

Fuzzy based binary feature profiling for modus operandi analysis

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It is a well-known fact that some criminals follow perpetual methods of operations, known as modi operandi (MO) which are commonly used to describe the habits in committing something especially in the context of criminal investigations. These modi operandi are then used in relating criminals to other crimes where the criminals have not yet been recognized. This paper presents a method which is focused on identifying the perpetual modi operandi of criminals by analyzing their previous convictions. The method involves in generating a feature matrix for a particular suspect based on the flow of events of his/her previous convictions. Then, based on the feature matrix, two representative modi operandi are generated: complete modus operandi and dynamic modus operandi. These two representative modi operandi will be compared with the flow of events of the crime in order to investigate and relate a particular criminal. This comparison uses several operations to generate two other outputs: completeness probability and deviation probability. These two outcomes are used as inputs to a fuzzy inference system to generate a score value which is used in providing a measurement for the similarity between the suspect and the crime at hand. The method was evaluated using actual crime data and ten other open data sets. Then ROC analysis was performed to justify the validity and the generalizability of the proposed method. In addition, comparison with nine other classification algorithms showed that the proposed method performs competitively with other related methods.

1 **Fuzzy based binary feature profiling for modus operandi**
2 **analysis**

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28 Abstract

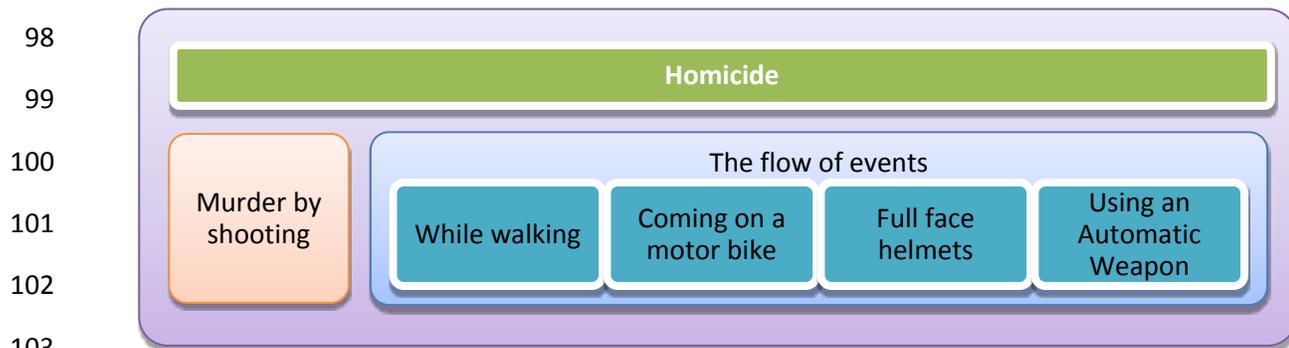
29 It is a well-known fact that some criminals follow perpetual methods of operations, known as
30 modi operandi (MO) which are commonly used to describe the habits in committing something
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32 relating criminals to other crimes where the criminals have not yet been recognized. This paper
33 presents a method which is focused on identifying the perpetual modi operandi of criminals by
34 analyzing their previous convictions. The method involves in generating a feature matrix for a
35 particular suspect based on the flow of events of his/her previous convictions. Then, based on
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38 compared with the flow of events of the crime in order to investigate and relate a particular
39 criminal. This comparison uses several operations to generate two other outputs: completeness
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41 inference system to generate a score value which is used in providing a measurement for the
42 similarity between the suspect and the crime at hand. The method was evaluated using actual
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44 validity and the generalizability of the proposed method. In addition, comparison with nine
45 other classification algorithms showed that the proposed method performs competitively with
46 other related methods.

47 Introduction

48 Scientists have long played a role in examining deviant behavior in society. "Deviance
49 behaviour" is a term used by scientists to refer to some form of "rule-breaking" behaviour [1]. It
50 can be the behaviour of violating a social norm or the law. Criminal behaviour is also a form of
51 deviance, one that is defined as the breaking of legal rules. Nevertheless, there is a difference
52 between deviance and crime. Deviance involves breaking a norm and evoking a negative
53 reaction from others. Crime is a deviance that breaks a law, which is a norm stipulated and
54 enforced by government bodies [1]. However, crimes affect the society negatively. Therefore,
55 law enforcement authorities take necessary actions to mitigate crimes in an environment
56 where high crime frequencies are observed each year. In this exercise the application of
57 technology for crime analysis is being widened in the world. Locard's Exchange principle states
58 that every contact of the perpetrators of a crime scene leaves a trace. The perpetrators will
59 both bring something into the scene and leave with something from the scene [2]. However,
60 the cognitive abilities of criminals will always make them minimize their risks of apprehension
61 by conducting the perfect crime and maximizing their gain [3]. Modus operandi or method of
62 operation such as preparation actions, crime methods and weapons are frequently used in
63 criminal profiling because the past crime trends show that, after criminals get used to a certain
64 method of operation, they try to use the same modus operandi in committing his/her next
65 crime [4].

66 The criminals develop a set of actions during the performance of a series of crimes which we
67 refer to as “modus operandi” (MO). MO is developed with the crimes he/she commits and the
68 nature of trying to stick with the developed MO that has worked throughout the previous
69 crimes [5]. In any criminal career, the MO happens to evolve, no matter what the
70 circumstances. Also, it is a common behaviour that serial offenders tend to exhibit significant
71 behaviour known as his/her signature. Therefore, MOs of criminals plays a major role in
72 investigating crimes [5]. It is a known fact that features such as criminal signature and physical
73 appearance are used in crime investigations in almost all the police departments around the
74 world. Sri Lanka police also use MOs of criminals to identify the suspects who have conducted
75 crimes. Currently Sri Lanka Police use a manual crime recording and investigation system. This
76 manual system has many problems such as data redundancy, inefficiency, tediousness, inability
77 to support crime investigation and many other problems which are associated with a
78 conventional manual system. To overcome these problems, a web-based framework was
79 proposed with geographical information support containing a centralized database for crime
80 data storage and retrieval, named SL-CIDSS: Sri Lanka Crime Investigation and Decision Support
81 System [6]. The proposed system accompanies a collection of data mining algorithms which
82 effectively support the crime investigation process. Fuzzy based binary feature profiling (BFPM)
83 for modus operandi analysis is one novel algorithm which is integrated with the system to
84 provide an effective way to find the similarity between crimes and criminals.

85 According to the penal code of Sri Lanka first enacted in 1882 and amended subsequently
86 several times in later years [7], Sri Lanka police classifies crimes into two categories: Grave
87 crimes and Minor offences. Until 2014, grave crimes were classified under 21 crime categories
88 and in 2015 another 5 new crime categories were introduced, making it 26 categories of grave
89 crime types. Kidnapping, Fraud or mischief causing damage greater than 25000 rupees,
90 Burglary, Grievous hurt, Hurt by sharp weapon, Homicide, Rape, Robbery, Cheating by trust,
91 Theft are 10 of the most frequent crime types. To identify the patterns involved in crimes, a
92 collection of subtypes were identified under these 26 crime types. These subtypes have been
93 created mainly for the purpose of modus operandi analysis. Most frequent behaviors of
94 criminals/crimes are considered as crime subtypes. When a crime is logged in the Grave Crime
95 Record (GCR) book, it is classified under one of the 26 main categories. But, under the section
96 of “nature of crime” in the GCR book, the police officers record the flow of the crime incident
97 including the subtypes.



104 **Figure 1.** Relationship between main crime type, subtypes and crime flows

105 A subtype is a sub category of one of the main crime types. This flow of crime is written in the
106 GCR book in the nature of the crime section as a description. For investigation, the nature of
107 the crimes is broken into subtypes and flows according to their frequency of occurrence and
108 uniqueness. These sub categorizations have been introduced mainly to minimize the broadness
109 of main type and to improve clarity. Fig.1. depicts the relationship of the subtypes and flows
110 where there can be a flow of events to a crime recorded as one of the 26 main crime types. For
111 the simplicity and easy handling of data, the investigators have provided subtype codes and
112 flow codes. The flow of events provides a modus operandi which is most of the time unique to
113 an offender. Each subtype is provided with a code under the main type, to make the crime
114 investigation process easier. For example, ROB/S001 denotes a subtype that is Highway
115 robbery; here ROB denotes the main type under which the corresponding subtype appears. In
116 this case, it is Robbery. Crime types are further subdivided into sub types to make the analysis
117 and processing simpler. In this manner, crime subtypes and flows have been identified under all
118 the 26 crime types. The space for adding more subtypes and flows under these crime types
119 exists. A new subtype or a flow is introduced to a particular main crime, if the same subtype or
120 the flow happens to persist for a prolonged time.

121 This paper proposes a novel method of criminal profiling using the modus operandi which can
122 be used to identify associations between crimes and chronic criminals. The method is based on
123 a new technique named, “binary feature vector profiling”. Key relationships between a criminal
124 and the conducted crimes are analyzed using binary feature profiling and association rule
125 mining techniques. Due to the impreciseness and vagueness of these extracted attributes, a
126 fuzzy inference system is used in making the final decision. The newly proposed method was
127 adapted into a classification algorithm in order to test its accuracy. An actual crime data set
128 which was obtained from Sri Lanka Police was used in testing the performance of the newly
129 proposed method and it was compared against nine well established classification algorithms.
130 Comparisons were done using Friedman’s rank tests. The results confirmed that the proposed
131 method produced competitive results compared to the other nine classification algorithms.

132 The rest of the paper is organized as follows. Related work section presents a summary of the
133 work that has been conducted on modus operandi analysis as well as a brief discussion on
134 crime investigation using link analysis and association mining in general. Materials and Methods
135 section discusses the main steps of the newly proposed algorithm. Next, Results and Discussion
136 section provides a validation and performance evaluation of the newly proposed method along
137 with a performance comparison with nine other classification algorithms. Finally, some
138 concluding remarks and future enhancements are outlined in the conclusion section.

