i, j$ satisfying

223
$$i \in \arg m(\alpha^k) ,\ \ j \in I_{low}(\alpha^k) ,$$
(22)

224
$$m(\alpha^k) + y_j \nabla f(\alpha^k)_j \ge \sigma(m(\alpha^k) + M'(\alpha^k)) > 0 .$$
(23)

225     4. Return $B = \{i, j\}$.

226 **4.2. The score function and sensitivity in the exponential mechanism**

227  In the exponential mechanism, the scoring function is an important guarantee for achieving DP. The rationality of
228 scoring function design is directly related to the execution efficiency of mechanism $M$. For one output $r$, the greater the
229 value of the scoring function, the greater the probability that $r$ will be selected. Based on the definition of the "maximal
230 violating pair", it is obvious that

$$m(\alpha^k) + M'(\alpha^k) \geq m(\alpha^k) + y_j \nabla f(\alpha^k)_j \,. \tag{24}$$

232 From Neq. (23) and (24), we conclude that

$$m(\alpha^k) + M'(\alpha^k) \geq m(\alpha^k) + y_j \nabla f(\alpha^k)_j \geq \sigma(m(\alpha^k) + M'(\alpha^k)) > 0 \,. \tag{25}$$

234 We design a simple scoring function $q(D, r)$ for the DPWSS algorithm based on WSS 4 and Neq. (25) as follows

$$1 \geq q(D,r) = \frac{m(\alpha^k) + y_j \nabla f(\alpha^k)_j}{m(\alpha^k) + M'(\alpha^k)} \geq \sigma \,, \tag{26}$$

236 where $r$ denotes the working set $B$, which contains violating pair $i$ and $j$. The larger the value of scoring function $q(D, r)$,
237 the closer the selected violation pair is to the maximal violation pair. The sensitivity of scoring function $q(D, r)$ is

$$\Delta q = 1 - \sigma \,, \tag{27}$$

239 and the value of $\Delta q$ is a small number, less than 1.

240  In the exponential mechanism, the output $r$ is selected randomly with probability

$$\Pr(r) = \exp\left(\frac{\varepsilon q(D,r)}{2\Delta q}\right) \Big/ \sum_{r' \in R} \exp\left(\frac{\varepsilon q(D,r')}{2\Delta q}\right). \tag{28}$$

242 **4.3. Privacy budget**

243  Privacy budget is a vital parameter in DP, which controls the privacy level in a randomized mechanism $M$. The
244 smaller the privacy budget, the higher the privacy level. When the allocated privacy budget runs out, mechanism $M$ will
245 lose privacy protection, especially for the iteration process. To improve the utilization of the privacy budget, every pair
246 of working sets is selected only once during the entire training process as in [25]. Meanwhile, in DPWSS every
247 iteration is based on the result of the last iteration, but not based on the entire original dataset. Therefore, there is no
248 need to split the privacy budget for every iteration.

249 **4.4. Description of DPWSS algorithm**

250  In the DPWSS algorithm, DP is achieved by privately selecting the working set with the exponential mechanism in
251 every iteration. We first present an overview of the DPWSS algorithm and then elaborate on the key steps. Finally, we
252 describe an SMO-type decomposition method using the DPWSS algorithm in detail.

253  The description of the DPWSS algorithm is shown below.

254

---

**Algorithm 1** DPWSS

---

**Input:** $G$: gradient array; $y$: array of every instance labels with $\{+1, -1\}$; $l$: number of instances; $\alpha$: dual vector; $I$: the violating pair selected flag bool matrix; $\sigma$: constant-factor; $\varepsilon$: privacy budget; $eps$: stopping tolerance;

**Output:** $B$: working set;

---

**Begin**

1:    initialize $m(\alpha)$ and $M'(\alpha)$ to -INF;

2:    find $m(\alpha)$ by Eq. (20) for $t$ in [0:$l$-1] and $t$ in $I_{up}(\alpha)$;

3:    set $i = t$;

4:    find $M'(\alpha)$ by Eq. (21) for $t$ in [0:$l$-1] and $t$ in $I_{low}(\alpha)$;

5:    for $t = 0$ to $l$-1

6:        if $I[i][t] == false$ and $t$ in $I_{low}(\alpha)$ and $m(\alpha)+y[t]*G[t] > eps$ then

7:            compute scoring function $q(D, r_t)$ by Neq. (26);

8:            if $q(D, r_t) \geq \sigma$ then

9:                $q'(D, r_t) \leftarrow q(D, r_t)$;

10:           end if

11:       end if

12:   end for

13:   compute the probability $\Pr(B)$ for every violating pair by Eq. (28);

14:   randomly select a violating pair as a working set with probability $\Pr(B)$;

15:   Return $B = \{i, j\}$;

**End**

255

256    The DPWSS algorithm selects multiple violating pairs that meet the constraints based on WSS 4, and then randomly
257    selects one with a certain probability by the exponential mechanism to satisfy DP. Firstly, the DPWSS algorithm
258    computes $m(\alpha)$ and $M'(\alpha)$ for the scoring function $q$ from Line 1 to Line 4 and determines $i$ as one element of the
259    violating pair. Secondly, it computes the scoring function $q$ from Line 5 to Line 12. The constraints in Line 6 represent
260    that the violating pair $\{i, j\}$ has not been previously selected, meanwhile the value range of the other element $j$ and the
261    violating pair are valid for the changes of gradient $G$. The constraints in Line 8 represent that the scoring function value
262    is effective under constant-factor $\sigma$. Line 14 and Line 15 are key steps in the exponential mechanism, which randomly
263    select a violating pair with the chosen probability of the scoring function $q$. Lastly, the DPWSS algorithm outputs the
264    violating pair $\{i, j\}$ as the working set $B$ in Line 15. The time and memory complexity of DPWSS algorithm is $O(l)$.

265    In summary, a SMO method using the DPWSS algorithm is shown below.

266

---

**Algorithm 2** A SMO method using DPWSS

**Input:** $Q$: kernel symmetric matrix; $y$: array of every instance labels with $\{+1, -1\}$; $l$: number of instances; $C$: upper bound of all dual variables;

**Output:** $\alpha$: dual vector;

**Begin**

1:    initialize gradient array $G$ to all -1, dual vector $\alpha$ to all 0, and violating pair selected flag bool matrix $I$ to all 0;

2:    find $\alpha^I$ as the initial feasible solution, set $k = 1$;

3:    **while** $k <$ max_iter

4:       **if** $\alpha^k$ is a stationary point **then**

5:          exit the loop;

6:       **else**

7:          select working set $B = \{i, j\}$ by DPWSS;

8:          if $B$ is *NULL* then

9:             exit the loop;

10:        end if

11:      **end if**

12:      set $k = k+1$;

13:      set $I[i][j] = \text{true}$;

14:      update $\alpha[i]$ and $\alpha[j]$;

15:      project $\alpha$ back to the feasible region;

16:      update gradient $G$;

17:  **end while**

18:  return $\alpha$;

**End**

267

268      Algorithm 2 is an iterative process, which first selects working set $B$ by DPWSS, then updates dual vector $\alpha$ and
269      gradient $G$ in every iteration. After the iterative process, the algorithm outputs the final $\alpha$. There are three ways to get
270      out of the iterative process. One is that $\alpha$ is a stationary point, another is that all violating pairs have been selected, and
271      the last one is that the number of iterations exceeds the maximum value. Using Algorithm 2, we privately release the
272      classification model of SVMs with dual vector $\alpha$ while satisfying the requirement of DP.

273      **4.5. Privacy analysis**

274      In the DPWSS algorithm, randomness is introduced by randomly selecting working sets with the exponential
275      mechanism. By using the exponential mechanism, a violating pair is selected randomly with a certain probability. The
276      greater the probability, the closer the selected violating pair is to the maximal violating pair. For every iteration, the
277      violating pair in the outputs of the DPWSS algorithm is uncertain. The uncertainty masks the impact of individual
278      record change on the algorithm results, thus protecting the data privacy.

279      According to the definition of DP mentioned in Section 3, we proved that the DPWSS algorithm satisfies DP strictly
280      by theorem 1 as shown below.

281      **Theorem 1**  DPWSS algorithm satisfies DP.

282      **Proof**  Let $M(D, q)$ be to select the output $r$ of the violating pair in one iteration, and $\varepsilon$ be the allocated privacy budget
283      in the DPWSS algorithm. Based on Eq. (28), we randomly select violating pair $r$ as a working set with the following
284      probability by the exponential mechanism. To accord with the standard form of the exponential mechanism, we use $q$
285      to denote $q'$ in the DPWSS algorithm.

286
$$\frac{\Pr(M(D,q)=r)}{\Pr(M(D',q)=r)}=\frac{\exp\left(\frac{\varepsilon q(D,r)}{2\Delta q}\right)\bigg/\sum_{r'\in O}\exp\left(\frac{\varepsilon q(D,r')}{2\Delta q}\right)}{\exp\left(\frac{\varepsilon q(D',r)}{2\Delta q}\right)\bigg/\sum_{r'\in O}\exp\left(\frac{\varepsilon q(D',r')}{2\Delta q}\right)}=\exp\left(\frac{\varepsilon(q(D,r)-q(D',r))}{2\Delta q}\right)\times\frac{\sum_{r'\in O}\exp\left(\frac{\varepsilon q(D',r')}{2\Delta q}\right)}{\sum_{r'\in O}\exp\left(\frac{\varepsilon q(D,r')}{2\Delta q}\right)}$$

287
$$\leq\exp\left(\frac{\varepsilon}{2}\right)\times\frac{\sum_{r'\in O}\exp\left(\frac{\varepsilon}{2}\right)\exp\left(\frac{\varepsilon q(D,r')}{2\Delta q}\right)}{\sum_{r'\in O}\exp\left(\frac{\varepsilon q(D,r')}{2\Delta q}\right)}=\exp\left(\frac{\varepsilon}{2}\right)\times\exp\left(\frac{\varepsilon}{2}\right)\times\frac{\sum_{r'\in O}\exp\left(\frac{\varepsilon q(D,r')}{2\Delta q}\right)}{\sum_{r'\in O}\exp\left(\frac{\varepsilon q(D,r')}{2\Delta q}\right)}=\exp(\varepsilon)$$

288      According to Definition 2, we prove that

289
$$\Pr(M(D,q)=r)\leq\exp(\varepsilon)\times\Pr(M(D',q)=r)\,.$$

290      Therefore, the DPWSS algorithm satisfies DP.

291      Algorithm 2 is an iterative process, in which DPWSS is a vital step to privately select a working set. As the DPWSS
292 algorithm satisfies DP, we perform the steps of updating dual vector $\alpha$ and gradient $G$ in every iteration without
293 accessing private data. To improve the utilization of the privacy budget, every pair of working sets is selected only once
294 during the entire training process. Meanwhile, in Algorithm 2 every iteration is based on the result of the last iteration,
295 but not based on the original datasets. Therefore, Algorithm 2 satisfies DP.

296 **5. Experiments**

297      In this section, we compared the performance of the DPWSS algorithm with WSS 2, which is a classical non-private
298 WSS algorithm and has been used in the software LIBSVM[5]. The comparison between WSS 2 and WSS 1 was done
299 in [20]. We do not compare the DPWSS algorithm with other private SVMs. One reason is that randomness is
300 introduced in different ways, and the other reason is that the DPWSS algorithm achieves classification accuracy and
301 optimized objective value almost the same as the original non-privacy SVM algorithm.

302 **5.1. Datasets and experimental environment**

303      The datasets are partly selected for the experiments as [19], [20], and [25]. All datasets are for binary classification,
304 and available at http://www.csie.ntu.edu.tw/~cjlin/libsvmtools/. The basic information of the datasets includes dataset
305 size, value range, number of features, and imbalance ratio, which is shown in Table 2 below. To make the figures look
306 neater in the experiments, we use breast to denote the breast-cancer dataset and german to denote the german.number
307 dataset.

308                                        **Table 2.**

309      To carry out the contrast experiments efficiently, we use LIBSVM (version 3.24) as an implementation of the
310 DPWSS algorithm in C++ language and GNU Octave (version 5.2). All parameters are set to default values.

311 **5.2. An example of a private classification model**

312      Unlike other privacy SVMs, which introduce randomness into the objective function or classification result by the
313 Laplace mechanism, in our method the randomness is introduced into the training process of SVM. It is achieved by
314 privately selecting the working set with the exponential mechanism in every iteration. We give an example of a private
315 classification model to show how privacy is protected in Figure 1. The data uses two columns of the heart dataset and
316 moves the positive and negative instances to each end for easier classification. The solid lines represent the original
317 non-private classification model and circles represent support vectors. The dotted lines represent a private classification
318 model by training SVM with the DPWSS algorithm. It is observed that the differences between the private and non-
319 private classification models are very small, and achieves similar accuracy of classification. All the classification
320 models generated are different from each other to protect the training data privacy.

321 **Figure 1.**

322 **5.3. Algorithm performance experiments**

323     In this section, we evaluated the performance of the DPWSS algorithm versus WSS 2 by experiments for the entire
324 training process. The metrics of performance include classification capability, algorithm stability, and execution
325 efficiency under different constant-factor $\sigma$ and privacy budget $\varepsilon$.

326     The classification capability is measured by *AUC*, *Accuracy*, *Precision*, *Recall*, *F1*, and *Mcc*.

327
$$AUC = \frac{\sum_{i \in positiveClass} rank_i - \frac{M(1+M)}{2}}{M \times N} \tag{29}$$

328 The $rank_i$ denotes the serial number of instance $i$ after sorting by the probability, $M$ is the number of positive instances
329 and $N$ is the number of negative instances. The higher the $AUC$, the better the usability of the algorithm. Other metrics
330 are calculated as shown below, and they are all based on confusion matrix.

331
$$Accuracy = \frac{TP+TN}{TP+TN+FP+FN} \tag{30}$$

332
$$Precision = \frac{TP}{TP+FP} \tag{31}$$

333
$$Recall = \frac{TP}{TP+FN} \tag{32}$$

334
$$F1 = \frac{2TP}{2TP+FP+FN} \tag{33}$$

335
$$Mcc = \frac{TP \times TN - FP \times FN}{\sqrt{(TP+FP)(TP+FN)(TN+FP)(TN+FN)}} \tag{34}$$

336     The algorithm stability is measured by the error of optimized objective value between DPWSS algorithm and WSS 2,
337 named *objError*.

338
$$objError = \left| obj_{DPWSS} - obj_{WSS2} \right| \tag{35}$$

339 The smaller the *objError*, the better the stability of the algorithm.

340     The execution efficiency of the algorithm is measured by the ratio of iteration between the two algorithms, named
341 *iterationRatio*.

342
$$iterationRatio = \frac{\#iteration\ with\ DPWSS}{\#iteration\ with\ WSS2} \tag{36}$$

343 The smaller the *iterationRatio*, the better the execution efficiency of the algorithm. We do not compare the training
344 time between the two algorithms as it is a millisecond class for the entire training process to most of the datasets.

345     To evaluate the influence of different constant-factor $\sigma$ and privacy budget $\varepsilon$ to the three metrics for algorithm
346 performance, we set $\sigma$ at 0.1, 0.3, 0.5 and 0.7 under $\varepsilon$ fixed at 1 and set $\varepsilon$ at 0.1, 0.5 and 1 under $\sigma$ fixed at 0.7. We do
347 not set $\sigma$ at 0.9, because under the circumstances, most of the violating pairs will be filtered out that the algorithm fails
348 to reach the final objective value.

349   Firstly, we measure the classification capability of the DPWSS algorithm versus WSS 2. The experiments for the
350 DPWSS algorithm were repeated 5 times under different $\sigma$ and $\varepsilon$, and the averages of the experimental results are
351 shown in Table 3. Observed from the results, the DPWSS algorithm achieves almost the same classification capability
352 as WSS 2 on all datasets. The maximum error between them is no more than 3%. Due to the repeated execution of the
353 iterative process, the DPWSS algorithm obtains a well private classification model. The classification capability is not
354 affected by the randomness of DP and the filtering effect of parameter $\sigma$ on violating pairs. The DPWSS algorithm
355 introduces randomness into the training process of SVMs, not into the objective function or classification result. There
356 are no requirements of the differentiability of the objective function and the complex sensitivity analysis, and the less
357 influence of high-dimensional data on noise. Therefore, the DPWSS algorithm achieves the target extremum through
358 the optimization process under the current condition. Meanwhile, the imbalance of dataset has little effect on the
359 classification capability of the DPWSS algorithm.

360                    **Table 3.**

361   Secondly, we compare the optimized objective values and measure the algorithm stability by *objError* between the
362 DPWSS algorithm and WSS 2. The experimental results are shown in Figure 2 to Figure 5. Observed from the results,
363 the DPWSS algorithm achieves similar optimized objective values with WSS 2 on all datasets under different $\sigma$ and $\varepsilon$.
364 The errors between the DPWSS algorithm and WSS 2 are very small (nearly within two). Due to the repeated
365 execution of the iterative process, the DPWSS algorithm converges stably to optimized objective values and is not
366 affected by the randomness of DP and the filtering effect of parameter $\sigma$ on violating pairs. With the increase of $\sigma$, the
367 errors also tends to increase.

