# Low-diameter topic-based pub/sub overlay network construction with minimum–maximum node degree

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## ABSTRACT

In the construction of effective and scalable overlay networks, publish/subscribe (pub/sub) network designers prefer to keep the diameter and maximum node degree of the network low. However, existing algorithms are not capable of simultaneously decreasing the maximum node degree and the network diameter. To address this issue in an overlay network with various topics, we present herein a heuristic algorithm, called the constant-diameter minimum–maximum degree (CD-MAX), which decreases the maximum node degree and maintains the diameter of the overlay network at two as the highest. The proposed algorithm based on the greedy merge algorithm selects the node with the minimum number of neighbors. The output of the CD-MAX algorithm is enhanced by applying a refinement stage through the CD-MAX-Ref algorithm, which further improves the maximum node degrees. The numerical results of the algorithm simulation indicate that the CD-MAX and CD-MAX-Ref algorithms improve the maximum node-degree by up to 64% and run up to four times faster than similar algorithms.

## INTRODUCTION

In publish/subscribe (pub/sub) systems, publishers forward different types of messages to specific subscribers in a decoupled mode. Publishers broadcast information through logical channels, while subscribers receive them based on their topic interests. Pub/sub systems are divided into two different categories, namely topic- and content-based categories.

In topic-based pub/sub systems, publishers broadcast their messages based on the topic of the message. Each topic exclusively pertains to a specific logical channel. Subsequently, as stated in Yang and Garcia-Molina (2003), subscribers receive all messages associated with the topics to which they have subscribed to . Consequently, as stated in Milo et al. (2007), all messages about those topics will be sent to every user who has joined that particular group . Publishers take the responsibility of classifying the messages that subscribers receive.

In contrast, in content-based pub/sub systems, subscribers only receive messages whose attributes match with their interests; hence, Carvalho et al. (2005) states that these attributes characterize the logical channels . In this category, the matching algorithm between the publishers and the subscribers is based on the attribute values referred to as the content. In other words, the receivers decide which messages they would receive. In both cases, however, publish/subscribe systems show similarities with Information Centric Networks in terms of sharing/distributing information among users.

45      Pub/sub systems have a variety of use cases. As explained by ORiordan (2021), a simple use case of a
46 pub/sub system may be a chat application where a participant can subscribe to any of the chat rooms which
47 has a designated pub/sub topic. When a user sends a message to a chat room, the message is published
48 on that topic of the chat room. The subscribers of the topic/chat room receive the message. As stated
49 by GoogleCloud (2021), the pub/sub systems fit best, when there is a need for durable message storage
50 and real-time delivery for those messages with high availability for massive scale. These features are the
51 foundation of cloud computing where pub/sub systems are heavily used. Balancing load among network
52 clusters, implementation of asynchronous workflows, distributing event notifications, data streaming from
53 various processes or devices are examples to pub/sub systems. Apache Kafka (2021), Azure-SB (2021),
54 GoogleCloud (2021) and AWS-SNS (2021) are examples of popular pub/sub systems.