139 **Related work**

140 Literature shows many methods which have been developed in the area of automated crime
141 investigation. Our major concern has been laid upon the research carried out on crime
142 investigation using association mining as our research considers on developing a model to find
143 the associations between the criminals and the crimes depending on the modes operandi. C
144 Bennell and DV Canter [8] have proposed a method to use statistical models to test directly the

145 police practice of utilizing modus operandi to link crimes to a common offender. The results
146 indicated that certain features such as the distance between burglary locations, lead to high
147 levels of predictive accuracy. Craig Bennell, et. al. [9] have tried to determine if readily
148 available information about commercial and residential serial burglaries, in the form of the
149 offender's modus operandi, provides a statistically significant basis for accurately linking crimes
150 committed by the same offenders. Benoit Leclerc, et al. [10] have reviewed the theoretical,
151 empirical, and practical implications related to the modus operandi of sexual offenders against
152 children. They have presented the rational choice perspective in criminology followed by
153 descriptive studies aimed specifically at providing information on modus operandi of sexual
154 offenders against children.

155 Clustering crimes, finding links between crimes, profiling offenders and criminal network
156 detection are some of the common areas where data mining is applied in crime analysis [11],
157 [12], [13]. Association analysis, classification and prediction, cluster analysis, and outlier
158 analysis are some of the traditional data mining techniques which can be used to identify
159 patterns in structured data. Offender profiling is a methodology which is used in profiling
160 unknown criminals or offenders. The purpose of offender profiling is to identify the socio-
161 demographic characteristics of an offender based on information available at the crime scene
162 [14] [15]. Association rule mining discovers the items in databases which occur frequently and
163 present them as rules. Since this method is often used in market basket analysis to find which
164 products are bought with what other products, it can also be used to find associated crimes
165 conducted with what other crimes. Here, the rules are mainly evaluated by the two probability
166 measures, support and confidence [16], [17]. Association rule mining can also be used to
167 identify the environmental factors that affect crimes using the geographical references [18].
168 Incident association mining and entity association mining are two applications of association
169 rule mining. Incident association mining can be used to find the crimes committed by the same
170 offender and then the unresolved crimes can be linked to find the offender who committed
171 them. Therefore, this technique is normally used to solve serial crimes like serial sexual offenses
172 and serial homicide [19].

173 Similarity-based association mining and outlier-based association mining are two approaches
174 used in incident association mining. Similarity-based association mining is used mainly to
175 compare the features of the crime with the criminal's behavioral patterns which are referred as
176 modus operandi or behavioral signature. In outlier-based association mining, crime associations
177 will be created on the fact that both the crime and the criminal have the possibility of having
178 some distinctive feature or a deviant behavior [20]. Entity association mining/link analysis is the
179 task of finding and charting associations between crime entities such as persons, weapons, and
180 organizations. The purpose of this technique is to find out how crime entities that appear to be
181 unrelated at the surface, are actually linked to each other [19]. Link analysis is also used as one
182 the most applicable methods in social network analysis [21] in finding crime groups, gate
183 keepers and leaders [22].

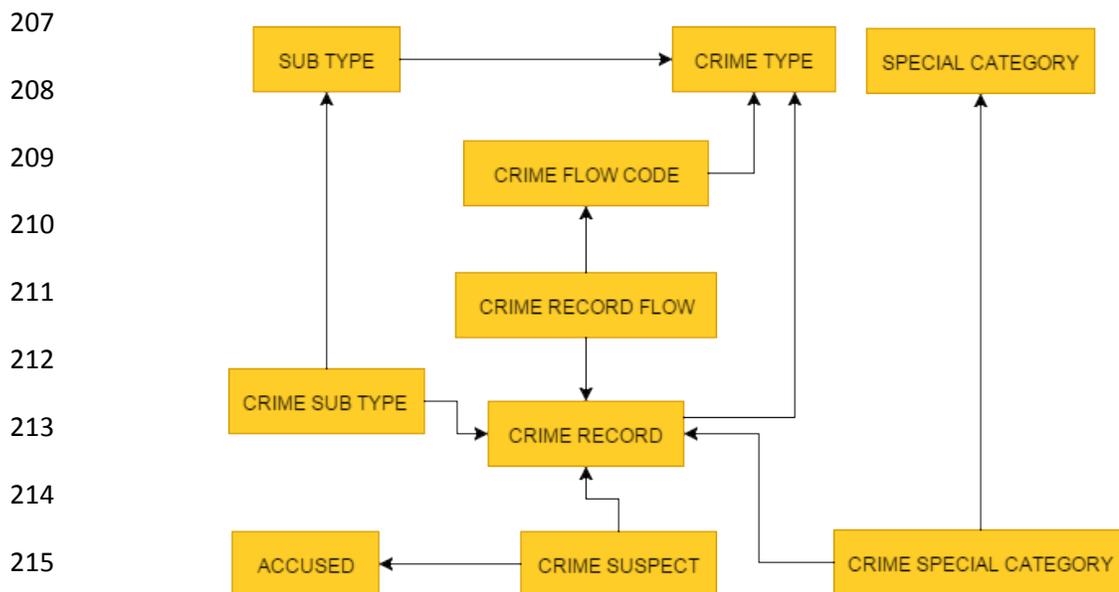
184 Attribution can be used to link crimes to offenders. If two offences in different places involve
185 the same specific type, those may be readily attributed to the same offender [11]. There are

186 three types of link analysis approaches, namely Heuristic-based, Statistical-based and Template-
 187 based [19]. Sequential pattern mining is also a similar technique to association rule mining. This
 188 method discovers frequently occurring items from a set of transactions occurred at different
 189 times [23]. Deviation detection detects data that deviates significantly from the rest of the data
 190 which is analyzed. This is also called outlier detection, and is used in fraud detection [23] [24].

191 In classification, the data points will be assigned to a set of predefined classes of data by
 192 identifying a set of common properties among them. This technique is often used to predict
 193 crime trends. Classification needs a reasonably complete set of training and testing data since a
 194 high degree of missing data would limit the prediction accuracy [23]. Classification comes under
 195 supervised learning method [19], [25] which includes methods such as Bayesian models,
 196 decision trees, artificial neural networks [26] and support vector machines. String comparison
 197 techniques are used to detect the similarity between the records. Classification algorithms
 198 compare the database record pairs and determine the similarity among them. This concept can
 199 be used to avoid deceptive offender profiles. Information of offenders such as name, address,
 200 etc. might be deceptive and therefore the crime database might contain multiple records of the
 201 same offender. This makes the process of identification of their true identity difficult [23].

202 Systems and Methods

203 This section provides a description about the systems and methods used in developing the
 204 fuzzy based binary feature profiling for modus operandi analysis. First, an overview about how
 205 SL-CIDSS captures the logics of modus operandi is explained. Then a detailed description about
 206 the steps of the newly proposed algorithm is explained.



216 **Figure 2.** Crime flow entity arrangement of SL-CIDSS

217 Figure 2 shows how SL-CIDSS database captures the crime types and subtypes. A crime record
 218 has a crime record flow. Typically, a crime is committed by a criminal and a particular accused

219 might commit one or more crimes. A CRIME RECORD can be of one the 26 crime types. A
220 particular CRIME RECORD will be considered under one main CRIME TYPE with the highest
221 precedence in the order of seriousness. For example, a crime incident that includes a murder
222 and a robbery will be categorized as a murder though a robbery has also taken place. But in the
223 nature of crime section, all crimes followed by the main type will be stated. Therefore, the
224 CRIME RECORD FLOW captures all the steps of the crime as a sequence of steps recorded. The
225 crime flows that have been previously registered are mapped under CRIME FLOW CODE. Also, a
226 particular CRIME RECORD instance can contain multiple SUB TYPES which are recorded as
227 CRIME SUB TYPE. The SPECIAL CATEGORY captures the crimes with special features such as
228 crimes occurring at the same location or retail shop. A crime may involve several special
229 categories which are saved in the CRIME SPECIAL CATEGORY. The ACCUSED entity records the
230 information of suspects and accused and they are related to crime through the CRIME SUSPECT
231 entity.

232 As the first step of the newly employed method, a feature matrix is generated, resulting in a
233 binary matrix representing the crime flows. This binary feature matrix is composed of the
234 binary patterns generated on previous convictions of a particular criminal/suspect. As the MOs
235 are represented in binary all the analyses are conducted on this binary feature matrix. This
236 binary form of the feature matrix provides a provision to direct application of computer
237 algorithms with methods such as Apriori based association rule mining as the matrix is already
238 in the binary format. The reduced complexity of the binary feature matrices provides an easy
239 manipulation over the categorical and continuous valued features. Figure 3 shows the steps of
240 the proposed MO analysis algorithm.

- 241
- 242 **Step 1:** Generate the feature matrix.
- 243 **Step 2:** Generate the dynamic MOs (DMO) of the criminals.
- 244 **Step 3:** Generate the complete MO profile (CMOP) of the criminals.
- 245 **Step 4:** Find deviation probability (DP) of CMOP from the crime MO under consideration (UMO).
- 246 **Step 5:** Find completeness probability of UMO against DMO.
- 247 **Step 6:** Use the two values obtained from step 4 and 5 as inputs of a fuzzy Inference system to
248 obtain the final similarity value (out of 100).
- 249 **Step 7:** Classify the UMO under the class with highest similarity score for validation.

250 **Figure 3.** Steps of the newly employed algorithm

251

252 **Generating the binary feature matrix**

253 Table 1 shows how the feature matrix is generated and provides the way to generate modus
 254 operandi of criminals as binary sequences. According to the table, events of the crime scene are
 255 observed starting from its crime type. After a particular crime type is identified, the feature
 256 matrix is updated with ones for each subtype and flow code that is available in the crime or
 257 suspect modus operandi. The matrix will be filled by zeros in places which the modus operandi
 258 does not have any contact with. The column names to the feature matrix are generated in such
 259 a way that it covers the collection of main types, sub types, crime flows and special categories
 260 at hand. For example, if we consider the list of crime types, subtypes, crime flows and the
 261 special category in Table 1, it results in a feature matrix of a 21-bit vector shown in the last two
 262 columns.