368 **Figure 2.**

369 **Figure 3.**

370 **Figure 4.**

371 **Figure 5.**

372   Lastly, we compare the iterations and measure the execution efficiency by *iterationRatio* between the two algorithms.
373 The experimental results are shown in Figure 6 to Figure 21. Observed from the results, the DPWSS algorithm
374 achieves higher execution efficiency with fewer iterations versus WSS 2 on all datasets under different $\sigma$ and $\varepsilon$.
375 Because the DPWSS algorithm introduces randomness into the WSS process, the iterations will increase more or less.
376 However, with the increase of constant-factor $\sigma$, the iterations are affected by the filtering effect of it on violating pairs
377 larger and larger. When $\sigma$ increases to 0.3, the execution efficiency of the DPWSS algorithm is already higher than
378 WSS 2 for most datasets. When $\sigma$ increases to 0.7, the iterations of the DPWSS algorithm are far less than WSS2 for all
379 datasets except ijcnn1. Therefore, our method should set larger $\sigma$ for big datasets. While the privacy budget $\varepsilon$ has little
380 effect on iterations under a fixed constant-factor $\sigma$.

381 **Figure 6.**

382 **Figure 7.**

383 **Figure 8.**

384 **Figure 9.**

385 **Figure 10.**

386 **Figure 11.**

387 **Figure 12.**

388 **Figure 13.**

389    **Figure 14.**

390    **Figure 15.**

391    **Figure 16.**

392    **Figure 17.**

393    **Figure 18.**

394    **Figure 19.**

395    **Figure 20.**

396    **Figure 21.**

397    In the above experiments, we compared the average results of 5 times running of the DPWSS algorithm with the
398    WSS 2 algorithm. It can be seen from the experimental results that the two algorithms have similar classification
399    capability and optimized objective value under different parameter combinations. Under the same set of parameters,
400    the experimental results of DPWSS algorithm differ little each time, and the main difference lies in the iterations.
401    These slight differences show that the DPWSS algorithm has good usability while satisfying DP. Due to the
402    limitations of the paper, we have not listed each running result in the experiments.

403    **6. Conclusions**

404    In this paper, we study the privacy leakage problem of the traditional SVM training methods. The DPWSS algorithm
405    was proposed to release a private classification model of SVM and theoretically proved to satisfy DP through utilizing
406    the exponential mechanism to privately select working sets in every iteration. The extended experiments show that the
407    DPWSS algorithm achieves similar classification capability and the optimized objective value as the original non-
408    privacy SVM under different parameters. Meanwhile, the DPWSS algorithm has a higher execution efficiency by
409    comparing iterations on different datasets. In the DPWSS algorithm, randomness is introduced in the training process.
410    The most prominent advantages include that there are no requirements for differentiability of the objective function and
411    complex sensitivity analysis compared with objective perturbation or output perturbation methods. And a number of
412    training set selection methods can be easily combined with the DPWSS algorithm for large-scale training problems that
413    require large memory and enormous amounts of training time.  Because the DPWSS algorithm doesn't change the
414    training process of the classical non-privacy SVMs, it is also suitable for multi-class classification. It's a challenge that
415    parameter setting of the constant-factor $\sigma$ for different datasets. The idea of introducing randomness into the
416    optimization process can be easily extended to other privacy-preserving machine learning algorithms, and how to
417    ensure that the method meets the DP requirements is another challenge. Furthermore, the DPWSS algorithm is valid to
418    release a private classification model for linear SVM, while invalid for other non-linear kernel SVM as the privacy
419    disclosure problem of the support vectors in kernel function. In future work, we will study how to release a private
420    classification model for non-linear kernel SVMs.

421

422    **References**

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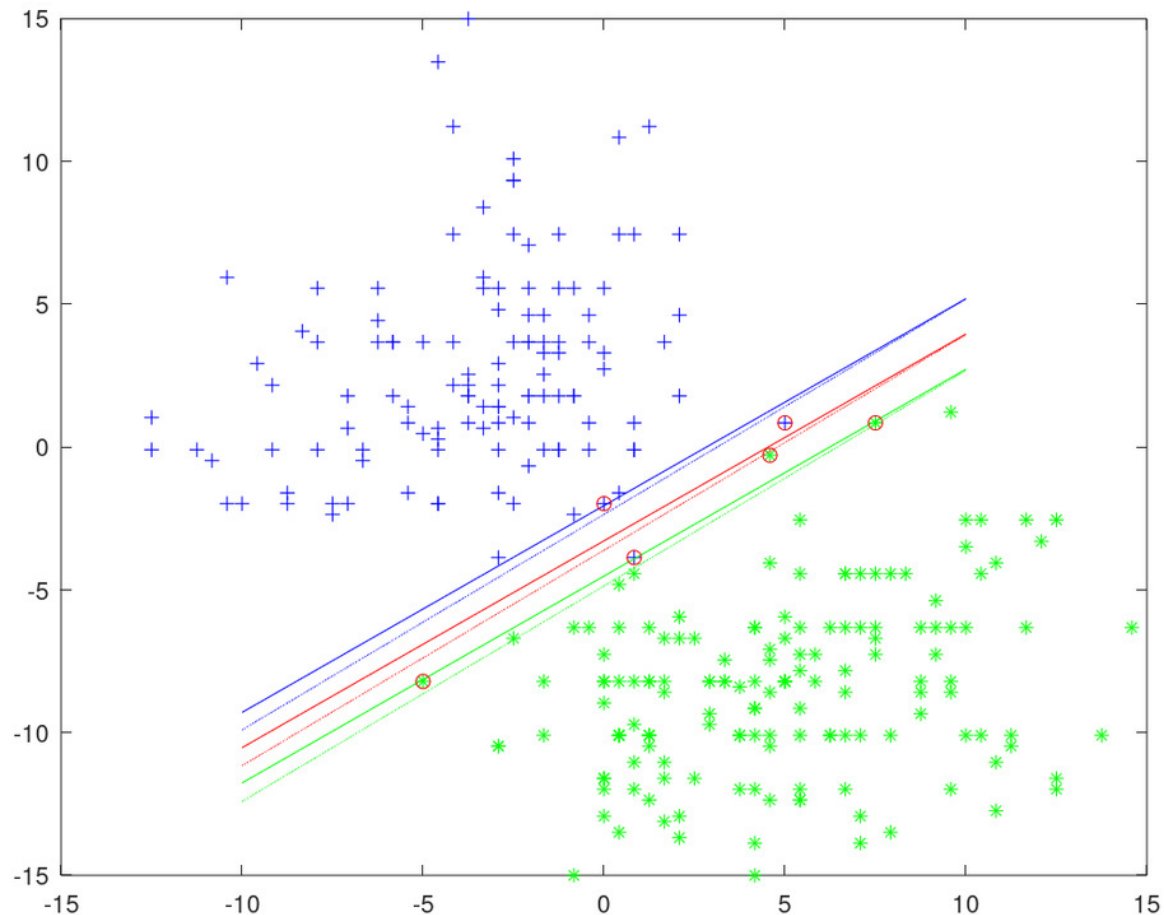
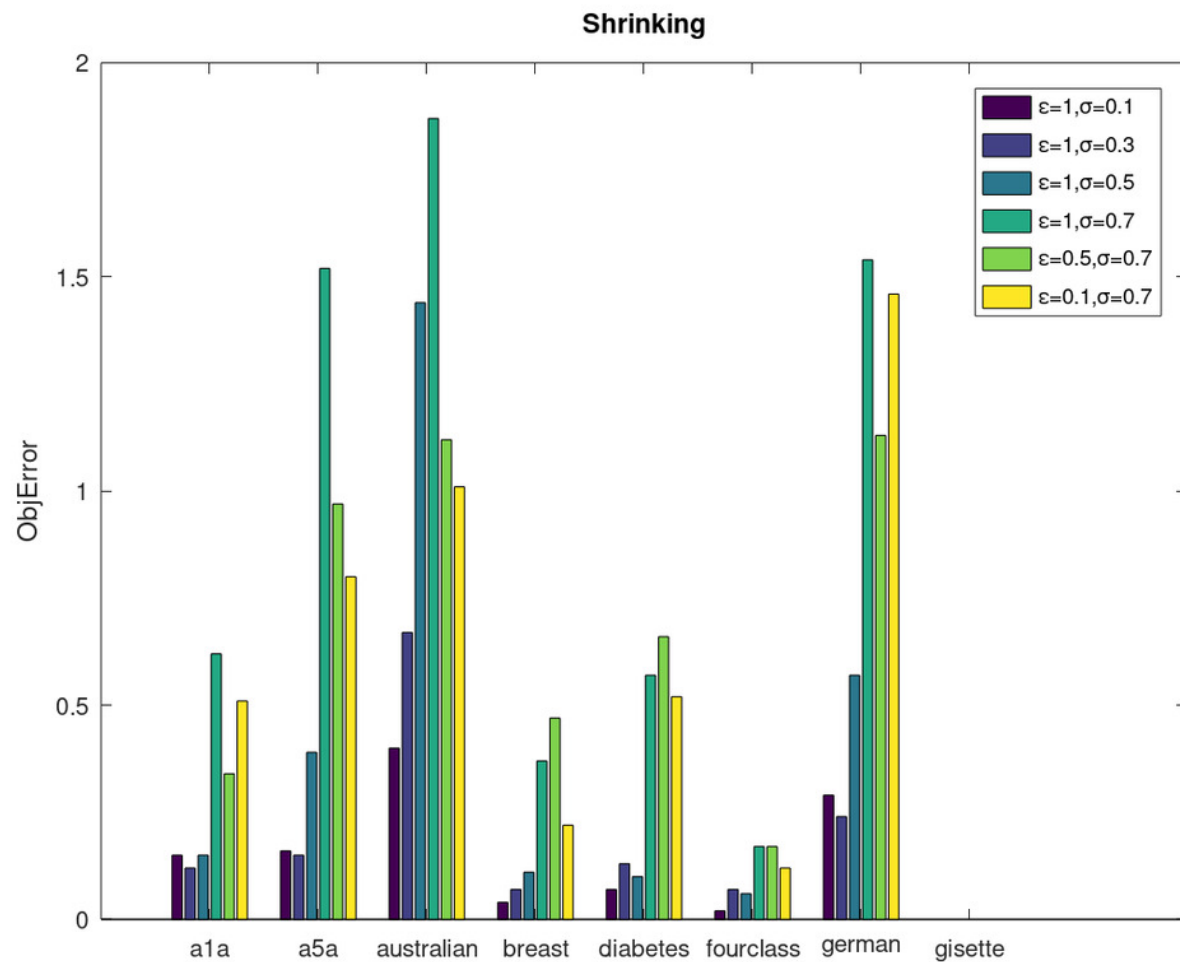
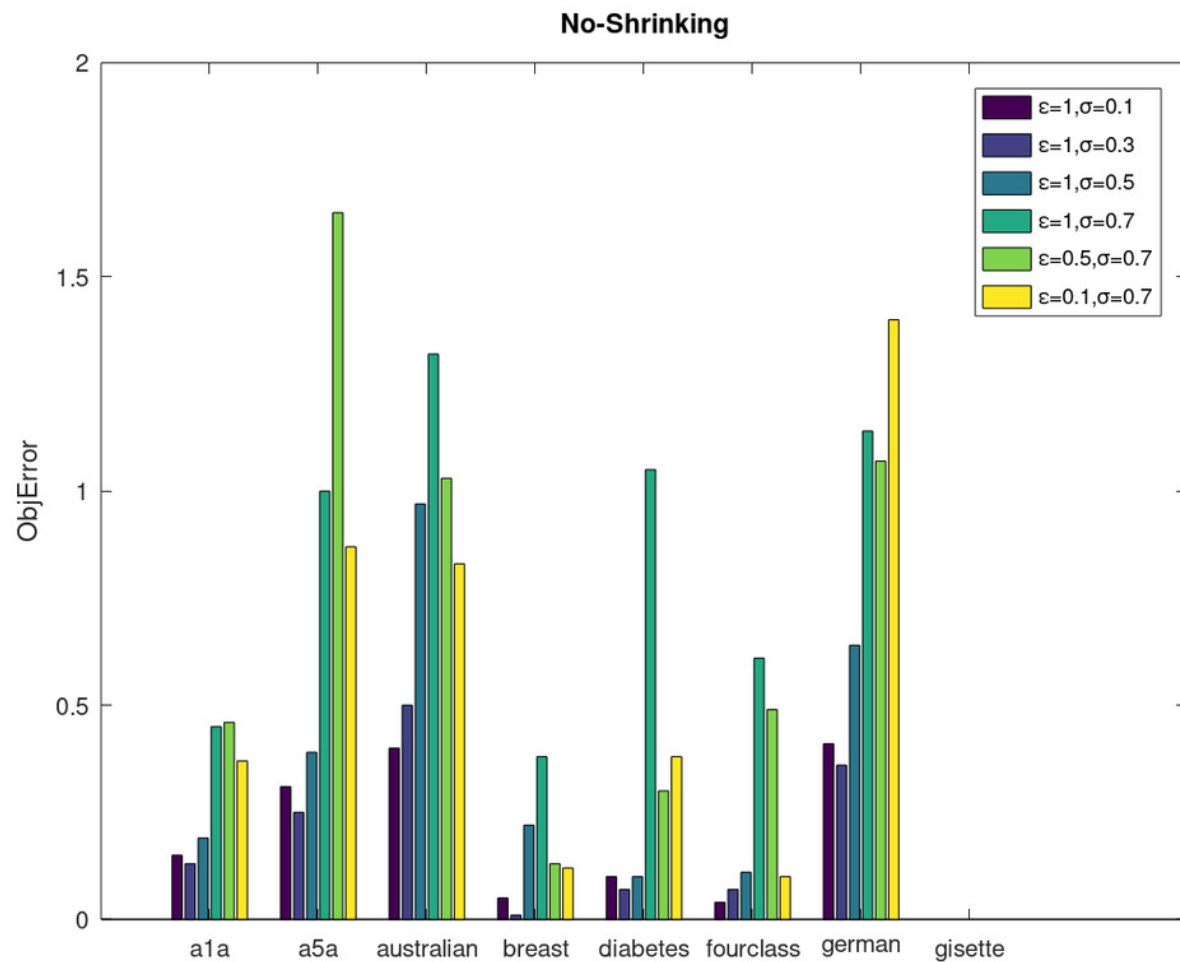
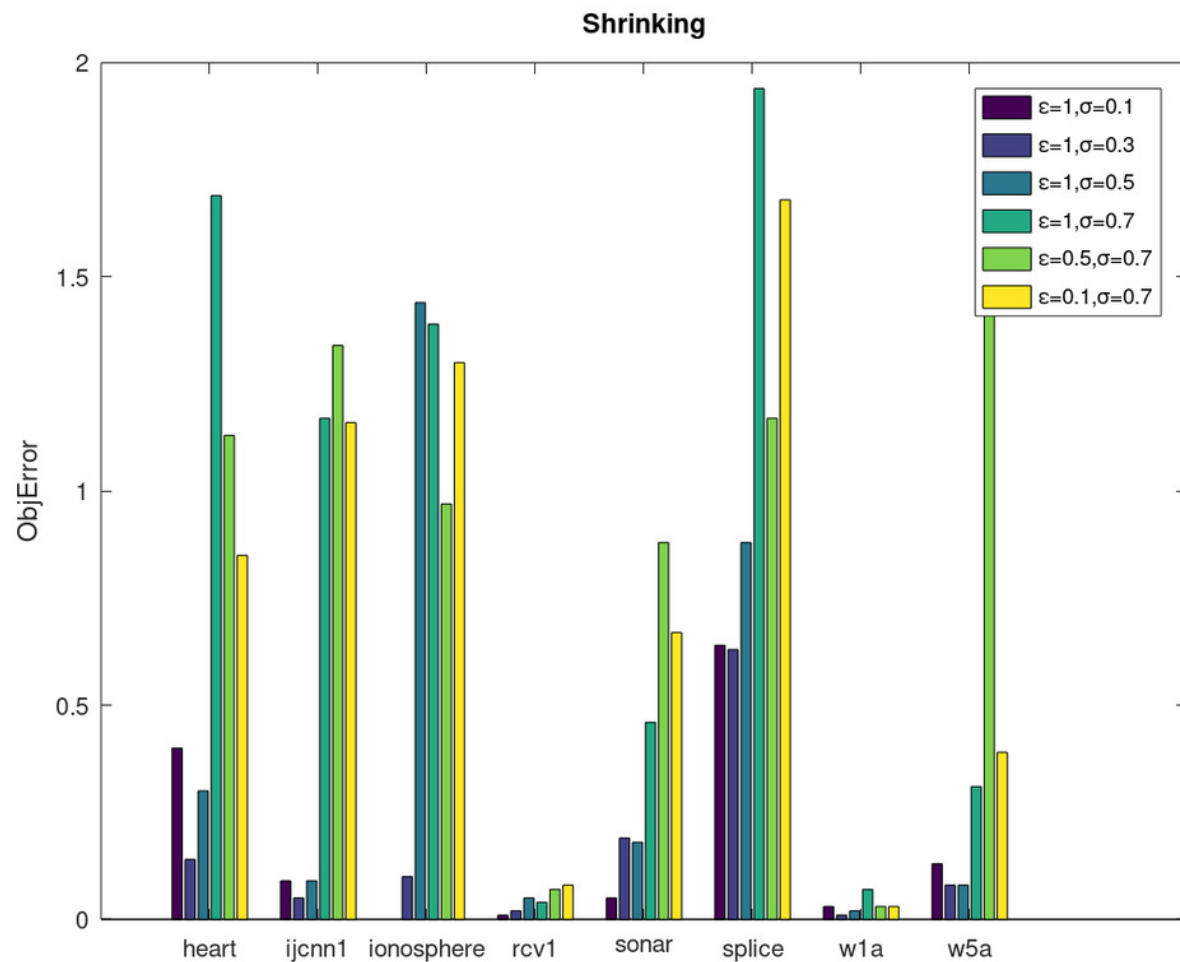
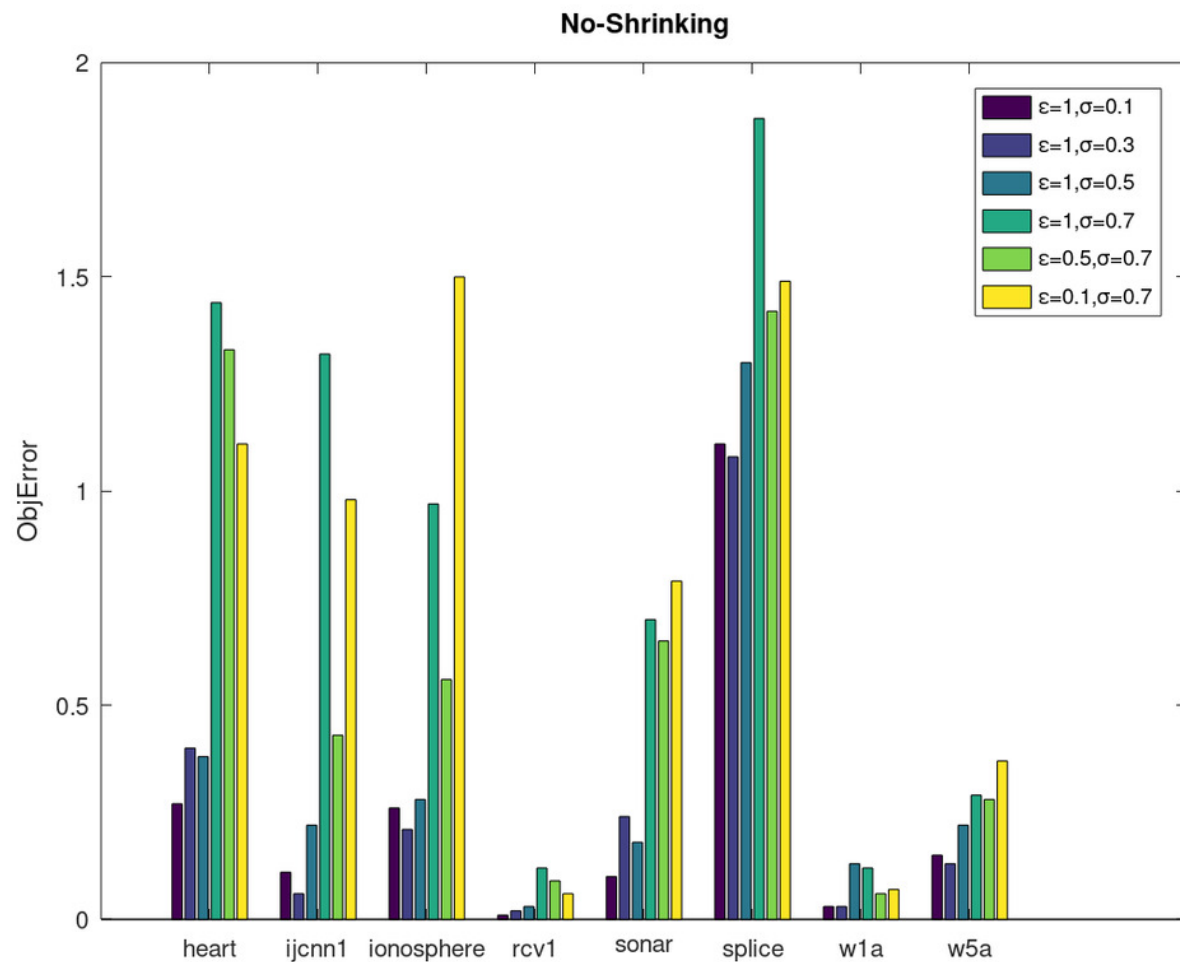
# Figure 1

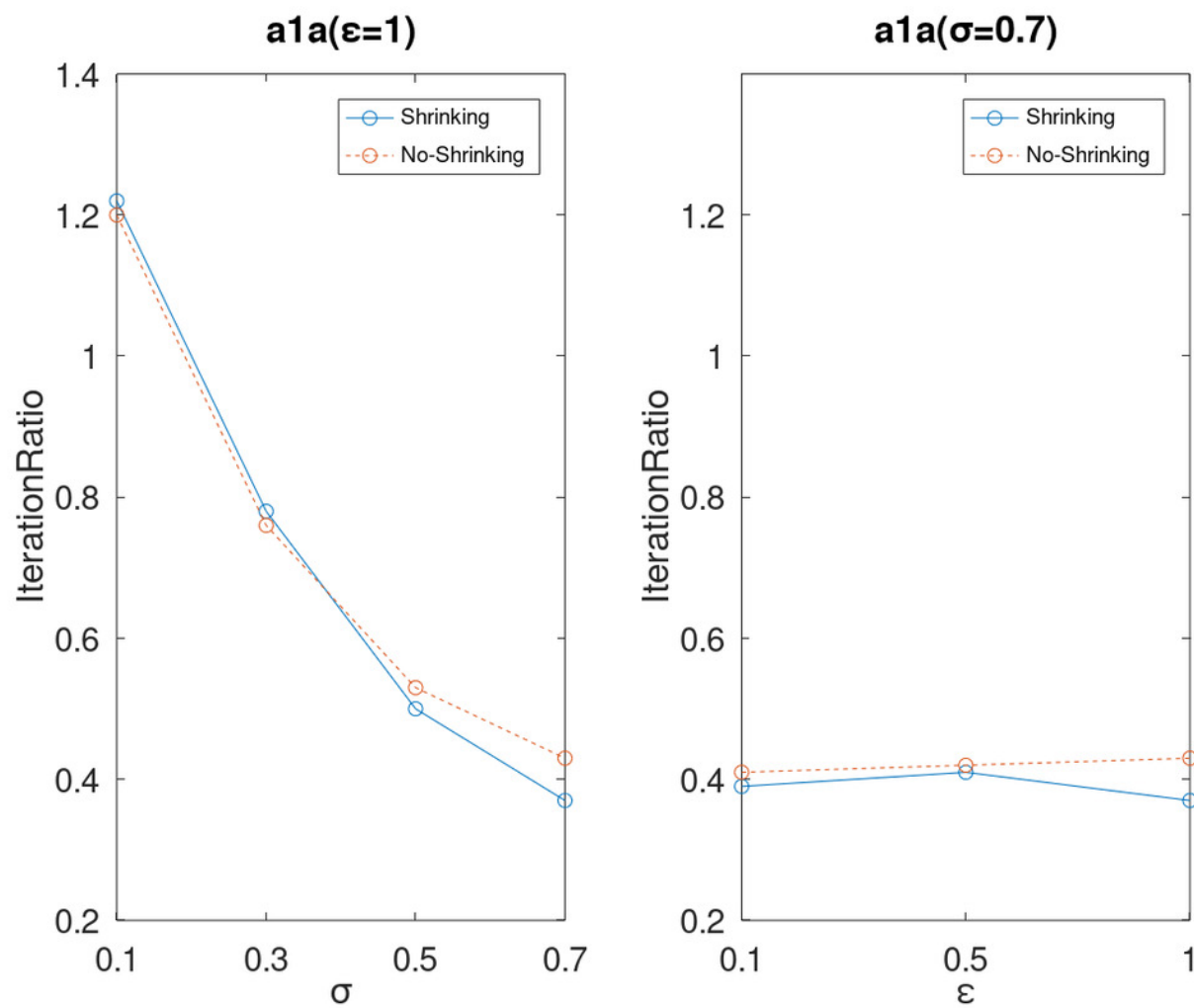
An example of a private classification model.

# Figure 2

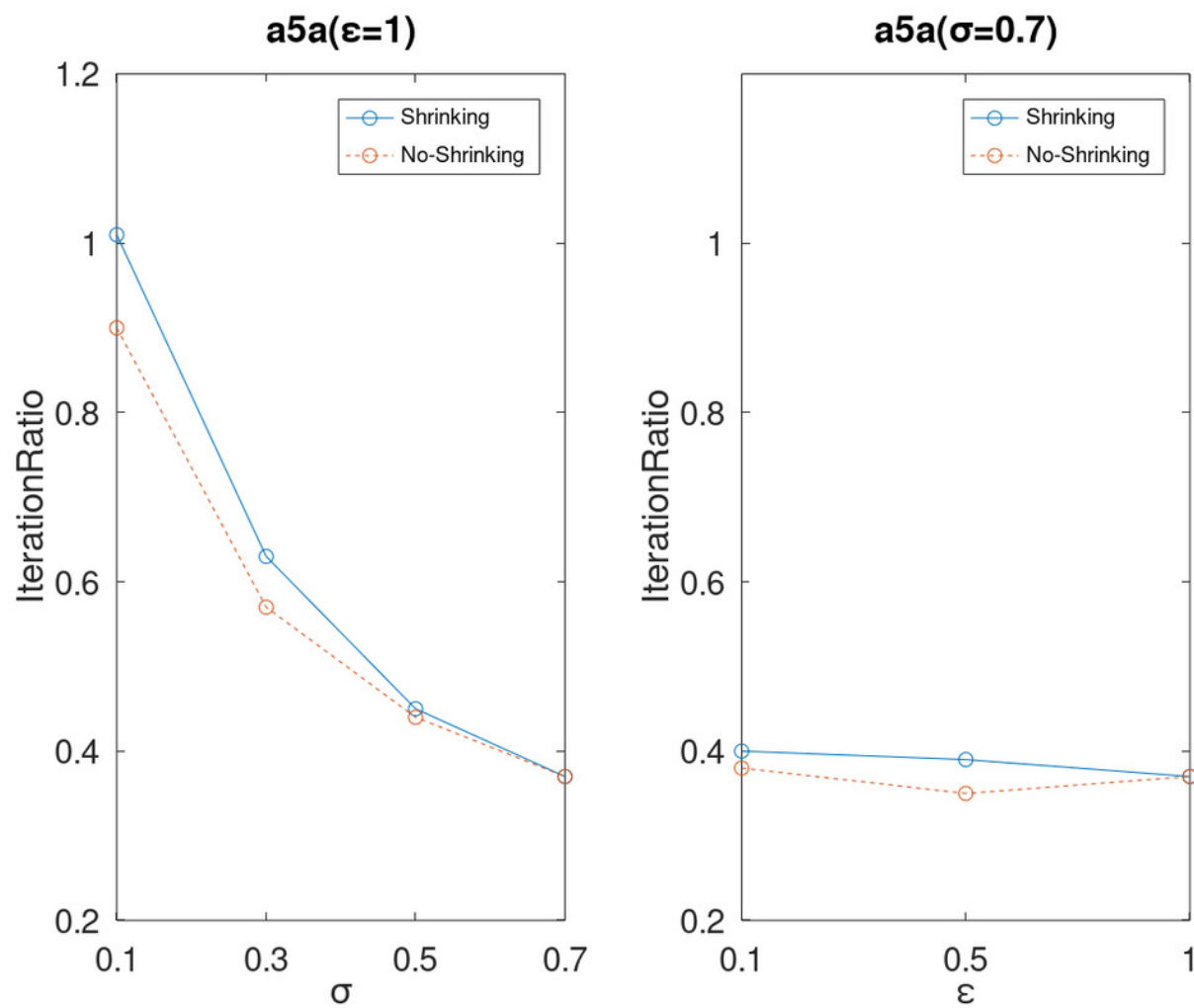
The algorithm stability of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset 1 to 8 with shrinking.

# Figure 3

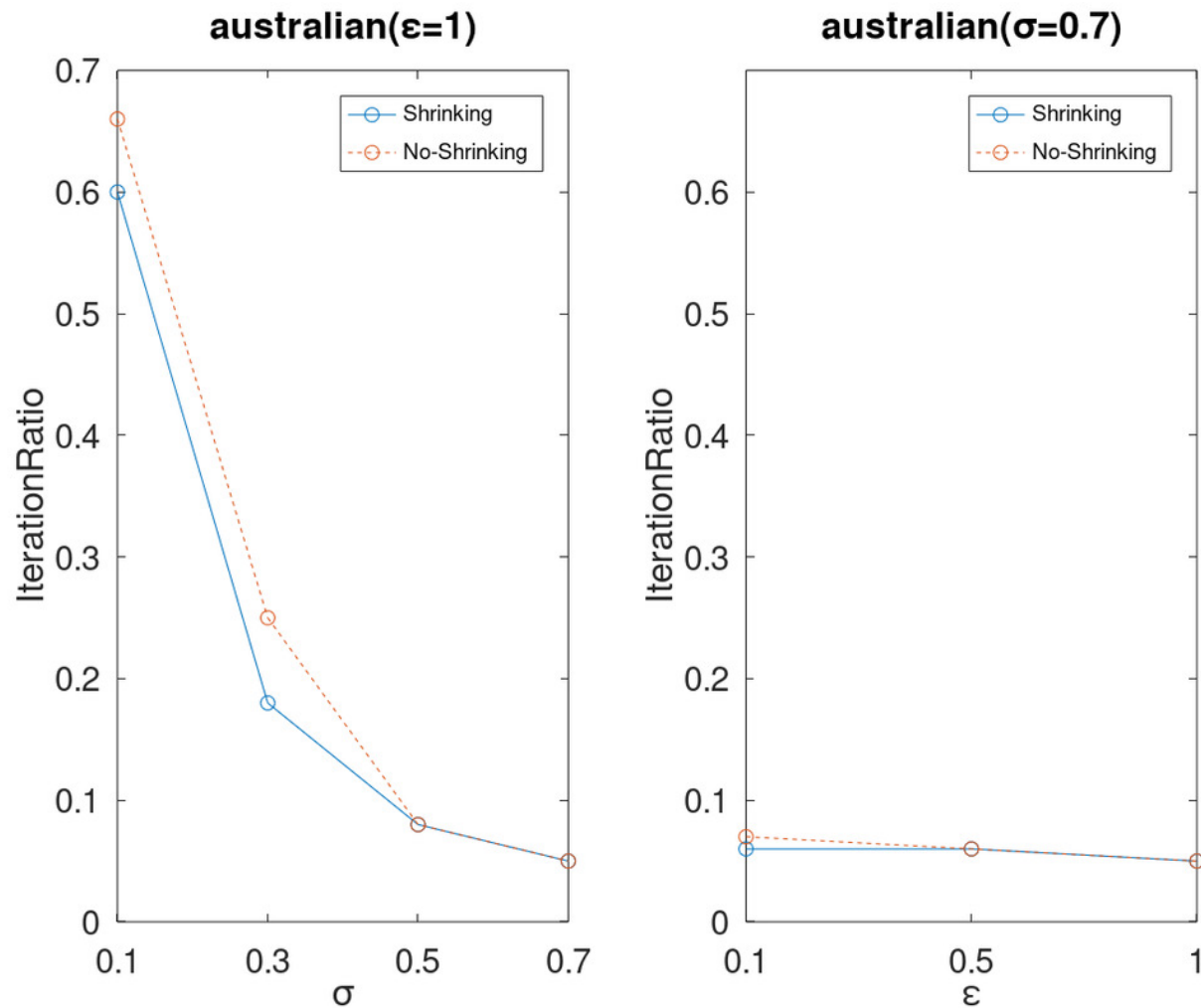
The algorithm stability of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset 1 to 8 without shrinking.