263 **Table 1.** *An instance of feature selection for the feature matrix generation*

Main Semantic	Crime flow element code	Description	Suspect 1	Suspect 2
Crime types	HB	House Breaking	0	1
	HK	Hurt by Knife	0	0
	RB	Robbery	1	0
	TH	Theft	0	0
Sub types	ABD/S003	Abduction from the legal guardian	1	0
	ABD/S004	Abducting to marry	0	0
	ABD/S005	Abducting for sexual harassment	0	0
	BGL/S004	Use of stealth	0	1
	BGL/S011	Burglary in business places	0	0
	ROB/S001	Organized vehicle robbery	1	0
Crime Flows	BGL/F001	Entering from the window	0	1
	BGL/F002	Entering from the Fanlights	0	0
	BGL/F003	Removing grills	0	1
	BGL/F004	Breaking glasses	0	0
	ROB/F001	Showing identity cards	1	0

	ROB/F003	Wearing uniforms	1	0
	ROB/F004	Robbery using identity cards, uniforms and chains	0	1
	ROB/F009	Seizing inmates	0	0
	ROB/F010	Appearing as CID officers	0	0
Special Category	Retailer 1	Attacking/ robbing retailer 1's stores	0	0
	Retailer 2	Attacking/ robbing retailer 2's stores	0	0

264 In this manner we can produce binary MO patterns based on the crimes committed by different
 265 criminals as shown in the last two columns of Table 1. According to Table 1, Accused 1 has
 266 committed a robbery with the subtypes, ABD/S003 (an abduction of a child from the legal
 267 guardian), ROB/S001 (an organized vehicle robbery) and the flows, ROB/F001 (Identity cards
 268 have been shown), ROB/F003 (accused has been wearing uniforms). Accused 2 has committed a
 269 house breaking with the sub type BGL/S011 (use of stealth), and the flows, BGL/F001 (Entering
 270 from the window), BGL/F003 (Removing Grills).

271 In this manner, depending on the complete crime MO under consideration, it may generate
 272 modus operandi of different lengths. According to the full crime MO, criminal based MOs can
 273 be generated and taken into a full feature matrix of binary patterns. *ct*, *st*, *fl* and *sc* in Table 2
 274 represent the abbreviations for crime type, sub type, flow and special category respectively.

275 **Table 2.** Feature matrix generated using the selected modus operandi attributes in Table 1.

	ct1	ct2	ct3	ct4	st1	st2	st3	st4	st5	st6	fl1	fl2	fl3	fl4	fl5	fl6	fl7	fl8	fl9	sc1	sc2
Accused 1	0	0	1	0	1	0	0	0	0	1	0	0	0	0	1	1	0	0	0	0	0
Accused 2	1	0	0	0	0	0	0	1	0	0	1	0	1	0	0	0	1	0	0	0	0

276

277 **Generating the dynamic MOs (DMOs) of the criminals**

278 Dynamic MO is a binary feature vector which is generated on bit patterns of the feature matrix
 279 of a particular criminal. The main purpose of the DMO is to obtain a criminal specific crime flow
 280 which captures the crime patterns which are frequently followed by a particular criminal. It
 281 was named as the dynamic modus operandi as it is subject to change when the new crime flows
 282 are added to the feature matrix. Therefore, this addresses the changing nature of the patterns
 283 used by the criminals in committing crimes. First, a frequency threshold is generated using
 284 characteristic features of the sub matrix at hand which is the matrix of all crimes committed by
 285 the same criminal under consideration. If the corresponding accused is convicted for four
 286 crimes, the four bit patterns related to those four crimes will be available in the feature matrix.

287 The matrix shown in Table 3 is an example to a situation of a feature matrix generated on the
 288 previous convictions of a criminal. For simplicity let's consider a feature matrix of 10 columns.

289 **Table 3.** Feature matrix generated on four previous convictions of a criminal

A	B	C	D	E	F	G	H	I	J
1	0	0	0	1	1	1	0	1	0
1	0	0	1	0	1	1	0	1	0
1	0	0	0	1	1	0	1	1	0
1	0	0	1	0	1	1	1	1	0

290

291 If we consider A to J of Table 3 as crime flow features of the corresponding MOs, we can
 292 understand that in the first MO the criminal has followed a flow of A-E-F-G-I. The same criminal
 293 has followed a crime flow of A-D-F-G-I in his second crime. Likewise the other two flows are, A-
 294 E-F-H-I and A-D-F-G-H-I respectively.

295 The DMO of a particular criminal is generated using the Apriori method [27]. Apriori method is
 296 used to find the crime entities with the frequency threshold (frt) which is generated according
 297 to Equation 2. Apriori is a method to find frequent item sets in transactions in association rule
 298 mining [27]. A demonstration of the generation of D in Equation 1 on the properties of feature
 299 matrix is shown in Table 4.

$$D = \left\{ d \mid d = \sum_{i=1}^n y_i \right\} \quad (1)$$

$$frt = M_D / n \quad (2)$$

300 Where,

301 y_i = cells in each column

302 M_D = Median of D,

303 $n = \sum f$ = number of values or total frequencies,

304 c = cumulative frequency of the median class

305 h = class interval size.

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313 **Table 4.** Column-wise addition of the feature matrix of the suspect under consideration

A	B	C	D	E	F	G	H	I	J
1	0	0	0	1	1	1	0	1	0
1	0	0	1	0	1	1	0	1	0
1	0	0	0	1	1	0	1	1	0
1	0	0	1	0	1	1	1	1	0
4	0	0	2	2	4	3	2	4	0



314

315 The column-wise addition of the matrix shown in Table 4 gives 4, 0, 0, 2, 2, 4, 3, 2, 4 and 0. The
 316 distinct numbers are selected from the resulting vector which results in 4, 0, 2. The vector with
 317 the distinct numbers is named as D. The median of D is then divided by the number of instances
 318 (rows) in the matrix as the frt, which is $2.5/4 = 0.625$ for the above case. Therefore, frt will
 319 range from 0 to 1. This value provides an insight to a fair threshold value for the Apriori method
 320 to generate the dynamic modus operandi with the most frequent elements. frt is used as the
 321 frequency threshold in finding the lengthiest MO with a probability of 0.625 because this value
 322 suggests that there is a moderate possibility of one feature having 0.625 probability in each of
 323 MO. This results in a dynamic modus operandi (DMO) as shown in Equation 4, because the only
 324 transaction of crime attributes which provides a support of 0.625 is $\sigma(A,F,G,I)$ as shown in
 325 Equation 3.

$$s = \frac{\sigma(A,F,G,I)}{|T|} = \frac{3}{4} = 0.75 \quad (3)$$

326

$$DMO = [1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0] \quad (4)$$

328

329 Generating the complete MO profile (CMOP) of the criminals

330 The complete MO profile (CMOP) is obtained by the OR operation between the bits of each
 331 column of the feature matrix of the corresponding criminal. CMOP guarantees the provision of a
 332 composite crime flow by considering all of the previous crime flow entities of a particular criminal. For
 333 example, the complete profile for the feature matrix shown in Table 3 is obtained as shown in
 334 Table 5.

335 **Table 5.** OR operation on the columns to obtain the complete MO profile

A	B	C	D	E	F	G	H	I	J
1	0	0	0	1	1	1	0	1	0
1	0	0	1	0	1	1	0	1	0
1	0	0	0	1	1	0	1	1	0
1	0	0	1	0	1	1	1	1	0
1	0	0	1	1	1	1	1	1	0



336 Therefore, $CMOP = [1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0]$. CMOP contains 1s for each place for which a
 337 particular crime flow entity has taken place at least once.

338 **Finding deviation probability (DP) of CMOP from the crime MO under** 339 **consideration (UMO)**

340 First, the deviation of each individual flow in the criminals feature matrix is obtained according
 341 to Equation 5. As the binary feature vectors are commonly used to represent patterns, many
 342 methods have been invented to find their similarity and distance [28]. Euclidean distance,
 343 Hamming distance, Manhattan distance, Jaccard, Sorensen and Tanimoto are few of the
 344 frequently used measures in that domain [28]. This probability value which is named as the
 345 deviation probability (DP), is used to obtain a measurement as to what extent of information is
 346 available in the UMO, extra to what is already available in the CMOP of a particular criminal.
 347 Let's assume that the bit pattern to be compared with the suspect's modus operandi profile
 348 under consideration is $UMO = [1\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1]$. Therefore, DP provides the probability of 1s
 349 which are available in UMO but not in CMOP.

350 The deviation probability, DP can be given as,

$$351 \quad DP = \frac{\sum_{i=1}^n x_i - y_i}{n}, \text{ for } x_i = 1, y_i = 0; i = 1, 2, \dots, n \quad (5)$$

352 Where,

353 $x_i = \text{elements of the UMO}$

354 $y_i = \text{elements of the CMOP}$

355 If we consider the feature matrix on Table 5,

$$356 \quad Deviation = [1\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1] - [1\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0]$$

$$357 \quad Deviation = [0\ 0\ 0\ -1\ 0\ 0\ 0\ -1\ -1\ 1] \quad (6)$$

358 Define $AD = 1$, where AD is the number of positive 1s.

359 Therefore, $DP = 1/10 = 0.1$

360 As it appears in Expression 6, it produces positive 1s for the places with the features available in
 361 UMO but not in CMOP. The higher the DP, higher the amount of extra information available in
 362 UMO. Hence, a DP value close to 0 indicates the absence of extra features in UMO.

363 **Finding completeness probability (CP) of UMO against DMO**

364 For the same feature matrix which was considered in Table 3, the CP is obtained according to
 365 Equation 7. Here, the UMO is compared with DMO to obtain a probability to determine what

366 extent of features in CP is available in UMO. Therefore, it is derived by the percentage of
367 attributes which are present in both UMO and DMO.

368 Let $DMO = \{x_i\}_{i=1}^n$ and $UMO = \{y_j\}_{j=1}^n$ be two binary sequences.

369 Define, $z_k = \begin{cases} 1; & x_i = y_j \\ 0; & otherwise \end{cases}$. Then, $CP = \frac{\sum_{k=1}^n z_k}{n}$ is the completeness probability. (7)

370 For example, if we consider $DMO = [1\ 0\ 0\ 0\ 0\ 1\ 1\ 0\ 1\ 0]$, then for the
371 $UMO = [1\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1]$ a CP of $3/10 = 0.3$ is generated as in the 1st, 6th and 7th positions
372 there are ones in both DMO and UMO. Higher the CP value, the more the UMO is composed of
373 crime flow entities which are available in the DMO. Therefore, a CP value close to 1 indicates
374 that the completeness of UMO compared to DMO is 100%.

375 **Building a fuzzy inference system to obtain the final similarity score**

376 The vagueness of the two measurements CP and DP generates a difficulty in calculating a
377 similarity score using crisp logic. Therefore, the two parameters CP and DP were adapted into a
378 fuzzy inference system which accepts two inputs and provides a score for the similarity
379 between the DMO and UMO. Figure 4 shows a block diagram of the proposed fuzzy inference
380 system. Mamdani fuzzy inference was proposed as an attempt to solve a control problem by a
381 set of linguistic rules obtained from experienced human operators [29]. First, the rule base of
382 the fuzzy controller was defined by observing the variations of CP and DP. The membership
383 functions of the inputs and outputs were then adjusted in such a way that, the parameters
384 which seem to be wrong can be fine-tuned, which is a common practice in defining fuzzy
385 inference systems [30]. Literature shows many methods used in fine tuning the fuzzy
386 parameters. Usage of adaptive networks [31] and Neuro-fuzzy systems [32] in fine tuning the
387 fuzzy parameters have received more attention. The problem at our hand was to generate a
388 fuzzy inference system which generates the highest similarity score when the DP value goes
389 down and CP value goes up. We conducted a manual mapping procedure for the fuzzy
390 membership functions. Therefore, the input and output space of the two inputs CP and DP and
391 the output were partitioned into 3 subsets. Namely, LOW, MODERATE and HIGH. Center of
392 gravity was used as the defuzzification strategy of the fuzzy controller. Mamdani fuzzy
393 inference was especially selected for the similarity score generation procedure, for the highly
394 intuitive knowledge base it offers due to the fact that both antecedents and the consequents of
395 the rules are expressed as linguistic constraints [33]. First, we selected all of these membership
396 functions with 50% overlap. Then the tuning procedure was conducted during which we
397 adjusted either the left and/or right spread and/or overlapping to get the best possible
398 similarity score for the given DP and CP. This procedure was conducted until the FIS generated
399 satisfactory results.

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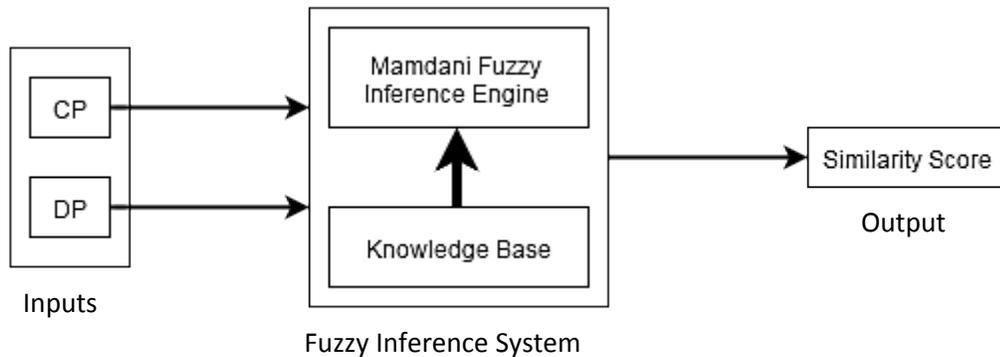


Figure 4. Block diagram of the proposed fuzzy inference system.

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Figures 5 and 6 show the fuzzy inputs of the Fuzzy Inference System (FIS) which correspond to CP and DP values respectively. Figure 7 depicts the fuzzy output of the FIS. As the Figures 5, 6 and 7 depict, all the different levels of membership functions under each input and the output are selected to be triangular and trapezoidal functions as triangular or trapezoidal shapes are simple to implement and computationally efficient [34].

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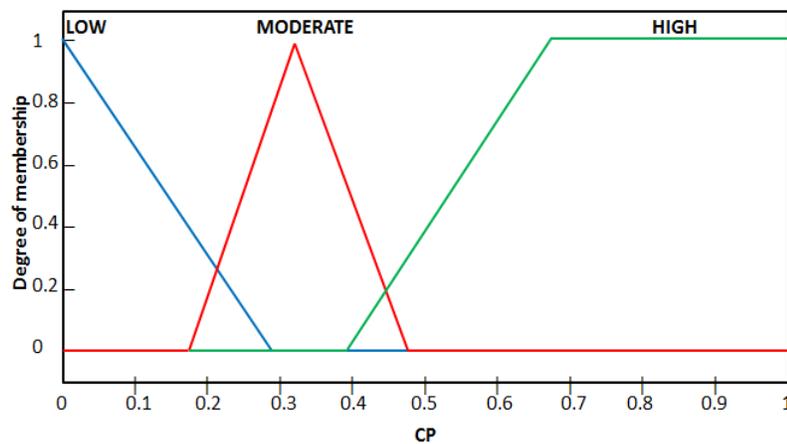


Figure 5. Input fuzzy variable 1: CP

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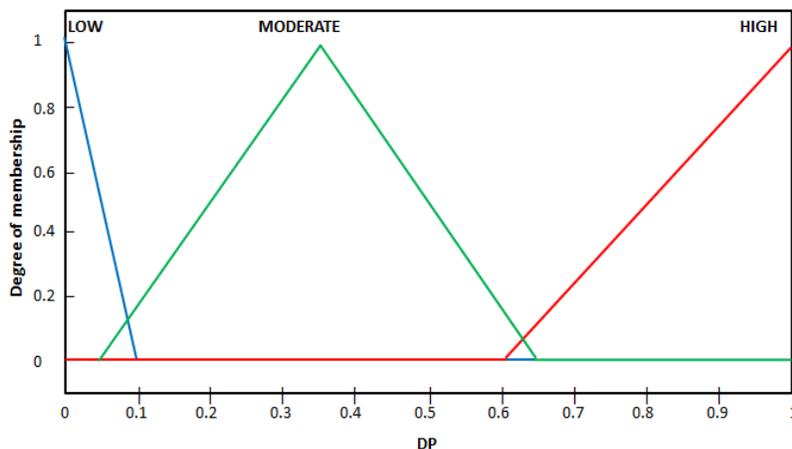
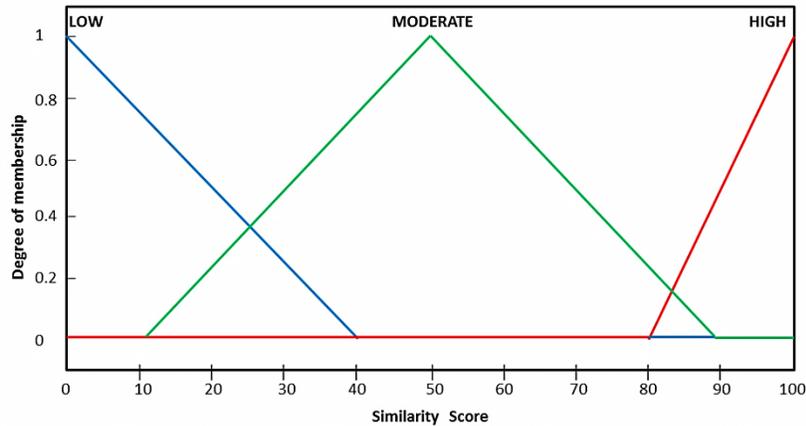


Figure 6. Input fuzzy variable 2: DP

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Figure 7. Output fuzzy variable: similarity score.

441 As shown in Figure 7, the universe of discourse of similarity score (fuzzy output) ranges from 0
442 to 100. The defuzzifier score which is generated from the FIS is considered as the measurement
443 for how close the modus operandi under consideration is to a particular suspect's profile. A
444 higher score value close to 100, which is generated from the FIS provides a good indication
445 about a high similarity between the modus operandi of the crime and suspect under
446 consideration.

447 The fuzzy rule derivation of the fuzzy controller is heuristic in nature. According to the
448 calculations of the two inputs, higher values of CP, close to 1 and lower values of DP close to 0,
449 positively affect the final similarity score. Therefore, the rule base of the fuzzy model is
450 generated accordingly. Our rule base provides a non-sparse rule composition of 9 combinations
451 as illustrated in Figure 8.

452

DP \ CP	LOW	MODERATE	HIGH
LOW	MODERATE	LOW	MODERATE
MODERATE	HIGH	MODERATE	LOW
HIGH	HIGH	MODERATE	LOW

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Figure 8. Fuzzy rule set of the rule base of the inference system.

458 The rule surface of the fuzzy controller depicted in Figure 9, shows the variation of the score
459 value with the changes of the two inputs CP and DP. According to the figure it's perfectly
460 visible, for higher values of CP (close to 1) and for lower values of DP (close to 0), the fuzzy
461 controller generates higher values for the score which are close to 100.