# Figure 4

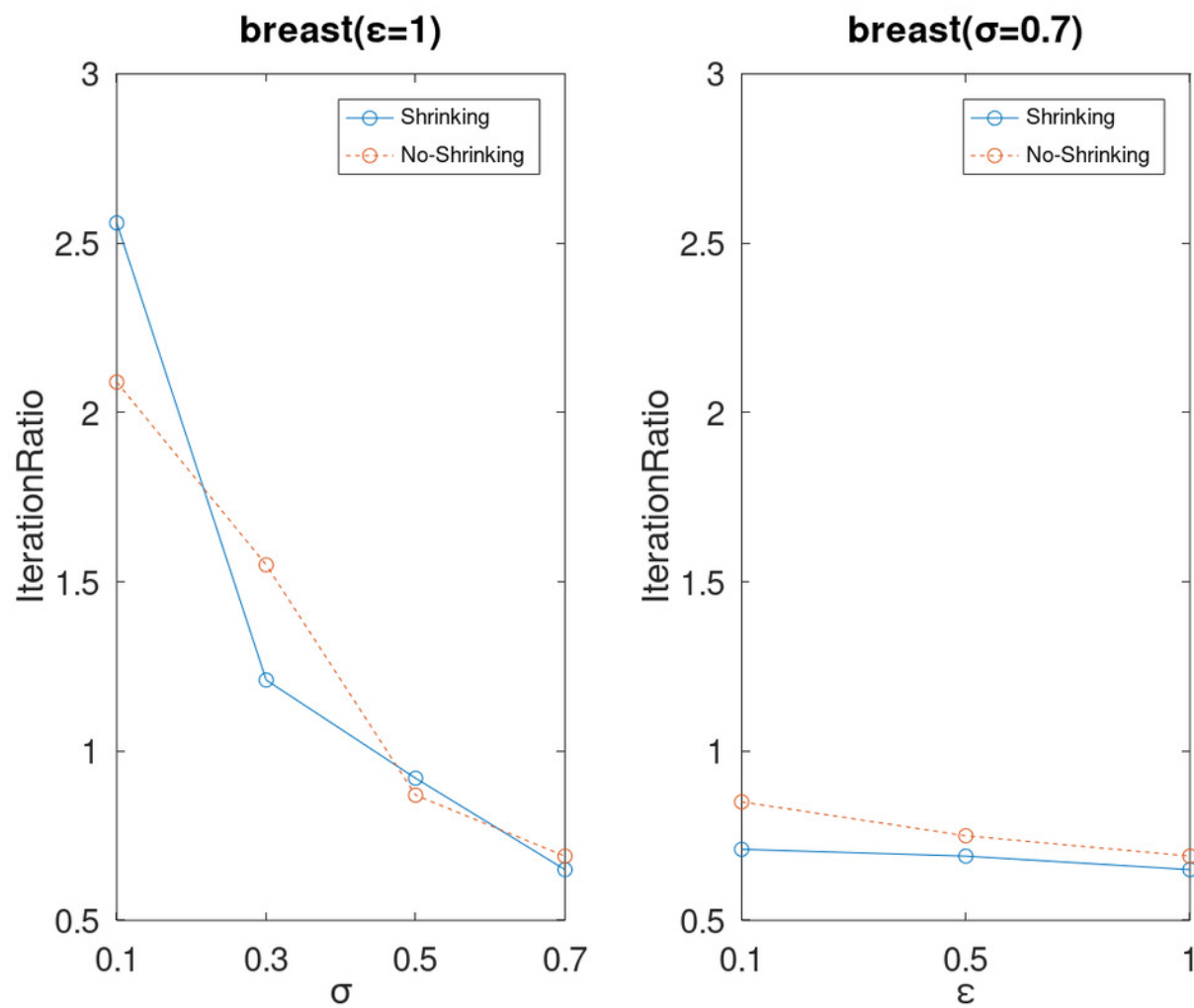
The algorithm stability of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset 9 to 16 with shrinking.

# Figure 5

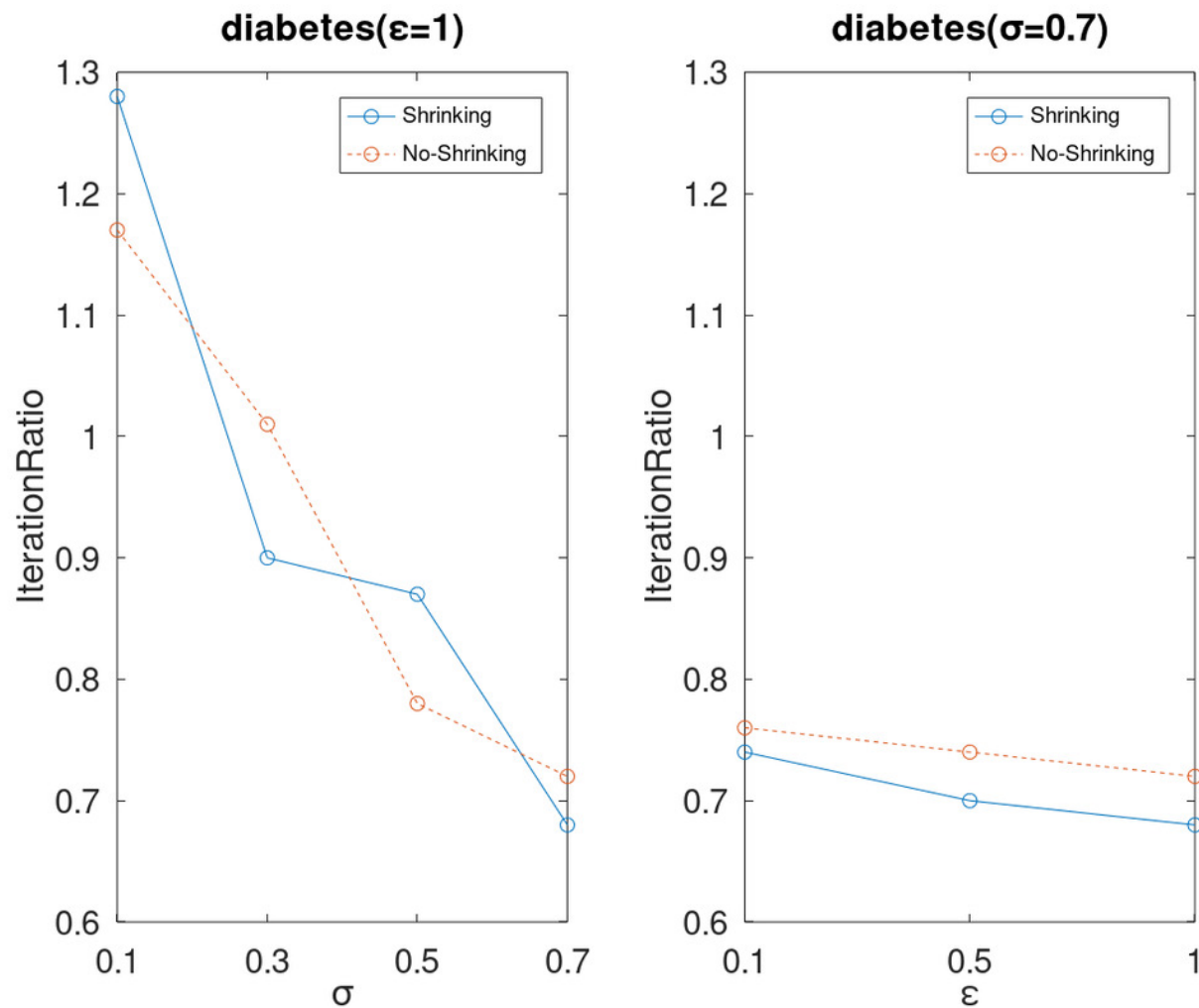
The algorithm stability of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset 9 to 16 without shrinking.

# Figure 6

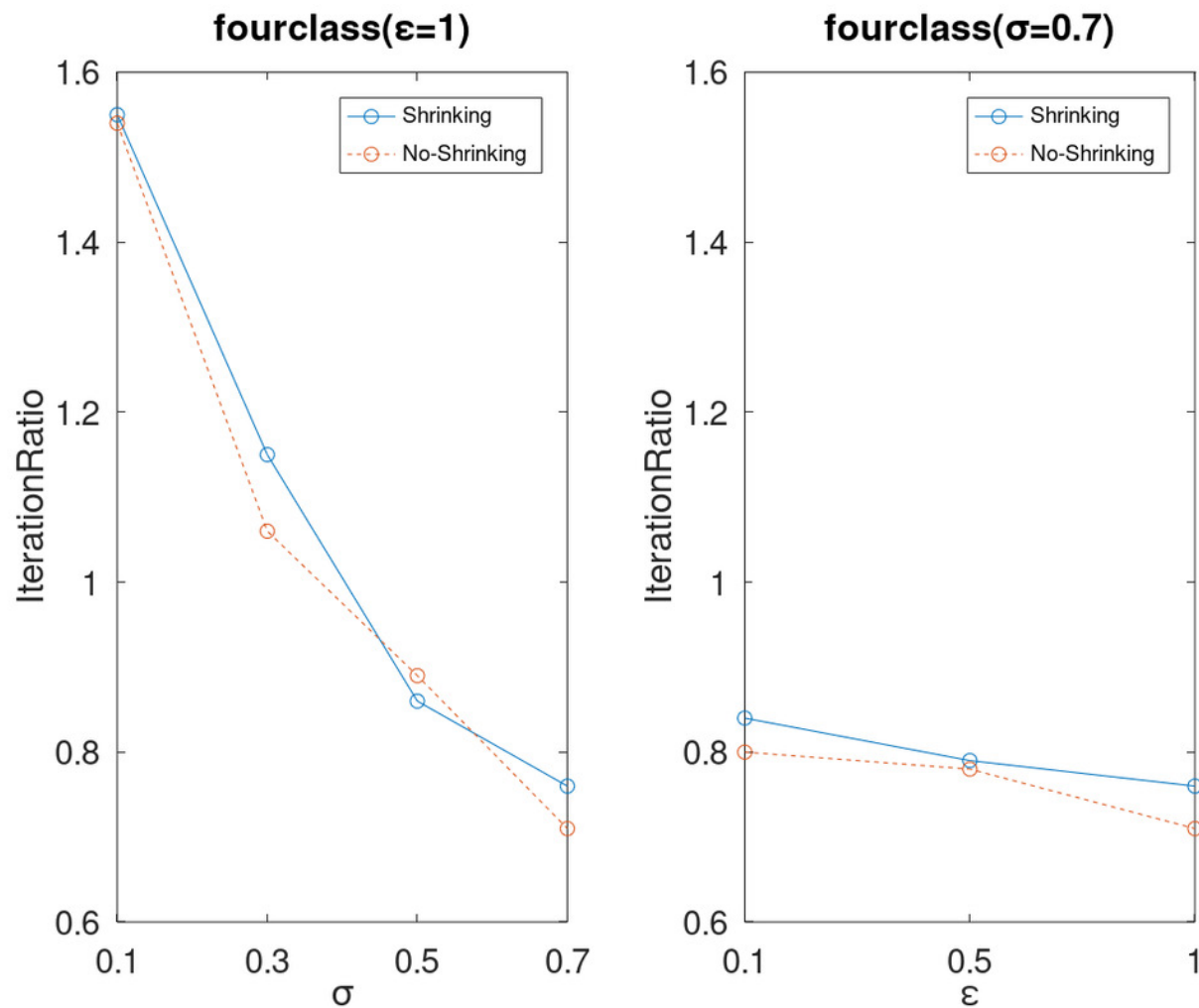
The execution efficiency of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset a1a.

# Figure 7

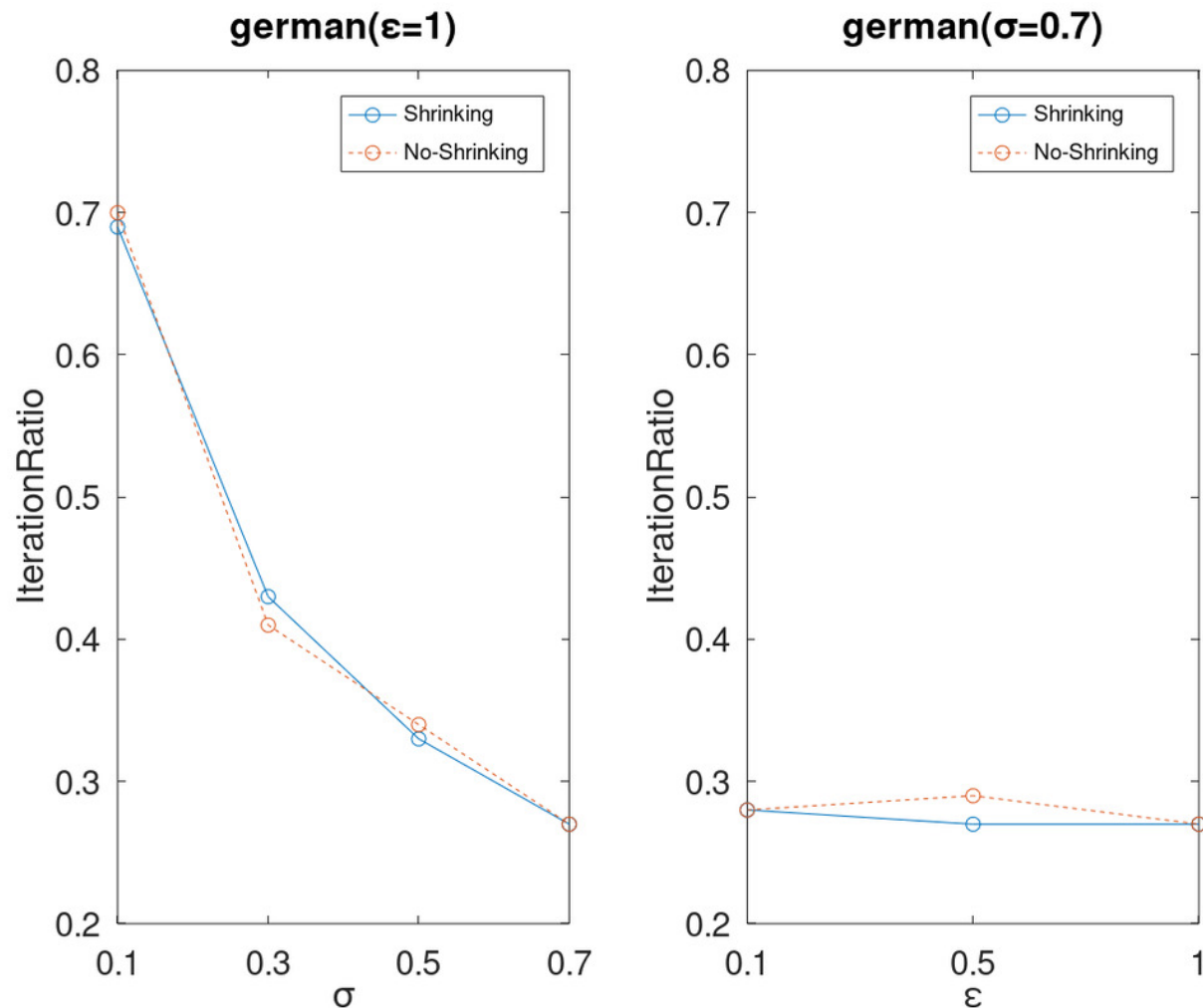
The execution efficiency of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset a5a.

# Figure 8

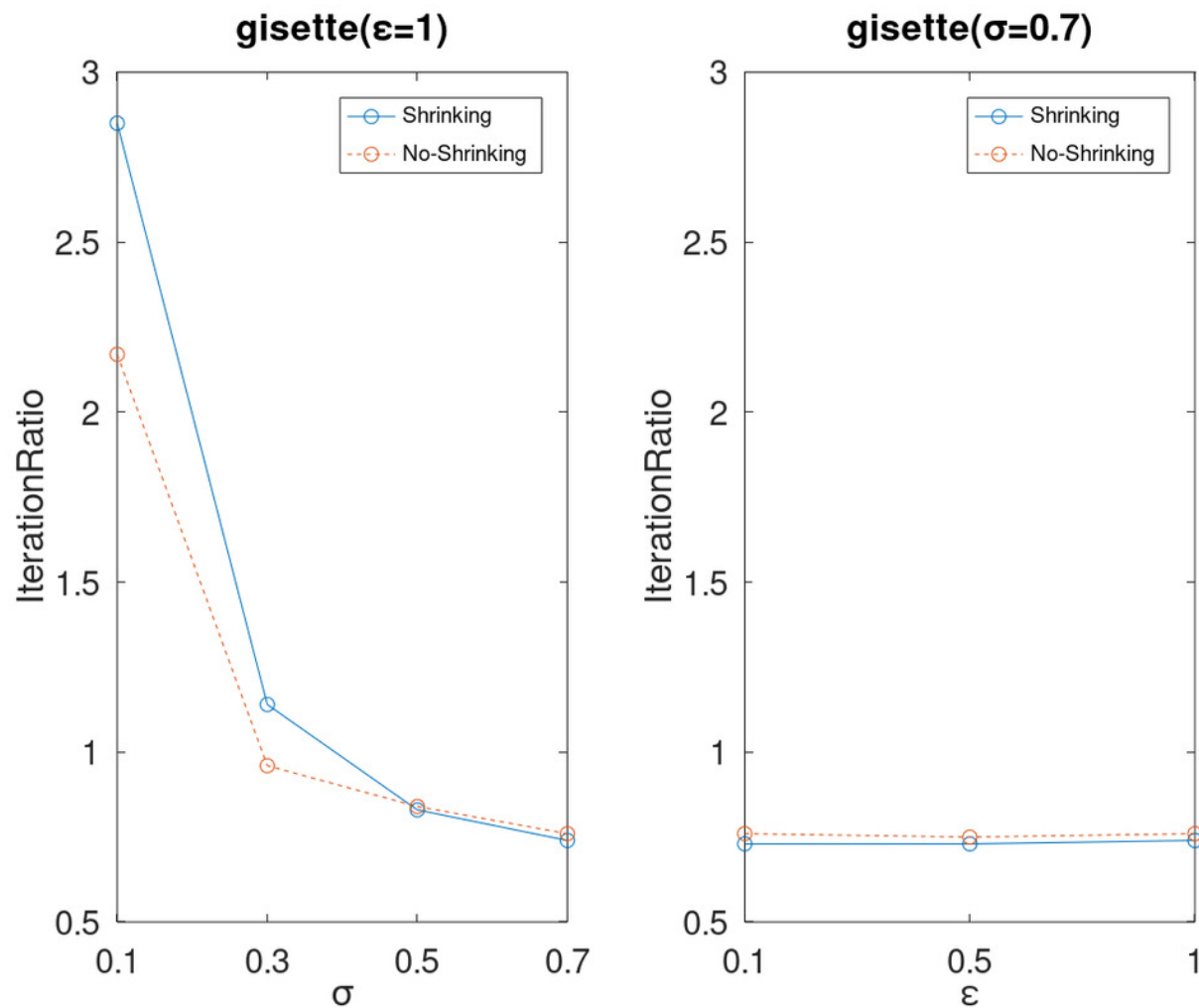
The execution efficiency of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset australian.