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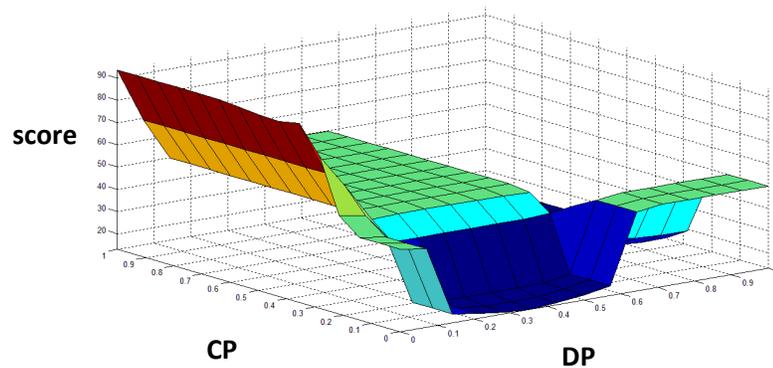
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Figure 9. Rule surface of the fuzzy controller.

469 Classification of the UMO under the class with the highest similarity

470 score

471 When the algorithm is used to find associations between modus operandi of criminals and
472 modus operandi of crimes, the similarity score which is generated from the newly proposed
473 method can be used directly. A similarity score which is close to 100 would suggest that the
474 criminal has a very high tendency to have committed the crime which is under investigation.
475 Therefore, the similarity scores generated can be used to classify a particular modus operandi
476 to a most probable suspect with the highest similarity score.

477 The proposed method was developed by using MATLAB 7.12.0 (R2011a) [35]. All the necessary
478 implementations were conducted using the MATLAB Script editor [36] apart from the FIS which
479 was implemented using the MATLAB fuzzy toolbox [37]. The nine classification algorithms
480 which were used for the performance comparison were classification algorithms which are
481 already packaged with the WEKA 3.6.12 tool [38].

482 Results and Discussion

483 The method was tested with a crime data set obtained from Sri Lanka Police. Figure 10 shows
484 the crime frequencies in Sri Lanka by the crime types from 2005 to 2011. It shows only 21 crime
485 types because the 5 new crime types were introduced in 2015. 4th column denoting House
486 Breaking and Theft shows the highest number of occurrences. 14: Theft of property, 10:
487 Robbery, 13: Cheating/ Misappropriation, 6: Hurt by Knife, 7: Homicide, 8: Rape/ Incest, 5:
488 Grievous Hurt, 3: Mischief over Rs. 5000/=, 1: Abduction/Kidnapping comes next. For the
489 validation of the algorithm, 7 crime types out of these 10 types were selected for the testing
490 data set. They are, House Breaking and Theft, Theft of property, Robbery, Homicide, Rape/

491 Incest, Grievous Hurt, Abduction/Kidnapping. 31 crime flows were selected which are common
 492 to the seven selected crime types. The data set is also composed of 8 sub types and 2 special
 493 categories. Altogether the data set consisted of 67 instances in which each instance is
 494 composed of 48 attribute values. The data set is distributed over 20 classes (criminals).

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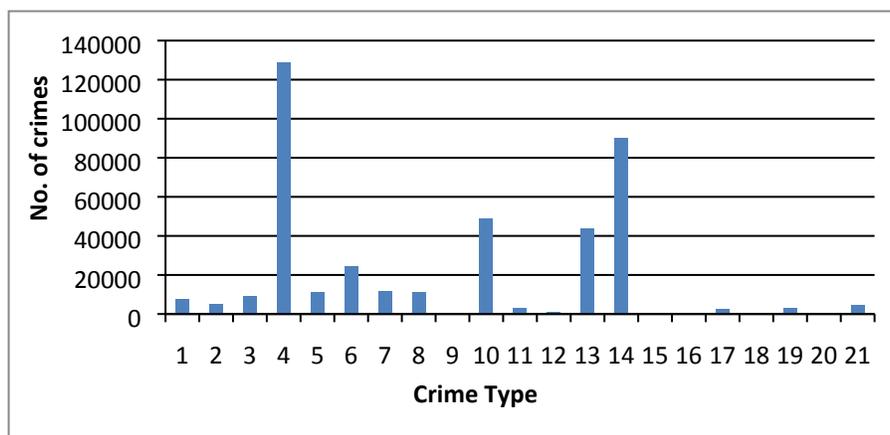
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Figure 10. *Frequency of different crime types from year 2005-2011*

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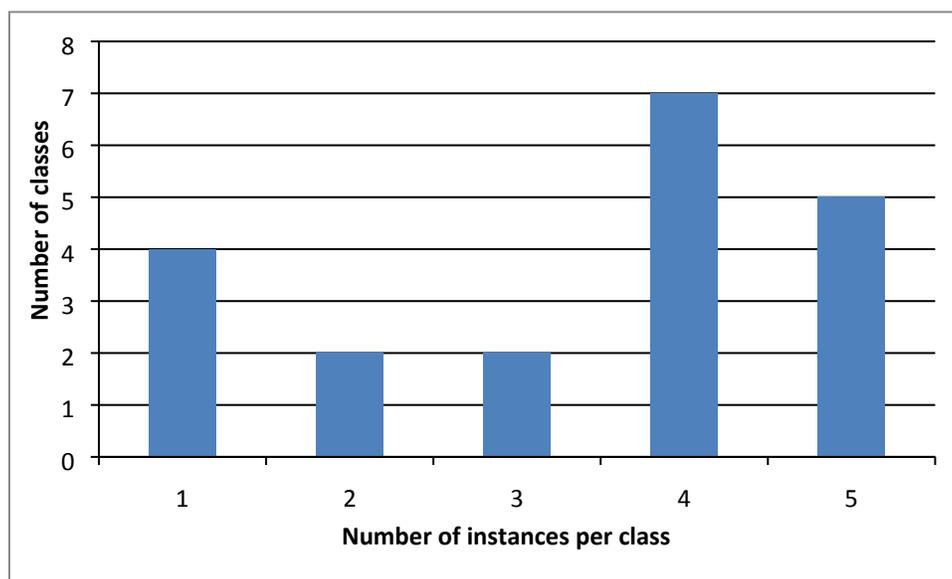
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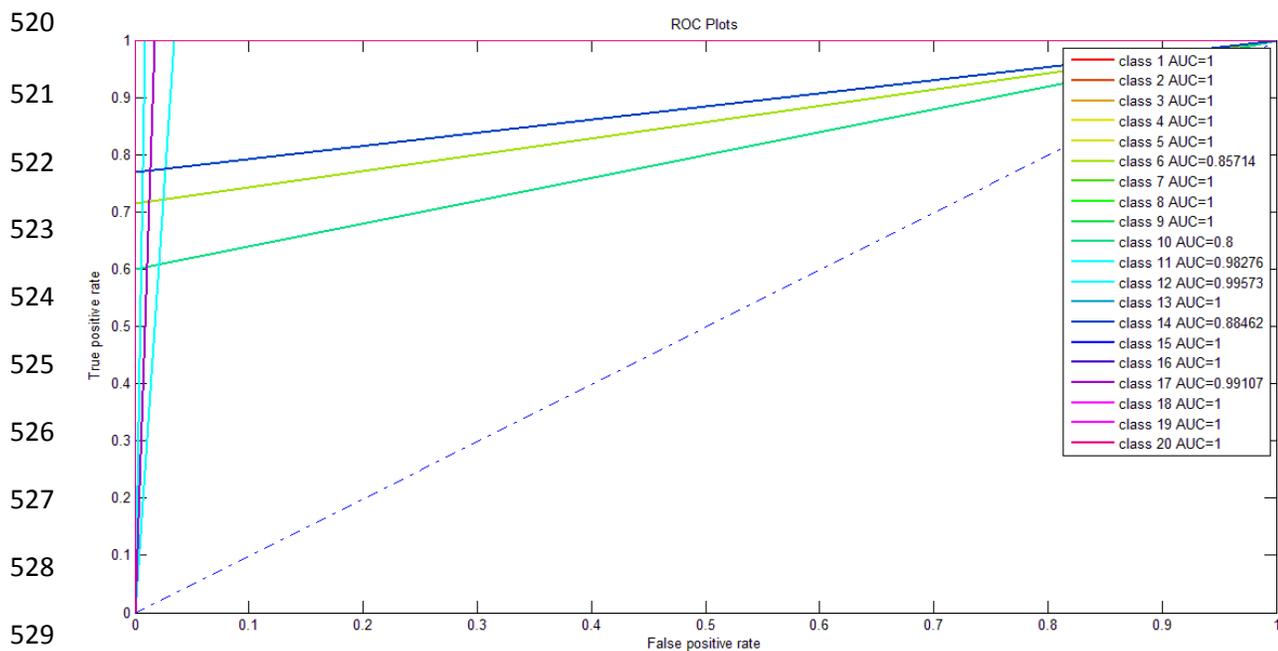
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Figure 11. *Distribution of modus operandi instances over the classes of the dataset*

513 All the tests were performed in a Windows computer with Intel (R) Core (IM) i7-2670QM CPU of
 514 2.20 GHz and a RAM of 8GB. The histogram of the instance distribution over the classes is
 515 shown in Figure 11.

516 10 fold cross validation [39] was used on the data set for a fair testing procedure. In 10-fold
 517 cross validation, the data set is divided into 10 subsets, and the holdout method is repeated 10

518 times. Each time, one of the 10 subsets is used as the test set and the other 9 subsets are put
519 together to form a training set. Then the average error across all 10 trials is computed [39].



530 **Figure 12.** ROC curves returned by the newly proposed method on the 20 classes of crime data set

531 The test results of modus operandi classifications in Area Under Curve (AUC) [40], Root Mean
532 Squared Error (RMSE) and time elapsed for the classification are shown in Table 6. A Receiver
533 Operating Characteristic (ROC) curve is a two dimensional graphical illustration of the trade-off
534 between the true positive rate (sensitivity) and false positive rate (1-specificity). Figure 12
535 depicts the ROC curve plotted on the classification results obtained by the newly proposed
536 method on the crime data set. In the particular instance which is shown in Figure 12, all the
537 ROC curves related to the crime data set are plotted well over the diagonal line and all of them
538 have returned AUC values which are either equal to 1 or very close to 1, providing a very good
539 classification.

540 The sole intention of this research was to find out relationships among the modus operandi of
541 criminals with the modus operandi found in the crime scenes to find the associations. To
542 prepare the data set which was used under this research, a crime data set of around 3000
543 instances was analysed. Due to limitations of the real crime data set, it was quite a complex
544 task to prepare a data set with a collection of sufficient modus operandi where each instance
545 has a considerable flow of crime flows. Therefore, only a sample of 67 instances could be
546 filtered from the population to generate a representative data set and it was verified by a
547 domain expert before being used in the analysis. As the number of instances was around 67, it
548 can be assumed to be an under-represented data sample. Another reason for the data set to

549 become under- represented was the challenge in finding classes/criminals with more than one
 550 crime committed. The actual crime data set which is used for the testing purposes is
 551 imbalanced as it is apparent in Figure 11. For example the data set is composed of 4 single
 552 instance classes while there are seven classes with four instances each. With a classification
 553 data set like this, there is a very high tendency of getting biased results. To make the
 554 classification process unbiased, we used the concept of oversampling. Oversampling and
 555 under-sampling are two concepts which are used in overcoming class imbalance problems in
 556 input data sets. Oversampling and under-sampling are two different categories of resampling
 557 approaches, where in oversampling the small classes are incorporated with repeated instances
 558 to make them reach a size close to larger classes, whereas in under-sampling, the number of
 559 instances is decreased in such a way that the number of instances reach a size close to the
 560 smaller classes [41].

561 Table 6 shows the results returned by the fuzzy based binary feature profiling which was
 562 conducted on the actual crime data set. As shown in the table, there is an increase in the
 563 accuracy when the input data set undergoes oversampling. Since the maximum number of
 564 instances available under one suspect is equal to 5, under-sampling does not provide a good
 565 accuracy. The results prove that the new algorithm works well for a balanced data set as the
 566 new method showed an increase in performance when the data set is subjected to an
 567 oversampling greater than or equal to 5 which is the highest number of instances under a
 568 particular class.

569 **Table 6.** Results returned by the fuzzy based binary feature profiling for the *modus operandi* analysis on
 570 actual data

Data set (Number)	Oversampling or Under-sampling value	AUC	Root Mean Squared Error	Average time elapsed
1	2	0.5417	0.6986	0.0015
2	3	0.5562	0.4969	0.0011
3	4	0.5965	0.4303	0.0014
4	N/A	0.6937	0.7000	0.0010
5	5	0.6612	0.4126	0.0011
6	6	0.7063	0.2959	0.0011
7	10	0.8033	0.1396	0.0012
8	20	0.9339	0.0941	0.0013
9	30	0.9661	0.0853	0.0014
10	40	0.9637	0.0551	0.0015
11	50	0.9756	0.0578	0.0016
12	60	0.9626	0.0638	0.0018
13	70	0.9365	0.0792	0.0019
14	80	0.9391	0.0784	0.0023

15	90	0.9671	0.0752	0.0029
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571 Figure 13 shows the change in AUC with the increase of sampling which starts from under-sampling of 2
572 and goes on to an over sampling of 90. According to the plot it can be observed that the ROC values are
573 increased when the oversampling is increased.

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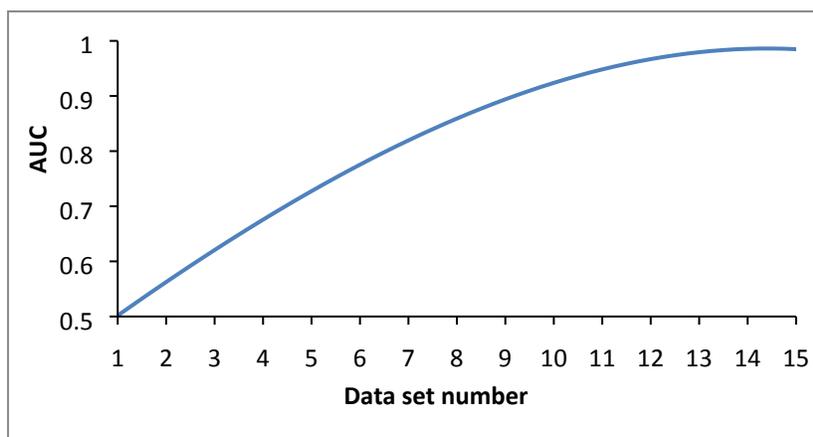
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Figure 13. Change of ROC values with oversampling

583 Figure 14 depicts the change of RMSE with the increase of sampling. The plot clearly illustrates
584 that the RMSE values are decreased with the oversampling.

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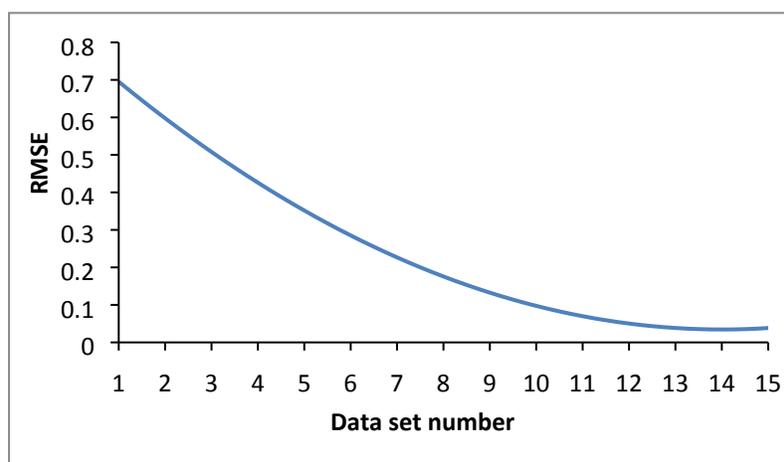
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Figure 14. Change of RMSE value with oversampling

593 The execution time of the algorithm was 0.001s when there is no oversampling or under-
594 sampling. The maximum execution time is 0.0031 when there is an oversampling of 90.
595 According to the plot shown in Figure 15, it is clear that there is an increase of execution time
596 as the oversampling size increases. But, the overall execution time is always remained under 3
597 milliseconds.

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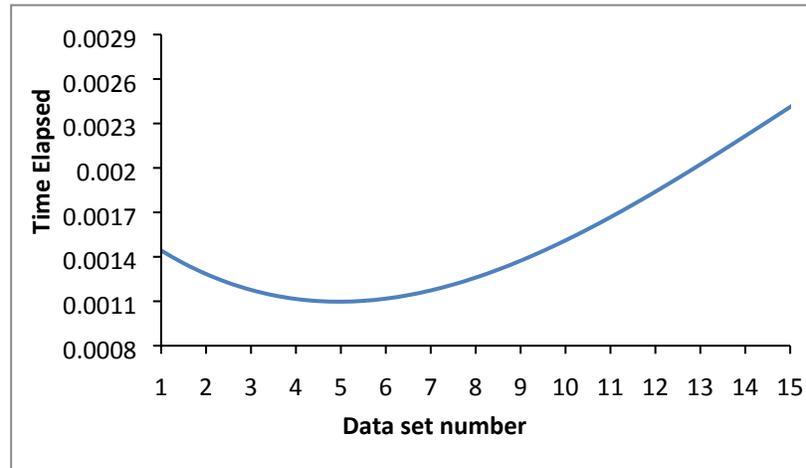


Figure 15. Change of time elapsed for the 15 data sets.

609 Overview of the classification algorithms used for the comparison

610 It is a known fact that there is no single algorithm which can be categorized as the best to solve
611 any problem. Different classification algorithms may perform differently in different situations
612 [42]. Therefore, the newly proposed method was then tested against ten other open
613 classification data sets and the performance was evaluated against the results obtained with
614 nine well-known classification techniques, thereby assessing the quality of the newly proposed
615 method. The nine classification techniques used for this purpose include, Logistic Regression,
616 J48 Decision Tree, Radial Basis Function Network (RBFNetwork), Multi-Layer Perceptron, Naive
617 Bayes Classifier, Sequential Minimal Optimization (SMO) algorithm, KStar instance based
618 classifier, Best-first decision tree (BFTree) classifier, and LMT (Logistic Model Tree) classifier.
619 These classifiers represent four classes of classification algorithms. Namely, function based
620 classifiers, Tree based classifiers, Bayesian classifiers and Lazy classifiers.

621 Logistic Regression learns conditional probability distribution. Relating qualitative variables to
622 other variables through a logistic cumulative distribution functional form is logistic regression
623 [43]. J48 is an open source java implementation of the C4.5 decision tree algorithm [44]. A
624 decision tree consists of internal nodes that specify tests on individual input variables or
625 attributes that split the data into smaller subsets, and a series of leaf nodes assigning a class to
626 each of the observations in the resulting segments. C4.5 algorithm constructs decision trees
627 using the concept of information entropy [45]. Neural networks are flexible in being modeled
628 virtually for any non-linear association between input variables and target variables [46]. Both
629 Radial basis function networks and Multilayer perceptron (MLP) networks are neural networks

630 [47]. Bayesian classifiers assign the most likely class to a given example described by its feature
 631 vector [48]. SMO is an implementation of John Platt's sequential minimal optimization
 632 algorithm for training a support vector classifier. It globally replaces all missing values and
 633 transforms nominal attributes into binary one. It also normalizes all attributes by default [49]
 634 [50]. KStar (K*) is an instance-based classifier which uses an entropy –based distance function
 635 [51]. BFTree (Best- First Decision Tree) uses binary split for both nominal and numeric attributes
 636 [52]. LMT is a classifier for building 'logistic model trees', which are classification trees with
 637 logistic regression functions at the leaves [53], [54].