# Figure 9

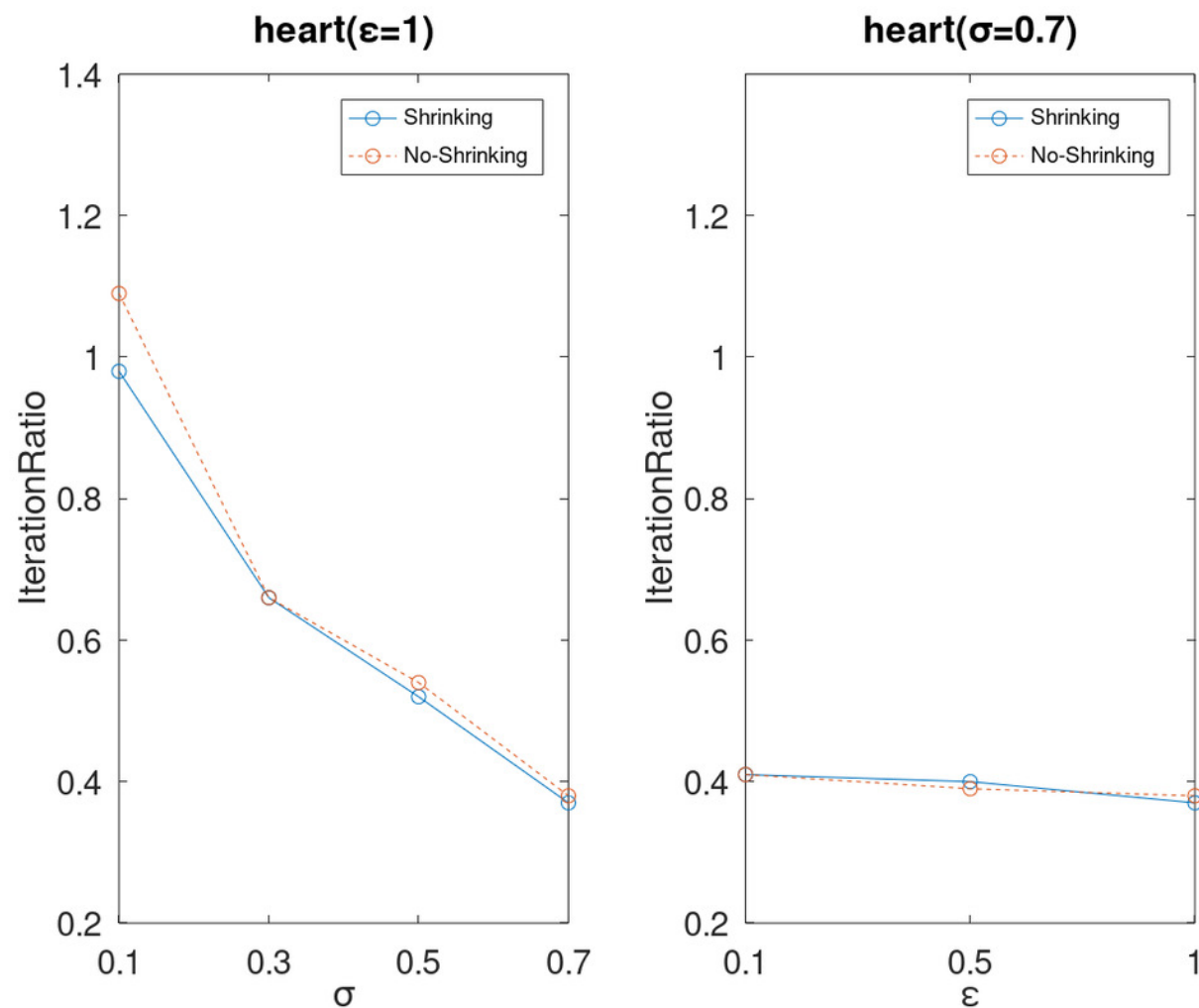
The execution efficiency of DPWSS for different ε and σ versus WSS 2 on dataset breast.

# Figure 10

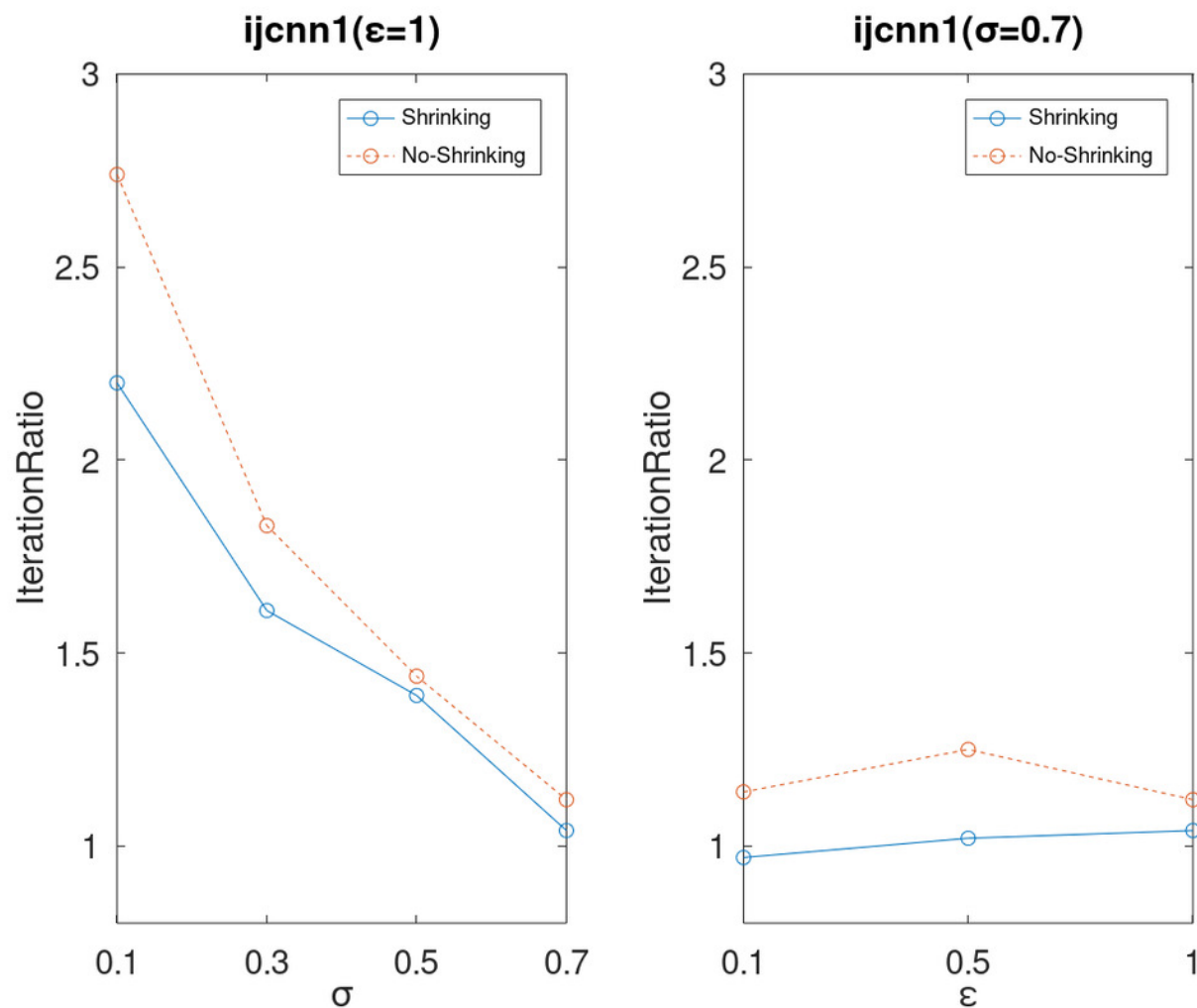
The execution efficiency of DPWSS for different ε and σ versus WSS 2 on dataset diabetes.

# Figure 11

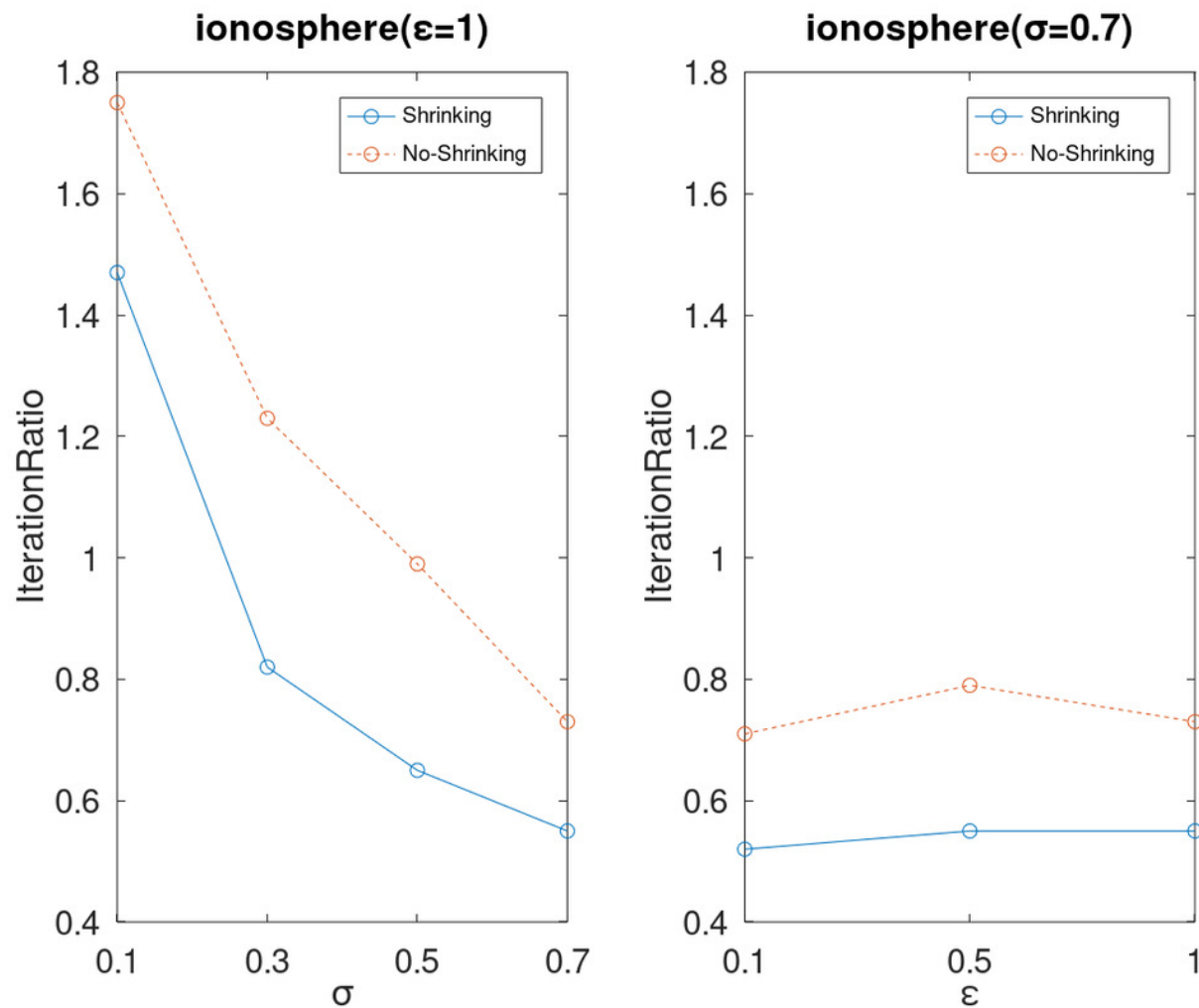
The execution efficiency of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset fourclass.

# Figure 12

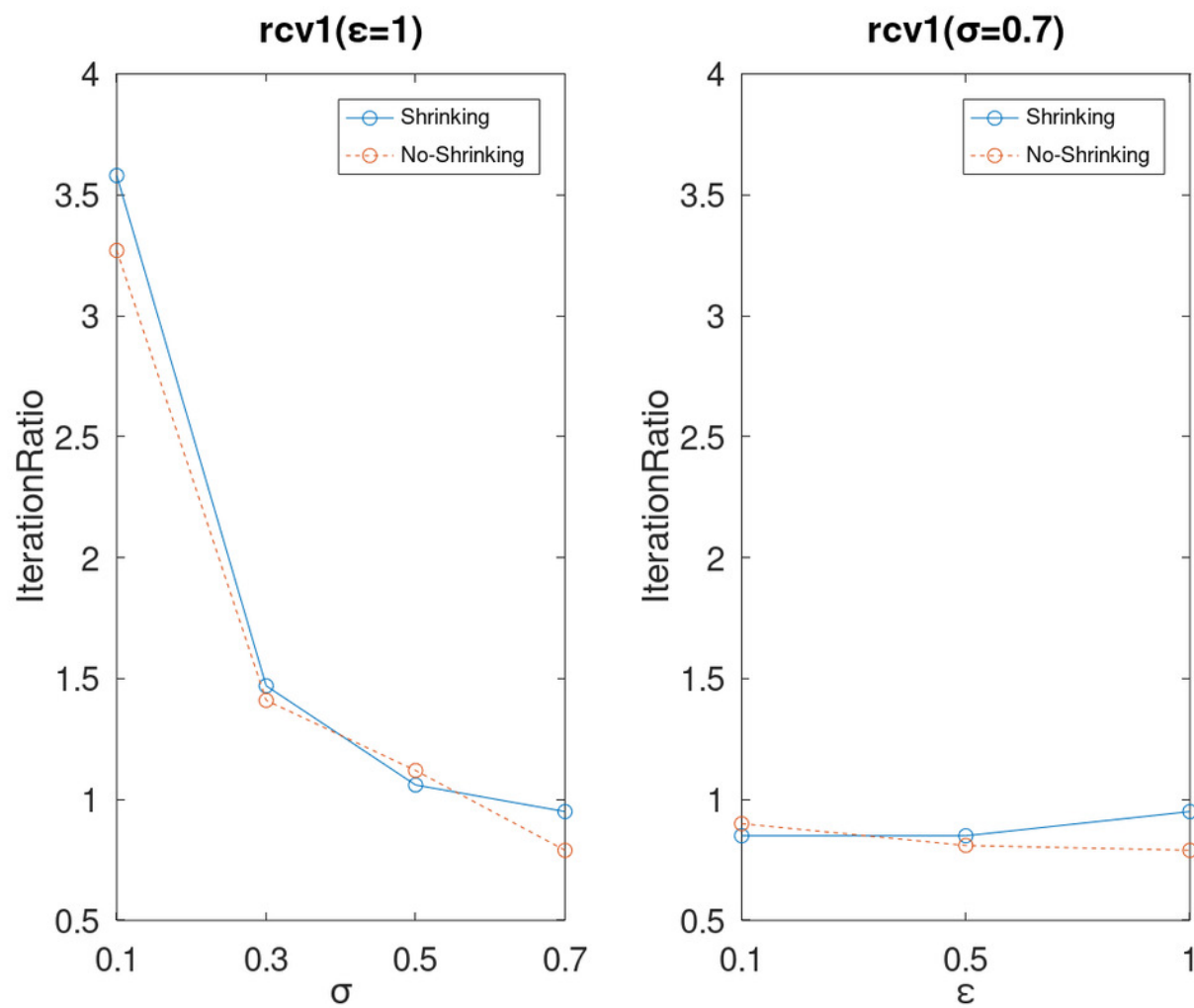
The execution efficiency of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset german.