638 **Table 7.** *Description of the classification data sets for performance comparison*

Data set	Description	Number of Instances	No of Attributes
Dermatology Data Set [55]	This database has been created on a dermatology test carried out on skin samples which have been taken for the evaluation of 22 histopathological features. The values of the histopathological features have been determined by an analysis of the samples under a microscope. In the dataset constructed for this domain, the family history feature has the value 1 if any of these diseases has been observed in the family, and 0 otherwise. Every other feature (clinical and histopathological) was given a degree in the range of 0 to 3. Here, 0 indicates that the feature was not present, 3 indicates the largest amount possible, and 1, 2 indicate the relative intermediate values.	336	33
Balance Scale Data Set [56]	This data set has been generated to model psychological experimental results. Each example is classified as having the balance scale tip to the right, tip to the left, or be balanced. The attributes are the left weight, the left distance, the right weight, and the right distance. The correct way to find the class is the greater of (left-distance * left-weight) and (right-distance * right-weight). If they are equal, it is balanced. There are 3 classes (L,B,R), five levels of Left-Weight (1,2,3,4,5), five levels of Left-Distance (1,2,3,4,5), five levels of Right-weight (1,2,3,4,5) and five levels of Right-Distance (1,2,3,4,5).	625	4
Balloons Data Set [57]	This data set has been generated using an experiment of stretching a collection of balloons carried out on a group of adults and children [58]. In the data set, Inflated is true if (color=yellow and size = small) or (age=adult and act=stretch). In the data set there are two main output classes, namely T if inflated and F if not inflated, two colors yellow and purple, two sizes, large and small, two act types,	20	4

	stretch and dip, and two age groups, adult and child.		
Car Evaluation Data Set [59]	Car Evaluation Database has been derived from a simple hierarchical decision model originally developed for the demonstration of DEX by M. Bohanec and V. Rajkovic [60]. The Car Evaluation Database contains examples with information that is directly related to CAR. They are buying, maint, doors, persons, lug_boot and safety. The attribute buying is the buying price which is considered to have four levels v-high, high, med, low. Maint is the price of the maintenance which contains the four levels, v-high, high, med, low. Doors have the four levels 2, 3, 4, 5-more. Person (capacity in terms of persons to carry), lug_boot (the size of luggage boot) and safety (estimated safety of the car) have 3 levels each.	1728	6
Soybean Data set [61] [62]	This is a small subset of the original soybean database. The data set is distributed over four classes, D1, D2, D3 and D4. The 35 categorical variables represent different levels of qualities of the soybean vegetable. These categorical variables include, plant-stand, precip, temp, hail, crop-hist, area-damaged, severity, seed-tmt, germination, lant-growth, leaves, leafspots-halo, leafspots-marg, leafspot-size, leaf-shread, leaf-malf, leaf-mild, stem, lodging, stem-cankers, canker-lesion, fruiting-bodies, external, mycelium, int-discolor, sclerotia, fruit-pods, fruit,seed, mold-growth, seed-discolor, seed-size, shriveling and roots. The number of levels represented by each variable varied from 2 to 3.	47	35
Lenses Data set [63]	Lenses data set is a small database about fitting contact lenses. The data set is composed of five attributes including the class variable. The data set has three classes. Age of the patient, spectacle prescription, astigmatic, tear production rate are the attributes of the data set. The attributes contain at least of two categories and at most of three categories.	24	4
Nursery Data set [64]	Nursery Database has been derived from a hierarchical decision model originally developed to rank applications for nursery schools. It has been used during several years in 1980's when there has been excessive enrollment to these schools in Ljubljana, Slovenia, and the rejected applications frequently needed an objective explanation. The final decision depended on three sub problems: occupation of parents and child's nursery, family structure and financial standing, and social and health picture of the family. The model has been developed within expert system shell for decision making [65].	12960	8
Tic-tac-toe	This database encodes the complete set of possible board	958	9

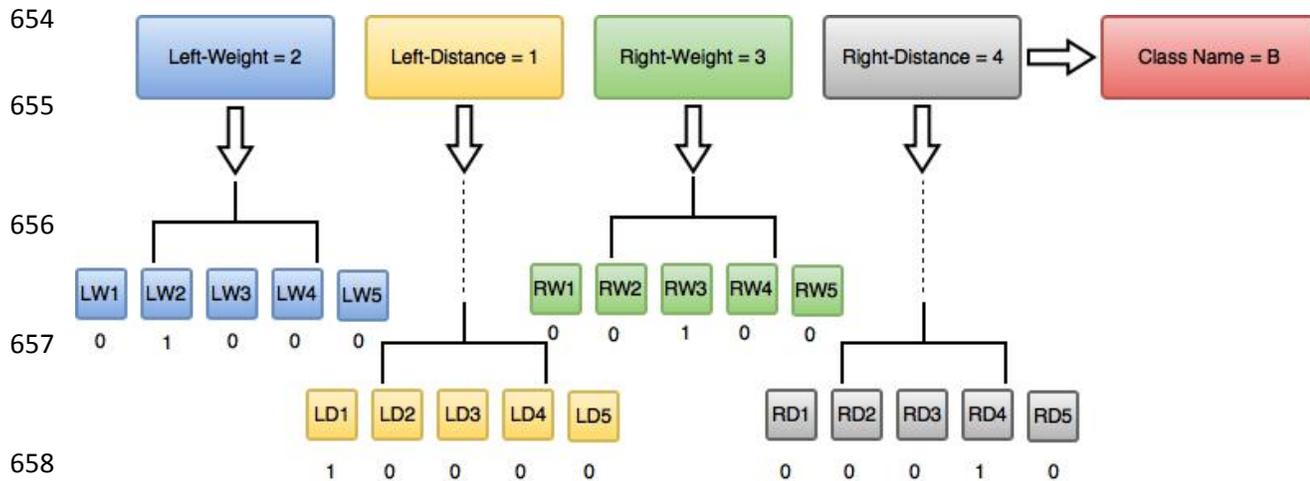
Data set [66]	configurations at the end of tic-tac-toe games, where "x" is assumed to have played first. The target concept is "win for x" (i.e., true when "x" has one of 8 possible ways to create a "three-in-a-row").		
SPECT Heart Data set [67]	The data set describes diagnosing of cardiac Single Proton Emission Computed Tomography (SPECT) images. Each of the patients is classified into two categories: normal and abnormal. The database of 267 SPECT image sets (patients) was processed to extract features that summarize the original SPECT images. It has 267 instances that are described by 23 binary attributes including the class variable.	267	22
MONK's Problems Data set [68]	The MONK's problems have been the basis of a first international comparison of learning algorithms. The result of this comparison is summarized in "The MONK's Problems. There are three MONK's problems. The domains for all MONK's problems are the same. The data set is composed of 7 attributes and a binary class variable.	432	7

639 As the newly proposed method accepts only binary input variables, the data sets which are
 640 used for the analysis must be preprocessed into the acceptable format. For example, the
 641 "balance scale" data set is composed of 4 attributes. Table 8 shows the attributes and their
 642 information of the balance scale data set.

643 **Table 8.** *Attribute information of the balance data set*

Attribute	Number of Categories	Categories
Class Name	3	L, B, R
Left-Weight	5	1,2,3,4,5
Left-Distance	5	1,2,3,4,5
Right-Weight	5	1,2,3,4,5
Right-Distance	5	1,2,3,4,5

644 Therefore, the data set was adjusted as shown in figure 16, prior to using it with the proposed
 645 method. Each category of a particular attribute is represented by a dummy variable. For
 646 example, Left-Weight attribute results in 5 attributes in the preprocessed data set and each
 647 attribute is represented using 5 binary variables as LW1, LW2, LW3, LW4 and LW5 where the
 648 presence of the attribute denotes 1 and 0 otherwise. As depicted in Figure 16, if Left-Weight
 649 has a value of 2 in an instance it results in 1 for the corresponding derived attribute that is LW2.
 650 Therefore, if there is an instance where Left-Weight=2, Left-Distance=1, Right-Weight=3 and
 651 Right-Distance=4, Class Name=B, it is represented as LW1=0,LW2=1, LW3=0, LW4=0, LW5=0,
 652 LD1=1, LD2=0, LD3=0, LD4=0, LD5=0, RW1=0, RW2=0, RW3=1, RW4=0, RW5=0, RD1=0, RD2=0,
 653 RD3=0, RD4=1, RD5=0, Class Name=B.



659 **Figure 16.** Schematic diagram for pre-processing of the balance dataset in such a way that it matches
 660 the format of inputs of the newly proposed method

661

662 The pre-processed data is then fed to the newly proposed algorithm and the nine other
 663 algorithms. Performances were compared based on AUC, root mean squared error (RMSE) and
 664 the processing time for model generation. 10 fold cross validation was used under each test for
 665 fair testing procedure. Then the receiver operating characteristic curves of the results were
 666 analysed. For simplicity the newly proposed modes operandi analysis algorithm was acronymed
 667 as BFPM (Binary feature profiling methodology).

668 As all the data sets which were used for the tests are composed of multi classes, weighted
 669 average AUC was used, where each target class is weighted according to its prevalence as given
 670 in Equation 8. Weighted average was used in order to prevent target classes with smaller
 671 instance counts from adversely affecting the results [69].

$$672 \quad AUC_{weighted} = \sum_{\forall c_i \in C} AUC(c_i) \times p(c_i) \quad (8)$$

673 Table 9 shows the weighted average AUC values obtained for each data set under each
 674 classification algorithm.

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680 **Table 9.** Weighted average AUC values obtained by the algorithms on classifying the data sets

	BFP M	Logistic Regressi on	J48	Radial Basis Funci on Netwo rk	Multi- Layer Perceptr on	Naive Bayes Classifi er	SMO	KStar	BFTre e	LMT
Dermatolo gy Data set	1	0.9990	0.975 0	0.9860	0.9980	0.9980	0.993 0	0.997 0	0.969 0	0.996 0
Balance scale Data set	0.794 5	0.9760	0.811 0	0.9680	0.9770	0.9710	0.883 0	0.951 0	0.813 0	0.981 0
Balloons Data Set	1	1	1	1	1	1	1	1	1	1
Car evaluation Data set	0.808 7	0.9900	0.976 0	0.9740	1	0.9760	0.955 0	0.997 0	0.994 0	0.999 0
Soybean Data set	1	1	0.986 0	1	1	0.9760	1	1	0.974 0	1
Lenses Data set	0.953 7	0.7470	0.840 0	0.9170	0.8390	0.8700	0.725 0	0.887 0	0.867 0	0.798 0
Nursery Data set	0.910 0	0.9880	0.995 0	0.9870	1	0.9820	0.964 0	0.998 0	0.999 0	0.999 0
Tic-tac-toe Data set	0.916 7	0.9960	0.897 0	0.7340	0.9940	0.7440	0.976 0	0.999 0	0.945 0	0.992 0
SPECT Heart Data set	0.785 7	0.8310	0.756 0	0.8400	0.7860	0.8490	0.707 0	0.785 0	0.723 0	0.841 0

MONK's Problems Data set	0.833 3	0.7050	0.994 0	0.8130	0.9980	0.7120	0.746 0	0.997 0	0.955 0	0.988 0
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681

682 Friedman's rank test is a nonparametric test analogous to a standard one-way repeated-
683 measures analysis of variance [70]. The Friedman's rank test results returned on the AUC test
684 data are shown in Table 10. According to the table, the highest mean rank is returned for MLP
685 while the lowest mean rank is returned for SMO, proving that MLP provides the best
686 performance while SMO provides the least performance for the 10 data sets tested. Therefore,
687 it indicates that the new model provides a better performance than BFTree, J48 and SMO
688 algorithms for the 10 data sets tested.

689 **Table 10.** Friedman's mean rank values returned on the data available in Table 9

Method	Mean Rank
MLP	7.70
LMT	7.10
KStar	6.95
LogisticRegression	5.95
RBFNetworks	5.05
NaiveBayesClassifier	5.05
BFPM	4.95
BFTree	4.50
J48	4.20
SMO	3.55

690

691 Table 11 shows the RMSE (Root Mean Squared Error) values obtained on the 10-fold cross
692 validation results obtained by each algorithm. Friedman's rank test was conducted on the RMSE
693 values obtained by the ten algorithms. The test returned the results shown in Table 12.
694 According to the mean rank values, the new model provides a better accuracy than
695 RBFNetworks, BFTree, KStar, NaiveBayesClassifier and SMO in the means of RMSE for the ten
696 data sets tested.

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702 **Table 11.** RMSE values returned by each algorithm on the classification of the ten data sets

	BFP M	Logistic Regressi on	J48	Radial Basis Func tion Netwo rk	Multi- Layer Percept r on	Naive Bayes Classifi er	SMO	KStar	BFTre e	LMT
Dermatolo gy Data set	0.088 1	0.0748	0.116 3	0.1091	0.0764	0.0876	0.311 0	0.093 2	0.132 7	0.088 9
Balance scale Data set	0.349 1	0.2092	0.369 9	0.2464	0.2055	0.2793	0.342 7	0.274 1	0.355 1	0.222 7
Balloons Data Set	0	0	0	0.0010	0.0264	0.2385	0	0.238 2	0	0.161 8
Car evaluation Data set	0.176 6	0.1520	0.171 8	0.1983	0.0440	0.2162	0.320 2	0.199 0	0.112 4	0.092 0
Soybean Data set	0	0	0.103 1	0	0.0329	0.1031	0.311 8	0.000 4	0.106 1	0.122 9
Lenses Data set	0.173 7	0.4714	0.324 9	0.3324	0.3812	0.3326	0.396 7	0.329 6	0.333 7	0.378 0
Nursery Data set	0.298 1	0.1456	0.095 1	0.1501	0.0174	0.1767	0.320 4	0.192 8	0.038 7	0.063 7
Tic-tac-toe Data set	0.417 2	0.1289	0.345 9	0.4391	0.1611	0.4319	0.129 2	0.278 8	0.241 4	0.205 5
SPECT Heart Data set	0.392 8	0.3529	0.386 6	0.3377	0.4069	0.4188	0.432 7	0.395 0	0.402 2	0.340 5

MONK's Problems Data set	0.388 1	0.4202	0.141 6	0.4067	0.0432	0.4179	0.503 6	0.302 6	0.278 7	0.195 5
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703

704 **Table 12.** Friedman's rank test values returned on the data available in Table 11

Method	Mean Rank
MLP	3.70
LogisticRegression	4.00
LMT	4.60
J48	5.15
BFPM	5.20
RBFNetworks	5.40
BFTree	5.60
KStar	6.00
NaiveBayesClassifier	7.25
SMO	8.10

705

706 The average processing times for each algorithm on the classification of the ten data sets are
 707 given in Table 13. Friedman's rank test on the data of Table 13 returned the results shown in
 708 Table 14 in which the mean rank values prove better efficiency for the new method than J48,
 709 LogisticRegression, SMO, RBFNetworks, BFTree, MLP and LMT.

710 **Table 13.** Average processing time for each algorithm on the classification of the ten data sets

	BFP M	Logistic Regressi on	J48	Radial Basis Funci on Netwo rk	Multi- Layer Perceptr on	Naive Bayes Classifi er	SMO	KStar	BFTr ee	LMT
Dermatol ogy Data set	0.00 27	0.3900	0.08 00	0.3800	2.7300	0.0500	0.140 0	0.28 00	0.110 0	2.9000
Balance scale Data set	0.00 30	0.0300	0.08 00	0.2200	0.4800	0	0.050 0	0	0.060 0	0.8400
Balloons Data Set	0	0	0	0	0.0200	0	0.020 0	0	0.020 0	0.0500

Car evaluation Data set	0.0048	0.4200	0.0300	0.2700	13.4000	0.0200	0.1900	0	0.4800	13.3400
Soybean Data set	0.0042	0.0500	0.0700	0.1800	0.4300	0	0.0500	0	0.2300	0.9200
Lenses Data set	0.0009	0	0	0.0300	0.0800	0	0.0300	0	0.0100	0.0300
Nursery Data set	0.0013	8.8600	0.2500	16.4300	127.8600	0.0300	23.0300	0	7.9900	240.4600
Tic-tac-toe Data set	0.0091	0.1900	0.0100	0.1300	18.1800	0.0100	0.6000	0	0.6100	54.6700
SPECT Heart Data set	0.0073	0.0600	0.0100	0.0700	3.8800	0	0.0400	0	0.2300	2.1500
MONK's Problem Data set	0.0058	0.0800	0.0100	0.0600	5.5100	0	0.1300	0	0.2100	1.7200

711

712 **Table 14.** Mean rank values returned by the Friedman's rank test on the time values available in Table 13

Method	Mean Rank
KStar	2.10
NaiveBayesClassifier	2.35
BFPM	2.75
J48	4.15
LogisticRegression	5.35
SMO	6.25
RBFNetworks	6.35
BFTree	6.90

MLP	9.30
LMT	9.50

713
714 Friedman's rank test results for the three measurements, AUC, RMSE and time elapsed
715 conclude that the newly proposed method provides acceptable results against the nine other
716 well established classification algorithms.

717 Conclusion

718 The studies of modus operandi help crime investigation by letting the police officers to solve
719 crimes by linking suspects to crimes. Though there are many descriptive studies available under
720 modus operandi analysis, a very little amount of work is available under computer science.
721 Many of these methods have been derived using the methods based on link analysis. But, the
722 accuracy of these methods is always compromised due to the cognitive biases of the criminals.

723 A novel Fuzzy based Binary Feature Profiling method (BFPM) to find associations between
724 crimes and criminals, using modus operandi is introduced. The newly proposed method
725 subjects not only the properties of the present, but also the properties of his/her previous
726 convictions. The concept of dynamic modus operandi which is available in the proposed
727 method considers all the modi operandi of his/her previous convictions to provide a fair
728 rectification to the errors which result due to the human cognition. Dynamic MO uses frequent
729 item set mining to result in a generalized binary feature vector. Complete MO profile also
730 encapsulates past modus operandi of a particular criminal by aggregating the modus operandi
731 of all of his/her previous convictions to one binary feature vector. This feature also guarantees
732 a usage of criminal's past crime record with more generalizability. Completeness probability
733 measures how much information is available in the new crime which is not available in the
734 complete MO profile. Therefore, this measurement provides the capability of measuring how
735 much extra amount of information is carried by the MO of the new crime. The deviation
736 probability provides a notion about how much the new MO deviates from the most frequent
737 crime flows which are available in the dynamic MO of a particular criminal. The vagueness and
738 the impreciseness prompted the fact that it is not possible to use crisp logic to generate the
739 similarity score. Therefore a fuzzy inference system was modeled to generate the similarity
740 score.

741 Due to the under-represented and imbalanced properties of the actual data set, the new
742 method has returned a lower performance when it was proposed to the data set without any
743 rectification on the data set. However, with the introduction of over sampling, the method
744 returned a very good performance, allowing one to arrive at the conclusion that the method
745 could provide acceptable results for a balanced data set. The method generated favorable
746 results in providing a good similarity measurement to suggest the connections between crimes
747 and criminals. Fuzzy controller of the new approach guarantees to resemble the human
748 reasoning process by confirming the usage of human operator knowledge to deal with
749 nonlinearity of the actual situation. The newly proposed method was then adapted into a

750 classification algorithm for the validation and comparison with other classification algorithms.
751 The comparison of the new method with the well-established classification algorithms
752 confirmed the generalizability of the new method.

753 The method only provides the capability to process the categorical data sets. If there are any
754 continuous variables in the data set, the values must be introduced with categories before
755 further processing. The method can be further extended to directly accept the continuous
756 attributes. As the center of gravity method is used for the defuzzification process, further
757 optimizations can be done by simplifying the defuzzification procedure. Adapting the fuzzy
758 inference engine to a Sugeno [71] type and converting the defuzzification method to a more
759 computationally efficient method such as the weighted average [72] method would provide a
760 less complex computation. This would result in even less processing time when the
761 sophistication of the data set rises.

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