# Figure 13

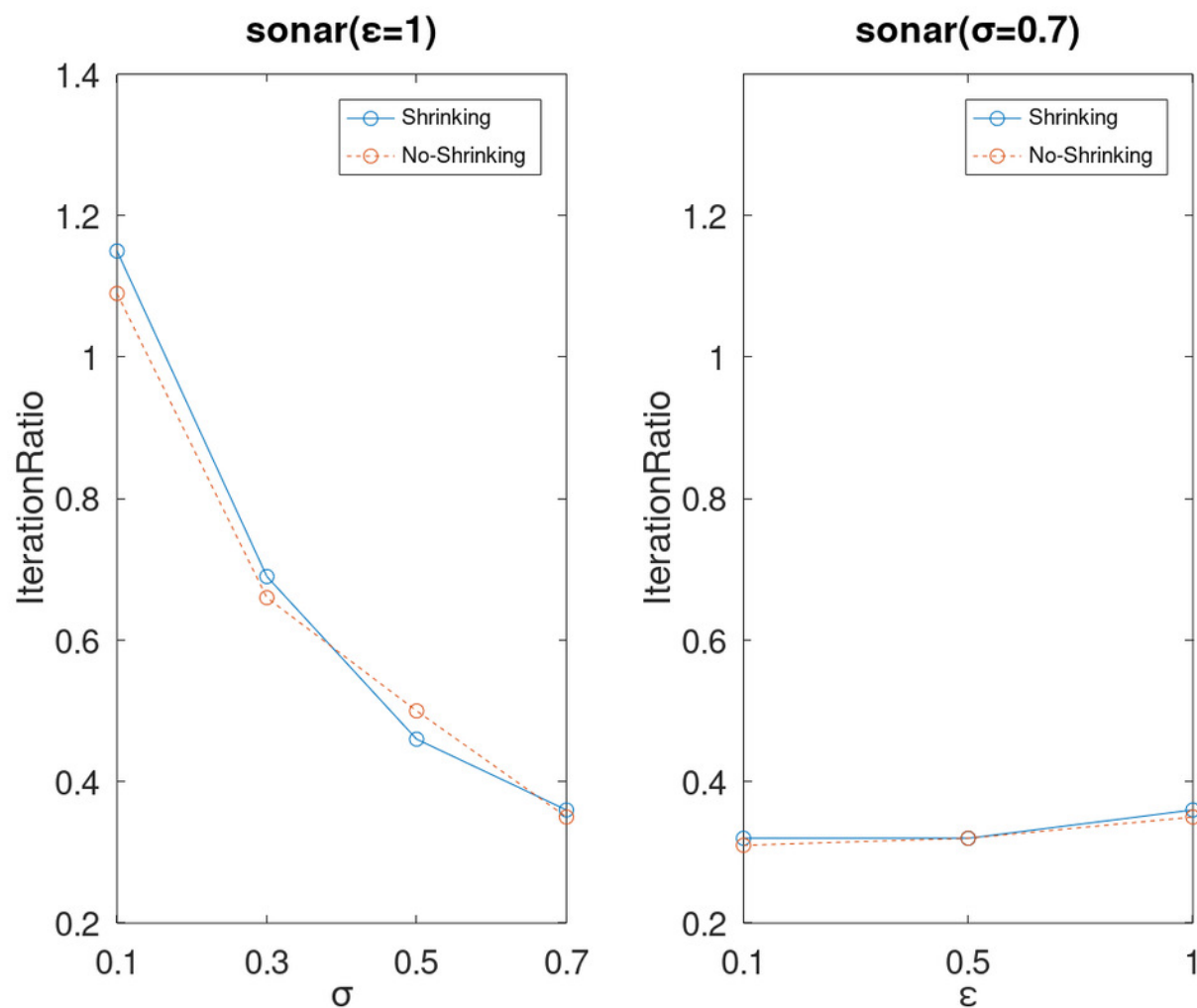
The execution efficiency of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset gisette.

# Figure 14

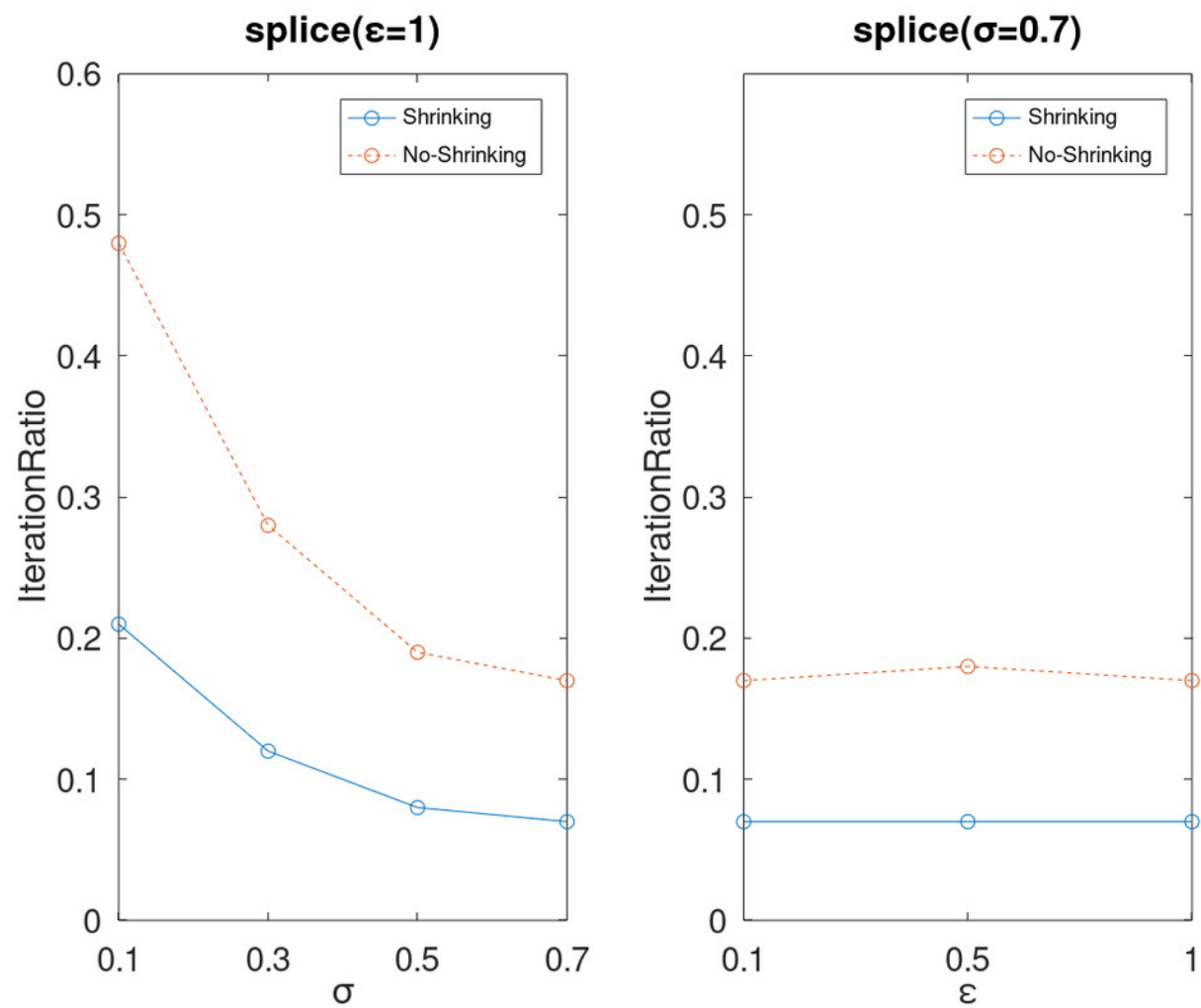
The execution efficiency of DPWSS for different ε and σ versus WSS 2 on dataset heart.

# Figure 15

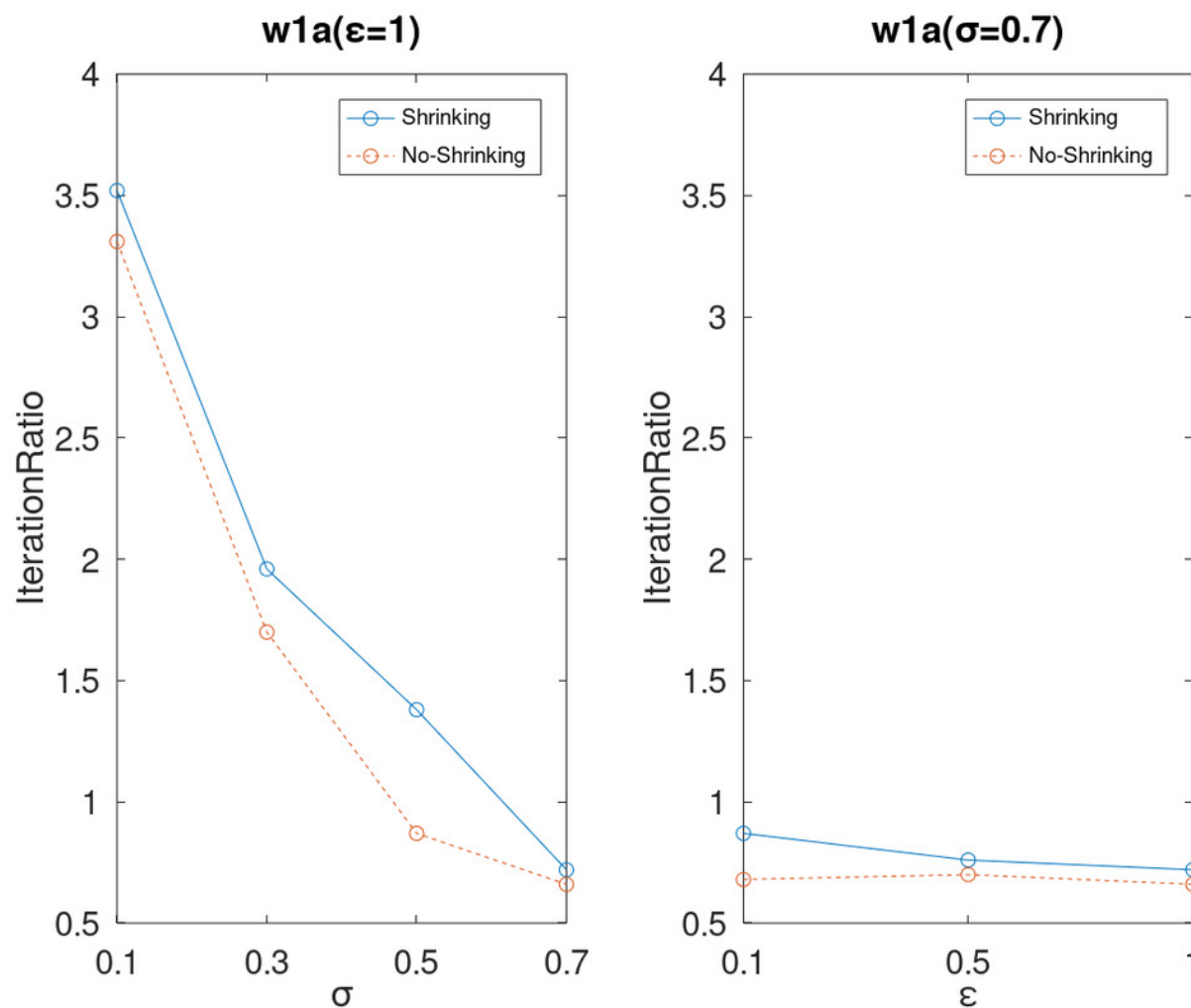
The execution efficiency of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset ijcnn1.

# Figure 16

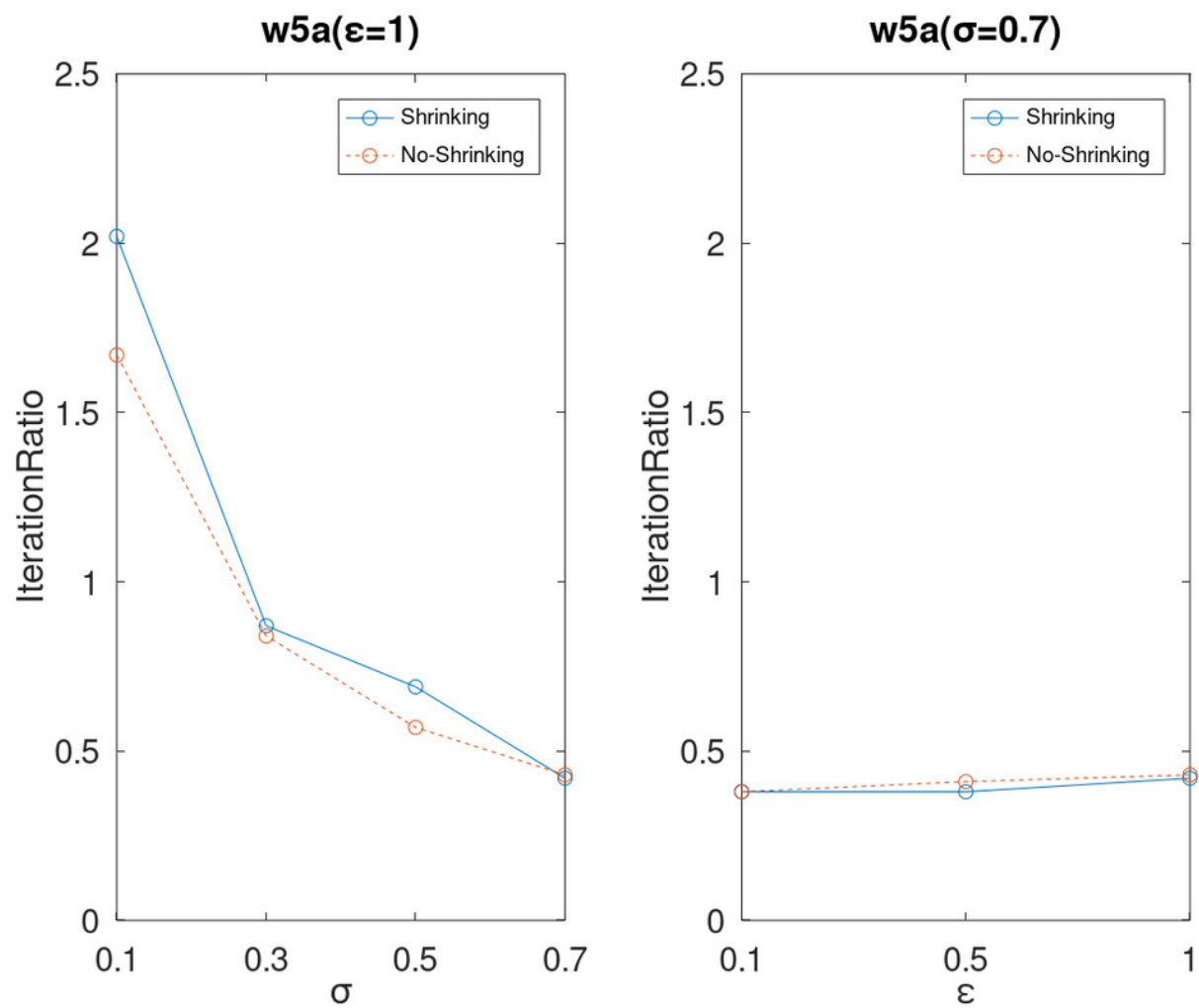
The execution efficiency of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset ionosphere.

# Figure 17

The execution efficiency of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset rcv1.

# Figure 18

The execution efficiency of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset sonar.

# Figure 19

The execution efficiency of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset splice.

# Figure 20

The execution efficiency of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset w1a.

# Figure 21

The execution efficiency of DPWSS for different $\varepsilon$ and $\sigma$ versus WSS 2 on dataset w5a.

Notations

| Symbol | Description |
|--------|-------------|
| $U$ | Universe |
| $D \subset U$ | Dataset to be trained |
| $D'$ | Neighbor dataset of $D$ |
| $x^i \in R^d$ | Train instance |
| $y^i \in \{1,-1\}$ | Label of train instance |
| $\alpha$ | Dual variable |
| $e$ | Vector of all ones |
| $C$ | Upper bound of all variables |
| $K$ | Kernel function |
| $Q$ | Symmetric matrix of kernel function |
| $B$ | Working set |
| $\tau$ | A small positive number |
| $\sigma$ | Constant-factor |
| $M$ | Mechanism |
| $Lap(\lambda)$ | Laplace distribution with mean 0 and scale factor $\lambda$ |
| $\varepsilon$ | Privacy budget |
| $f$ | Query function |
| $q(D, r)$ | Score function |
| $\Delta f, \Delta q$ | Sensitivity of function |
| $TP$ | True positive |
| $TN$ | True negative |
| $FP$ | False positive |
| $FN$ | False negative |

Basic information of the datasets

| Index | Dataset | #data | Range | #features | Imbalance ratio |
|-------|---------|-------|-------|-----------|-----------------|
| 1 | a1a | 1605 | [0,1] | 119 | 0.33 |
| 2 | a5a | 6414 | [0,1] | 122 | 0.32 |
| 3 | australian | 690 | [-1,1] | 14 | 0.8 |
| 4 | breast | 683 | [-1,1] | 10 | 1.86 |
| 5 | diabetes | 768 | [-1,1] | 8 | 1.87 |
| 6 | fourclass | 862 | [-1,1] | 2 | 0.55 |
| 7 | german | 1000 | [-1,1] | 24 | 0.43 |
| 8 | gisette | 6000 | [-1,1] | 5000 | 1 |
| 9 | heart | 270 | [-1,1] | 13 | 0.8 |
| 10 | ijcnn1 | 49990 | [-1,1] | 22 | 0.11 |
| 11 | ionosphere | 351 | [-1,1] | 34 | 1.79 |
| 12 | rcv1 | 20242 | [-1,1] | 47236 | 1.08 |
| 13 | sonar | 208 | [-1,1] | 60 | 0.87 |
| 14 | splice | 1000 | [-1,1] | 60 | 1.07 |
| 15 | w1a | 2477 | [0,1] | 300 | 0.03 |
| 16 | w5a | 9888 | [0,1] | 300 | 0.03 |

The performance of WSS2 and DPWSS for different $\varepsilon$ and $\sigma$

| dataset | shrinking | metrics | WSS2 | DPWSS | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | epsi=1 sigm=0.1 | epsi=1 sigm=0.3 | epsi=1 sigm=0.5 | epsi=1 sigm=0.7 | epsi=0.5 sigm=0.7 | epsi=0.1 sigm=0.7 |
| a1a | 1 | AUC | 0.9117 | 0.9119 | 0.9117 | 0.9113 | 0.9116 | 0.9109 | 0.9123 |
| | | Accuracy | 0.8623 | 0.8611 | 0.8629 | 0.8629 | 0.8592 | 0.8604 | 0.8636 |
| | | Precision | 0.7669 | 0.7654 | 0.7709 | 0.7709 | 0.76 | 0.7631 | 0.7667 |
| | | Recall | 0.6329 | 0.6278 | 0.6304 | 0.6304 | 0.6253 | 0.6278 | 0.6405 |
| | | F1 | 0.6935 | 0.6898 | 0.6936 | 0.6936 | 0.6861 | 0.6889 | 0.6979 |
| | | Mcc | 0.6104 | 0.6063 | 0.6115 | 0.6115 | 0.6012 | 0.6048 | 0.6148 |
| | | obj | -540.57 | -540.42 | -540.45 | -540.42 | -539.95 | -540.23 | -540.06 |
| | | iteration | 8649 | 10535 | 6735 | 4316 | 3239 | 3529 | 3406 |
| | 0 | AUC | 0.9117 | 0.9116 | 0.9116 | 0.9119 | 0.9122 | 0.9111 | 0.9117 |
| | | Accuracy | 0.8623 | 0.8623 | 0.8629 | 0.8629 | 0.8617 | 0.8623 | 0.8617 |
| | | Precision | 0.7669 | 0.7685 | 0.7692 | 0.7709 | 0.7645 | 0.7685 | 0.7645 |
| | | Recall | 0.6329 | 0.6304 | 0.6329 | 0.6304 | 0.6329 | 0.6304 | 0.6329 |
| | | F1 | 0.6935 | 0.6926 | 0.6944 | 0.6936 | 0.6925 | 0.6926 | 0.6925 |
| | | Mcc | 0.6104 | 0.61 | 0.612 | 0.6115 | 0.6088 | 0.61 | 0.6088 |
| | | obj | -540.57 | -540.42 | -540.44 | -540.38 | -540.12 | -540.11 | -540.2 |
| | | iteration | 7997 | 9566 | 6091 | 4252 | 3447 | 3379 | 3295 |
| a5a | 1 | AUC | 0.906 | 0.9059 | 0.9059 | 0.9057 | 0.9058 | 0.9058 | 0.9059 |
| | | Accuracy | 0.8506 | 0.8502 | 0.8499 | 0.8511 | 0.8486 | 0.8506 | 0.8503 |
| | | Precision | 0.7327 | 0.7317 | 0.7306 | 0.7354 | 0.7311 | 0.7334 | 0.7305 |

| | | Metric | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Recall | 0.6131 | 0.6119 | 0.6119 | 0.6112 | 0.6029 | 0.6119 | 0.615 |
| | | F1 | 0.6676 | 0.6664 | 0.666 | 0.6676 | 0.6608 | 0.6671 | 0.6678 |
| | | Mcc | 0.576 | 0.5746 | 0.5738 | 0.5768 | 0.5689 | 0.5758 | 0.5757 |
| | | obj | -2224.72 | -2224.56 | -2224.57 | -2224.33 | -2223.2 | -2223.75 | -2223.92 |
| | | iteration | 35752 | 36151 | 22590 | 15987 | 13240 | 14034 | 14275 |
| | 0 | AUC | 0.906 | 0.9058 | 0.9059 | 0.9058 | 0.9057 | 0.9058 | 0.906 |
| | | Accuracy | 0.8506 | 0.8506 | 0.8506 | 0.8514 | 0.8503 | 0.8502 | 0.85 |
| | | Precision | 0.7327 | 0.7323 | 0.7327 | 0.7373 | 0.7312 | 0.7331 | 0.7322 |
| | | Recall | 0.6131 | 0.6138 | 0.6131 | 0.6099 | 0.6138 | 0.6093 | 0.6099 |
| | | F1 | 0.6676 | 0.6678 | 0.6676 | 0.6676 | 0.6674 | 0.6655 | 0.6655 |
| | | Mcc | 0.576 | 0.5762 | 0.576 | 0.5773 | 0.5754 | 0.5741 | 0.5738 |
| | | obj | -2224.72 | -2224.41 | -2224.47 | -2224.33 | -2223.72 | -2223.07 | -2223.85 |
| | | iteration | 37578 | 33682 | 21592 | 16418 | 13926 | 12987 | 14318 |
| australian | 1 | AUC | 0.9393 | 0.9403 | 0.9378 | 0.9318 | 0.9141 | 0.9126 | 0.9202 |
| | | Accuracy | 0.8565 | 0.8565 | 0.8565 | 0.8565 | 0.8565 | 0.8565 | 0.8565 |
| | | Precision | 0.7873 | 0.7873 | 0.7873 | 0.7873 | 0.7873 | 0.7873 | 0.7873 |
| | | Recall | 0.9283 | 0.9283 | 0.9283 | 0.9283 | 0.9283 | 0.9283 | 0.9283 |
| | | F1 | 0.852 | 0.852 | 0.852 | 0.852 | 0.852 | 0.852 | 0.852 |
| | | Mcc | 0.7237 | 0.7237 | 0.7237 | 0.7237 | 0.7237 | 0.7237 | 0.7237 |
| | | obj | -199.65 | -199.25 | -198.98 | -198.21 | -197.78 | -198.53 | -198.64 |
| | | iteration | 10727 | 6438 | 1910 | 835 | 493 | 596 | 612 |

|        |   |           |          |          |          |          |          |          |          |
|--------|---|-----------|----------|----------|----------|----------|----------|----------|----------|
|        | 0 | AUC       | 0.9393   | 0.9397   | 0.9324   | 0.9111   | 0.923    | 0.9223   | 0.9316   |
|        |   | Accuracy  | 0.8565   | 0.8565   | 0.8565   | 0.8565   | 0.8565   | 0.8565   | 0.8565   |
|        |   | Precision | 0.7873   | 0.7873   | 0.7873   | 0.7873   | 0.7873   | 0.7873   | 0.7873   |
|        |   | Recall    | 0.9283   | 0.9283   | 0.9283   | 0.9283   | 0.9283   | 0.9283   | 0.9283   |
|        |   | F1        | 0.852    | 0.852    | 0.852    | 0.852    | 0.852    | 0.852    | 0.852    |
|        |   | Mcc       | 0.7237   | 0.7237   | 0.7237   | 0.7237   | 0.7237   | 0.7237   | 0.7237   |
|        |   | obj       | -199.65  | -199.25  | -199.15  | -198.68  | -198.33  | -198.62  | -198.82  |
|        |   | iteration | 10590    | 6978     | 2629     | 847      | 542      | 637      | 731      |
| breast | 1 | AUC       | 0.9962   | 0.9962   | 0.9963   | 0.9961   | 0.9962   | 0.9961   | 0.9962   |
|        |   | Accuracy  | 0.9707   | 0.9707   | 0.9707   | 0.9707   | 0.9707   | 0.9707   | 0.9707   |
|        |   | Precision | 0.9818   | 0.9818   | 0.9818   | 0.9818   | 0.9818   | 0.9818   | 0.9818   |
|        |   | Recall    | 0.973    | 0.973    | 0.973    | 0.973    | 0.973    | 0.973    | 0.973    |
|        |   | F1        | 0.9774   | 0.9774   | 0.9774   | 0.9774   | 0.9774   | 0.9774   | 0.9774   |
|        |   | Mcc       | 0.936    | 0.936    | 0.936    | 0.936    | 0.936    | 0.936    | 0.936    |
|        |   | obj       | -46      | -45.96   | -45.93   | -45.89   | -45.63   | -45.53   | -45.78   |
|        |   | iteration | 212      | 542      | 257      | 196      | 138      | 146      | 150      |
|        | 0 | AUC       | 0.9962   | 0.9962   | 0.9963   | 0.9963   | 0.9962   | 0.9962   | 0.9962   |
|        |   | Accuracy  | 0.9707   | 0.9707   | 0.9707   | 0.9707   | 0.9722   | 0.9722   | 0.9722   |
|        |   | Precision | 0.9818   | 0.9818   | 0.9818   | 0.9818   | 0.9819   | 0.9841   | 0.9819   |
|        |   | Recall    | 0.973    | 0.973    | 0.973    | 0.973    | 0.9752   | 0.973    | 0.9752   |
|        |   | F1        | 0.9774   | 0.9774   | 0.9774   | 0.9774   | 0.9785   | 0.9785   | 0.9785   |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Mcc | 0.936 | 0.936 | 0.936 | 0.936 | 0.9391 | 0.9393 | 0.9391 |
| | | obj | -46 | -45.95 | -45.99 | -45.78 | -45.62 | -45.87 | -45.88 |
| | | iteration | 212 | 443 | 329 | 184 | 146 | 160 | 181 |
| | | AUC | 0.8388 | 0.8393 | 0.839 | 0.8388 | 0.8383 | 0.8378 | 0.8385 |
| | | Accuracy | 0.776 | 0.7747 | 0.7734 | 0.7747 | 0.7708 | 0.7721 | 0.7721 |
| | | Precision | 0.7918 | 0.7904 | 0.789 | 0.7904 | 0.7893 | 0.7886 | 0.7897 |
| | | Recall | 0.89 | 0.89 | 0.89 | 0.89 | 0.884 | 0.888 | 0.886 |
| | 1 | F1 | 0.838 | 0.8373 | 0.8365 | 0.8373 | 0.834 | 0.8354 | 0.8351 |
| | | Mcc | 0.4878 | 0.4846 | 0.4813 | 0.4846 | 0.4759 | 0.4784 | 0.4788 |
| | | obj | -403.1 | -403.03 | -402.97 | -403 | -402.53 | -402.44 | -402.58 |
| | | iteration | 680 | 873 | 612 | 590 | 460 | 475 | 502 |
| diabetes | | AUC | 0.8388 | 0.8392 | 0.8391 | 0.8393 | 0.8384 | 0.8383 | 0.839 |
| | | Accuracy | 0.776 | 0.7747 | 0.776 | 0.7747 | 0.7695 | 0.7721 | 0.776 |
| | | Precision | 0.7918 | 0.7904 | 0.7918 | 0.7914 | 0.7858 | 0.7897 | 0.7918 |
| | | Recall | 0.89 | 0.89 | 0.89 | 0.888 | 0.888 | 0.886 | 0.89 |
| | 0 | F1 | 0.838 | 0.8373 | 0.838 | 0.8369 | 0.8338 | 0.8351 | 0.838 |
| | | Mcc | 0.4878 | 0.4846 | 0.4878 | 0.48449 | 0.4718 | 0.4788 | 0.4878 |
| | | obj | -403.1 | -403 | -403.03 | -403 | -402.05 | -402.8 | -402.72 |
| | | iteration | 680 | 793 | 687 | 529 | 490 | 502 | 516 |
| fourclass | 1 | AUC | 0.8266 | 0.8268 | 0.8262 | 0.8265 | 0.8255 | 0.8251 | 0.8261 |
| | | Accuracy | 0.7715 | 0.7715 | 0.7715 | 0.7715 | 0.7715 | 0.7726 | 0.7726 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Precision | 0.7455 | 0.7455 | 0.7455 | 0.7477 | 0.7477 | 0.7489 | 0.7489 |
| | | Recall | 0.544 | 0.544 | 0.544 | 0.5407 | 0.5407 | 0.544 | 0.544 |
| | | F1 | 0.629 | 0.629 | 0.629 | 0.6276 | 0.6276 | 0.6302 | 0.6302 |
| | | Mcc | 0.4818 | 0.4818 | 0.4818 | 0.4816 | 0.4816 | 0.4845 | 0.4845 |
| | | obj | -454.29 | -454.27 | -454.22 | -454.23 | -454.12 | -454.12 | -454.17 |
| | | iteration | 590 | 917 | 676 | 509 | 450 | 466 | 496 |
| | 0 | AUC | 0.8266 | 0.827 | 0.8256 | 0.8272 | 0.8245 | 0.8263 | 0.8257 |
| | | Accuracy | 0.7715 | 0.7691 | 0.7715 | 0.7691 | 0.7749 | 0.7726 | 0.7726 |
| | | Precision | 0.7455 | 0.7432 | 0.7455 | 0.7432 | 0.7653 | 0.7534 | 0.7467 |
| | | Recall | 0.544 | 0.5375 | 0.544 | 0.5375 | 0.5309 | 0.5375 | 0.5472 |
| | | F1 | 0.629 | 0.6238 | 0.629 | 0.6238 | 0.6269 | 0.6274 | 0.6316 |
| | | Mcc | 0.4818 | 0.4761 | 0.4818 | 0.4761 | 0.4894 | 0.4842 | 0.4847 |
| | | obj | -454.29 | -454.25 | -454.22 | -454.18 | -453.68 | -453.8 | -454.19 |
| | | iteration | 590 | 908 | 625 | 526 | 421 | 458 | 471 |
| | 1 | AUC | 0.8165 | 0.8163 | 0.8161 | 0.816 | 0.8161 | 0.8157 | 0.816 |
| | | Accuracy | 0.789 | 0.788 | 0.787 | 0.786 | 0.785 | 0.783 | 0.784 |
| | | Precision | 0.6943 | 0.6947 | 0.69 | 0.6886 | 0.6856 | 0.6861 | 0.6858 |
| german | | Recall | 0.53 | 0.5233 | 0.5267 | 0.5233 | 0.5233 | 0.51 | 0.5167 |
| | | F1 | 0.6011 | 0.597 | 0.5974 | 0.5947 | 0.5936 | 0.5851 | 0.5894 |
| | | Mcc | 0.469 | 0.4654 | 0.4638 | 0.4608 | 0.4586 | 0.4514 | 0.455 |
| | | obj | -519.05 | -518.76 | -518.81 | -518.48 | -517.51 | -517.92 | -517.59 |

|  |  |  | 13688 | 9415 | 5821 | 4533 | 3675 | 3741 | 3787 |
|---|---|---|---|---|---|---|---|---|---|
|  |  | iteration | 13688 | 9415 | 5821 | 4533 | 3675 | 3741 | 3787 |
|  |  | AUC | 0.8165 | 0.8163 | 0.8163 | 0.8155 | 0.8154 | 0.8159 | 0.8162 |
|  |  | Accuracy | 0.789 | 0.787 | 0.789 | 0.783 | 0.785 | 0.784 | 0.787 |
|  |  | Precision | 0.6943 | 0.6916 | 0.6978 | 0.6781 | 0.6856 | 0.6826 | 0.6933 |
|  |  | Recall | 0.53 | 0.5233 | 0.5233 | 0.5267 | 0.5233 | 0.5233 | 0.52 |
|  | 0 | F1 | 0.6011 | 0.5958 | 0.5981 | 0.5929 | 0.5936 | 0.5925 | 0.5943 |
|  |  | Mcc | 0.469 | 0.4631 | 0.4677 | 0.4548 | 0.4586 | 0.4563 | 0.4625 |
|  |  | obj | -519.05 | -518.64 | -518.69 | -518.41 | -517.91 | -517.98 | -517.65 |
|  |  | iteration | 13454 | 9367 | 5495 | 4576 | 3663 | 3842 | 3742 |
|  |  | AUC | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|  |  | Accuracy | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|  |  | Precision | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|  |  | Recall | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|  | 1 | F1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|  |  | Mcc | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| gisette |  | obj | -0.668 | -0.668 | -0.668 | -0.668 | -0.668 | -0.668 | -0.668 |
|  |  | iteration | 8157 | 23247 | 9312 | 6736 | 6002 | 5978 | 5979 |
|  |  | AUC | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|  |  | Accuracy | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|  | 0 | Precision | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
|  |  | Recall | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | F1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | | Mcc | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | | obj | -0.668 | -0.668 | -0.668 | -0.668 | -0.668 | -0.668 | -0.668 |
| | | iteration | 8246 | 17902 | 7933 | 6918 | 6258 | 6225 | 6250 |
| heart | 1 | AUC | 0.9282 | 0.9281 | 0.9287 | 0.9296 | 0.9287 | 0.9282 | 0.9296 |
| | | Accuracy | 0.8481 | 0.8444 | 0.8481 | 0.8481 | 0.8519 | 0.8593 | 0.8481 |
| | | Precision | 0.8376 | 0.8362 | 0.8376 | 0.8376 | 0.8509 | 0.8534 | 0.8376 |
| | | Recall | 0.8167 | 0.8083 | 0.8167 | 0.8167 | 0.8083 | 0.825 | 0.8167 |
| | | F1 | 0.827 | 0.822 | 0.827 | 0.827 | 0.8291 | 0.839 | 0.827 |
| | | Mcc | 0.6919 | 0.6843 | 0.6919 | 0.6919 | 0.6992 | 0.7144 | 0.6919 |
| | | obj | -92.47 | -92.07 | -92.33 | -92.17 | -90.78 | -91.34 | -91.62 |
| | | iteration | 1010 | 992 | 662 | 525 | 372 | 404 | 410 |
| | 0 | AUC | 0.9282 | 0.9278 | 0.9284 | 0.9292 | 0.9251 | 0.9278 | 0.9275 |
| | | Accuracy | 0.8481 | 0.8481 | 0.8444 | 0.8556 | 0.8556 | 0.8593 | 0.8481 |
| | | Precision | 0.8376 | 0.8435 | 0.8362 | 0.8462 | 0.8584 | 0.8596 | 0.8376 |
| | | Recall | 0.8167 | 0.8083 | 0.8083 | 0.825 | 0.8083 | 0.8167 | 0.8167 |
| | | F1 | 0.827 | 0.8255 | 0.822 | 0.8354 | 0.8326 | 0.8376 | 0.827 |
| | | Mcc | 0.6919 | 0.6917 | 0.6843 | 0.7069 | 0.7068 | 0.7143 | 0.6919 |
| | | obj | -92.47 | -92.2 | -92.07 | -92.09 | -91.03 | -91.14 | -91.36 |
| | | iteration | 1010 | 1097 | 671 | 542 | 380 | 390 | 411 |
| ijcnn1 | 1 | AUC | 0.918 | 0.918 | 0.918 | 0.9179 | 0.917 | 0.9184 | 0.9179 |

|  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|
|  | Accuracy | 0.9242 | 0.9242 | 0.9241 | 0.9242 | 0.9241 | 0.9241 | 0.9241 |
|  | Precision | 0.7579 | 0.758 | 0.7576 | 0.7581 | 0.767 | 0.7565 | 0.7598 |
|  | Recall | 0.3219 | 0.3215 | 0.3215 | 0.3217 | 0.314 | 0.3221 | 0.3188 |
|  | F1 | 0.4518 | 0.4515 | 0.4514 | 0.4517 | 0.4456 | 0.4518 | 0.4491 |
|  | Mcc | 0.4628 | 0.4626 | 0.4624 | 0.4628 | 0.4604 | 0.4625 | 0.4612 |
|  | obj | -8590.16 | -8590.07 | -8590.11 | -8590.07 | -8588.99 | -8588.82 | -8589 |
|  | iteration | 18382 | 40443 | 29635 | 25599 | 19150 | 18694 | 17842 |
|  | AUC | 0.918 | 0.9181 | 0.918 | 0.9181 | 0.9181 | 0.9183 | 0.9183 |
|  | Accuracy | 0.9241 | 0.924 | 0.9241 | 0.9241 | 0.9238 | 0.9242 | 0.924 |
|  | Precision | 0.7574 | 0.7573 | 0.7573 | 0.7572 | 0.7549 | 0.7569 | 0.7562 |
| 0 | Recall | 0.3217 | 0.3202 | 0.3215 | 0.3212 | 0.3179 | 0.3221 | 0.3208 |
|  | F1 | 0.4515 | 0.4501 | 0.4513 | 0.4511 | 0.4474 | 0.4519 | 0.4505 |
|  | Mcc | 0.4625 | 0.4614 | 0.4623 | 0.4621 | 0.4588 | 0.4626 | 0.4614 |
|  | obj | -8590.16 | -8590.05 | -8590.1 | -8589.94 | -8588.84 | -8589.73 | -8589.18 |
|  | iteration | 16469 | 45191 | 30133 | 23786 | 18416 | 20637 | 18750 |
|  | AUC | 0.9677 | 0.9684 | 0.9681 | 0.9679 | 0.9686 | 0.9688 | 0.9687 |
|  | Accuracy | 0.9373 | 0.9373 | 0.9373 | 0.9259 | 0.9345 | 0.9373 | 0.9345 |
|  | Precision | 0.9283 | 0.9283 | 0.9283 | 0.9234 | 0.9244 | 0.9356 | 0.9316 |
| ionosphere 1 | Recall | 0.9778 | 0.9778 | 0.9778 | 0.9644 | 0.9778 | 0.9689 | 0.9689 |
|  | F1 | 0.9524 | 0.9524 | 0.9524 | 0.9435 | 0.9503 | 0.952 | 0.9499 |
|  | Mcc | 0.8634 | 0.8634 | 0.8634 | 0.8379 | 0.8572 | 0.863 | 0.8567 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | obj | -73.41 | -73.41 | -73.31 | -71.97 | -72.02 | -72.44 | -72.11 |
| | | iteration | 1016 | 1489 | 834 | 664 | 555 | 562 | 525 |
| | | AUC | 0.9677 | 0.9674 | 0.9678 | 0.9673 | 0.9651 | 0.9667 | 0.965 |
| | | Accuracy | 0.9373 | 0.9288 | 0.9373 | 0.9288 | 0.9316 | 0.9288 | 0.9288 |
| | | Precision | 0.9283 | 0.9274 | 0.9283 | 0.9237 | 0.9277 | 0.9237 | 0.9274 |
| | | Recall | 0.9778 | 0.9644 | 0.9778 | 0.9689 | 0.9689 | 0.9689 | 0.9644 |
| | 0 | F1 | 0.9524 | 0.9455 | 0.9524 | 0.9458 | 0.9478 | 0.9458 | 0.9455 |
| | | Mcc | 0.8634 | 0.8441 | 0.8634 | 0.8442 | 0.8505 | 0.8442 | 0.8441 |
| | | obj | -73.41 | -73.15 | -73.2 | -73.13 | -72.44 | -72.85 | -71.91 |
| | | iteration | 770 | 1348 | 944 | 761 | 560 | 610 | 548 |
| | | AUC | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 |
| | | Accuracy | 0.9896 | 0.9896 | 0.9896 | 0.9896 | 0.9896 | 0.9896 | 0.9896 |
| | | Precision | 0.9896 | 0.9896 | 0.9896 | 0.9896 | 0.9896 | 0.9896 | 0.9897 |
| | | Recall | 0.9903 | 0.9903 | 0.9903 | 0.9903 | 0.9903 | 0.9903 | 0.9903 |
| | 1 | F1 | 0.9899 | 0.9899 | 0.9899 | 0.9899 | 0.9899 | 0.9899 | 0.99 |
| rcv1 | | Mcc | 0.9791 | 0.9791 | 0.9791 | 0.9791 | 0.9791 | 0.9791 | 0.9792 |
| | | obj | -1745.67 | -1745.66 | -1745.65 | -1745.62 | -1745.63 | -1745.6 | -1745.59 |
| | | iteration | 11639 | 41681 | 17129 | 12374 | 11029 | 9865 | 9945 |
| | | AUC | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 | 0.9989 |
| | 0 | Accuracy | 0.9896 | 0.9896 | 0.9896 | 0.9896 | 0.9896 | 0.9896 | 0.9896 |
| | | Precision | 0.9896 | 0.9896 | 0.9896 | 0.9896 | 0.9896 | 0.9897 | 0.9896 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Recall | 0.9903 | 0.9903 | 0.9903 | 0.9903 | 0.9903 | 0.9903 | 0.9903 |
| | F1 | 0.9899 | 0.9899 | 0.9899 | 0.9899 | 0.9899 | 0.99 | 0.9899 |
| | Mcc | 0.9791 | 0.9791 | 0.9791 | 0.9791 | 0.9791 | 0.9792 | 0.9791 |
| | obj | -1745.67 | -1745.66 | -1745.65 | -1745.64 | -1745.55 | -1745.58 | -1745.61 |
| | iteration | 11650 | 38114 | 16388 | 13014 | 9242 | 9419 | 10457 |
| | AUC | 0.9495 | 0.9491 | 0.9475 | 0.9467 | 0.9489 | 0.9459 | 0.9482 |
| | Accuracy | 0.8942 | 0.8894 | 0.8894 | 0.8894 | 0.8942 | 0.899 | 0.8894 |
| | Precision | 0.8641 | 0.8558 | 0.8558 | 0.8558 | 0.8641 | 0.8654 | 0.8558 |
| 1 | Recall | 0.9175 | 0.9175 | 0.9175 | 0.9175 | 0.9175 | 0.9278 | 0.9175 |
| | F1 | 0.89 | 0.8856 | 0.8856 | 0.8856 | 0.89 | 0.8955 | 0.8856 |
| | Mcc | 0.7896 | 0.7806 | 0.7806 | 0.7806 | 0.7896 | 0.7999 | 0.7806 |
| | obj | -65.67 | -65.62 | -65.48 | -65.49 | -65.21 | -64.79 | -65 |
| | iteration | 1492 | 1716 | 1035 | 687 | 544 | 478 | 473 |
| sonar | AUC | 0.9496 | 0.9501 | 0.9475 | 0.9464 | 0.9492 | 0.9455 | 0.9439 |
| | Accuracy | 0.8942 | 0.8894 | 0.8894 | 0.8894 | 0.899 | 0.899 | 0.8846 |
| | Precision | 0.8641 | 0.8558 | 0.8558 | 0.8558 | 0.8654 | 0.8585 | 0.8476 |
| | Recall | 0.9175 | 0.9175 | 0.9175 | 0.9175 | 0.9278 | 0.9381 | 0.9175 |
| 0 | F1 | 0.89 | 0.8856 | 0.8856 | 0.8856 | 0.8955 | 0.8966 | 0.8812 |
| | Mcc | 0.7896 | 0.7806 | 0.7806 | 0.7806 | 0.7999 | 0.8013 | 0.7717 |
| | obj | -65.67 | -65.57 | -65.43 | -65.49 | -64.97 | -65.02 | -64.88 |
| | iteration | 1397 | 1516 | 929 | 701 | 489 | 448 | 440 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| splice | 1 | AUC | 0.9173 | 0.9165 | 0.9164 | 0.9169 | 0.9165 | 0.9173 | 0.9162 |
| | | Accuracy | 0.842 | 0.84 | 0.842 | 0.845 | 0.841 | 0.84 | 0.844 |
| | | Precision | 0.8671 | 0.8665 | 0.8716 | 0.8724 | 0.8698 | 0.868 | 0.8737 |
| | | Recall | 0.8201 | 0.8162 | 0.8143 | 0.8201 | 0.8143 | 0.8143 | 0.8162 |
| | | F1 | 0.8429 | 0.8406 | 0.842 | 0.8455 | 0.8412 | 0.8403 | 0.844 |
| | | Mcc | 0.6853 | 0.6815 | 0.6859 | 0.6916 | 0.6838 | 0.6817 | 0.69 |
| | | obj | -375.19 | -374.55 | -374.56 | -374.31 | -373.25 | -374.02 | -373.51 |
| | | iteration | 95972 | 19779 | 11627 | 8079 | 6486 | 6847 | 6620 |
| | 0 | AUC | 0.9173 | 0.9164 | 0.9169 | 0.9156 | 0.9166 | 0.9163 | 0.9168 |
| | | Accuracy | 0.842 | 0.845 | 0.843 | 0.843 | 0.841 | 0.844 | 0.841 |
| | | Precision | 0.8671 | 0.8724 | 0.8719 | 0.8719 | 0.8698 | 0.8737 | 0.8698 |
| | | Recall | 0.8201 | 0.8201 | 0.8162 | 0.8162 | 0.8143 | 0.8162 | 0.8143 |
| | | F1 | 0.8429 | 0.8455 | 0.8432 | 0.8432 | 0.8412 | 0.844 | 0.8412 |
| | | Mcc | 0.6853 | 0.6916 | 0.6878 | 0.6878 | 0.6838 | 0.69 | 0.6838 |
| | | obj | -375.19 | -374.08 | -374.11 | -373.89 | -373.32 | -373.77 | -373.7 |
| | | iteration | 38987 | 18752 | 11058 | 7562 | 6501 | 6845 | 6785 |
| w1a | 1 | AUC | 0.9755 | 0.9759 | 0.9757 | 0.9757 | 0.9757 | 0.9756 | 0.9754 |
| | | Accuracy | 0.9927 | 0.9927 | 0.9927 | 0.9927 | 0.9927 | 0.9927 | 0.9927 |
| | | Precision | 0.9821 | 0.9821 | 0.9821 | 0.9821 | 0.9821 | 0.9821 | 0.9821 |
| | | Recall | 0.7639 | 0.7639 | 0.7639 | 0.7639 | 0.7639 | 0.7639 | 0.7639 |
| | | F1 | 0.8594 | 0.8594 | 0.8594 | 0.8594 | 0.8594 | 0.8594 | 0.8594 |

| | | Metric | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Mcc | 0.8628 | 0.8628 | 0.8628 | 0.8628 | 0.8628 | 0.8628 | 0.8628 |
| | | obj | -62.92 | -62.89 | -62.91 | -62.9 | -62.85 | -62.89 | -62.89 |
| | | iteration | 2565 | 9030 | 5034 | 3529 | 1835 | 1949 | 2224 |
| | 0 | AUC | 0.9755 | 0.9755 | 0.9758 | 0.9759 | 0.9766 | 0.9758 | 0.9734 |
| | | Accuracy | 0.9927 | 0.9927 | 0.9927 | 0.9927 | 0.9927 | 0.9927 | 0.9927 |
| | | Precision | 0.9821 | 0.9821 | 0.9821 | 0.9821 | 0.9821 | 0.9821 | 0.9821 |
| | | Recall | 0.7639 | 0.7639 | 0.7639 | 0.7639 | 0.7639 | 0.7639 | 0.7639 |
| | | F1 | 0.8594 | 0.8594 | 0.8594 | 0.8594 | 0.8594 | 0.8594 | 0.8594 |
| | | Mcc | 0.8628 | 0.8628 | 0.8628 | 0.8628 | 0.8628 | 0.8628 | 0.8628 |
| | | obj | -62.92 | -62.89 | -62.89 | -62.79 | -62.8 | -62.86 | -62.85 |
| | | iteration | 2547 | 8436 | 4326 | 2222 | 1691 | 1781 | 1737 |
| w5a | 1 | AUC | 0.9632 | 0.9632 | 0.9638 | 0.9632 | 0.9625 | 0.9653 | 0.9626 |
| | | Accuracy | 0.9889 | 0.9887 | 0.9888 | 0.9887 | 0.9887 | 0.9889 | 0.9887 |
| | | Precision | 0.9524 | 0.9424 | 0.9474 | 0.9424 | 0.9424 | 0.9524 | 0.9424 |
| | | Recall | 0.6406 | 0.6406 | 0.6406 | 0.6406 | 0.6406 | 0.6406 | 0.6406 |
| | | F1 | 0.766 | 0.7627 | 0.7643 | 0.7627 | 0.7627 | 0.766 | 0.7627 |
| | | Mcc | 0.7762 | 0.772 | 0.7741 | 0.772 | 0.772 | 0.7762 | 0.772 |
| | | obj | -291.68 | -291.55 | -291.6 | -291.6 | -291.37 | -290.15 | -291.29 |
| | | iteration | 15422 | 31105 | 13388 | 10568 | 6514 | 5906 | 5858 |
| | 0 | AUC | 0.9632 | 0.9636 | 0.9632 | 0.9633 | 0.9632 | 0.9623 | 0.9626 |
| | | Accuracy | 0.9889 | 0.9888 | 0.9887 | 0.9888 | 0.9889 | 0.9888 | 0.9887 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Precision | 0.9524 | 0.9427 | 0.9424 | 0.9474 | 0.9476 | 0.9474 | 0.9424 |
| Recall | 0.6406 | 0.6441 | 0.6406 | 0.6406 | 0.6441 | 0.6406 | 0.6406 |
| F1 | 0.766 | 0.7653 | 0.7627 | 0.7643 | 0.7669 | 0.7643 | 0.7627 |
| Mcc | 0.7762 | 0.7743 | 0.772 | 0.7741 | 0.7764 | 0.7741 | 0.772 |
| obj | -291.68 | -291.53 | -291.55 | -291.46 | -291.39 | -291.4 | -291.31 |
| iteration | 15511 | 25969 | 13074 | 8879 | 6630 | 6373 | 5838 |