55      For each topic $t \in T$ in a typical fully decentralized topic-based pub/sub system based on the peer-
56 to-peer (P2P) connected overlay, a sub graph is derived using the nodes interested in t. Hence, the
57 nodes interested in topic t do not need to rely on other nodes to send or receive their messages. Every
58 node must maintain its connections (e.g., checking the accessibility of neighbors) and monitor data
59 streaming through the connections; therefore, overlay networks with a low maximum number of links
60 emitting from a node and low network diameters are desirable. If a proper correlation exists between node
61 subscriptions, the connectivity of many topics subscribed by those two nodes will be satisfied by adding
62 only one edge between the two nodes. Hence, the maximum number of links of a node and the total
63 number of overlay connections will considerably diminish. The significance and the impact of the topic
64 correlation of nodes in optimizing the overlay networks were highlighted in a relevant paper by Chockler
65 et al. (2007a). A constructed sub-graph acts as a selective message delivery among different publishers
66 and subscribers belonging in the same interest group. In a sub-graph, messages are routed to the node
67 destination (subscriber) with the same topic interest. Generally, a sub-group of pub/sub system can be
68 modelled as a trimerous $< \Pi, \beta, \Sigma >$ collections of functions. The sets involved are determined based on
69 their functionality: $\Pi = p_0, ..., p_i - 1$ is a set of i processes in the system that act as publishers providing
70 information to those needing it. $\Sigma = C_0, ..., C_j - 1$ is a set of j processes referred to as subscribers that
71 are known as consumers of information provided by publishers. The set of publishers and the set of
72 subscribers can have non-zero intersection, which means that the process can also operate as a publisher
73 and a subscriber at the same time (decoupling). Pub-sub systems are decoupled; therefore, a process may
74 change anything about a publisher if it does not change the way it produces messages. Hence, there is
75 no need to change something about the downstream subscribers. Similarly, the opposite process is true
76 as well. Systems with decoupling mechanisms do not need to consider any issues such as addressing
77 and synchronization. $\beta = B_0, ..., B_k - 1$ presents a logical centralized service that enables publishers and
78 subscribers to connect. In other words, any publisher or subscriber in a sub group may exclusively send or
79 receive specific information through $\beta$. Not only does $\beta$ provide communication between publishers and
80 subscribers, publishers and subscribers are kept in a decoupled mode during the communication process.

81      Baldoni et al. (2007) and Lau et al. (2009) states that, reducing the maximum number of links of an
82 overlay can play a vital role in various network fields, such as survivable and wireless network design.
83 Chockler et al. (2007a) presented the concept of topic connectivity, in which an individual overlay network
84 connects nodes with similar topics. They introduced the Greedy Merge (GM) algorithm to construct
85 an overlay with the least possible number of connections. A number of other solutions for the overlay
86 design were also recently introduced by Carvalho et al. (2005). However, all of the existing methods
87 (Chen et al. (2010a), Chen et al. (2015), Chen et al. (2010b)) suffer from either a high diameter or a
88 high maximum-node-degree. In addition, all constant-diameter algorithms connect the nodes in a star
89 topology manner, resulting in the best possible diameter while giving rise to nodes with high node degrees.
90 Chockler et al. (2007b) states that, these nodes are responsible for managing a large number of connections
91 to their neighbors, which results in a high traffic overhead.

92      In this study, we propose an algorithm for constructing a scalable topic-connected overlay (TCO),
93 which has a low maximum node degree and a constant diameter of 2, to solve the above-mentioned
94 problems. The proposed algorithms (i.e., CD-MAX and CD-MAX-Ref) outperform the existing algorithms
95 in the literature in terms of constructing optimum overlays with a minimum node degree and a small
96 diameter. In addition, the performance of the proposed CD-MAX algorithm in terms of the required
97 running time for constructing overlays provides a suitable conformance on scalability requirements.

98      The remainder of this paper is presented as follows: Section 2 provides a summary of the previous
99 studies on pub/sub networks, including the concept of the GM algorithm and other algorithms proposed

100 for building overlays with a minimum number of edges; Section 3 presents the details of the proposed
101 CD-MAX and CD-MAX-Ref algorithms; Section 4 provides the comparative results of the CD-MAX and
102 CD-MAX-Ref algorithms against the most recent methods from the literature; and Section 5 presents our
103 drawn conclusion and the possible future directions.

## OVERLAY DESIGN ALGORITHMS

105 An effective publication routing protocol is needed in designing an efficient pub/sub system, and it
106 can play a vital role on the system performance. Therefore, Onus and Richa (2011) stated that the
107 quality of a constructed overlay can be assessed based on the complexity of the routing scheme applied.
108 The complexity can be minimized if all the nodes interested in a topic $t \in T$ can be organized into a
109 dissemination tree. In this case, as Chockler et al. (2007a) and Chockler et al. (2007b) states, the topic
110 dissemination trees should have the following issues:

111 • Each tree for topic m includes only the nodes interested in topic m

112 • The diameter of the topic trees should be low

113 The GM algorithm by Chockler et al. (2007a) solves the two issues by the low-diameter pub-
114 lish/subscribe overlay algorithms. Suppose that G is an overlay network, and the essential prerequisite
115 to solving issue (1) is to ensure topic connectivity, where a sub-graph connects all the nodes interested
116 in topic m. Chockler et al. (2007a) introduced the topic connectivity concept and the minimum topic-
117 connected overlay problem. They provided an approximation solution, called the GM algorithm, for the
118 problem with the minimum number of links.
119 The GM algorithm begins with the overlay network $G = (N, \emptyset)$. There are $m \in M | n : Int(n,m) = 1|$
120 individual topic-connected components of G for each topic $m \in M$. $Int(x,m)$ indicates whether or not
121 node x is interested in topic m. The algorithm continues by connecting two nodes at each repetition
122 until the resulting overlay comprises maximally one topic-connected component for each $m \in M$. The
123 CD-ODA-II algorithm by Onus and Richa (2011) initializes with $G = (N, \emptyset)$ as the overlay network. A
124 node u which has the maximum connection density is chosen in each iteration. Afterwards, edges are
125 added between u and its neighbors. Thereafter, the interest topics of u are removed from the set of topics.
126 Unlikely, the 2D-ODA algorithm by Onus and Richa (2016) starts with $G = (V, E)$ as the overlay network
127 and a topic set T is selected in which that topic is in the interest of node u and the selected topic has the
128 maximum node density. The node with maximum topic density for a topic is chosen at each iteration, then
129 together with the star structure, node is added to the network. After all, the topic is removed from the set.

### Publish/Subscribe Challenges

131 The following three main challenges must be handled in building an effective pub/sub system: (1)
132 expression of the interest to the topics by the subscribers, (2) organization of the notification service
133 to deliver interests to topics, and (3) delivery of messages to the subscribers by the publishers. These
134 states are strongly coupled, and their contribution can affect the system performance. For instance, as
135 described in Triantafillou and Aekaterinidis (2004), a rudimentary subscription algorithm may improve
136 the functionality of multicasting, but it facilitates a poor expression ability for subscribers to announce the
137 topics they are interested in. Eugster et al. (2003) states that the architecture of the pub/sub systems can
138 generally be divided into client–server groups and P2P. In client–server architectures, the servers are the
139 providers of information (publishers), while the clients are the subscribers. Intermediate nodes, called
140 brokers, have been introduced to decouple clients and servers and achieve a better performance. Therefore,
141 these architectures are referred to as broker-based architectures. Meanwhile, in P2P architectures, each
142 node performs as either subscribers or publishers or both. In a P2P paradigm, all nodes can operate under
143 various roles (e.g., subscriber, root, or internal node of a multicast tree) and play under a combination of
144 these roles. The P2P architectures are also called decentralized architectures. A typical pub/sub system
145 must hold two principle characteristics of P2P networks: i) scalability and ii) fault tolerance/reliability.
146 The following sub-sections briefly introduce both architectures.

### Preliminaries

148 An overlay is defined as an undirected graph G(V,E), where V is the set of nodes, and E is the set of edges.
149 The number of nodes interested in at least one topic, which node u is interested in, is called the node u
150 interest group, which is computed as $n_u = |\{v \in V | \exists t \in T, Int(v,t) = Int(u,t) = 1\}|$

151    T is the set of topics, which a subscriber can be interested in. $Int(u,t)=1$ indicates that subscriber
152 u is interested in topic t, while $Int(u,t)=0$ indicates otherwise. The degree of node u denoted by $d_u$ is
153 defined as the total number of edges incident to it and given as $d_u = |\{v \in V \,|\, (u,v) \in E\}|$

154    The degree of topic t is defined as the number of subscribers interested in that topic as $d_t = $
155 $|\{v \in V \,|\, Int(v,t)\}|$ The density of node u is given by $density_u = \frac{\sum_{t \in T} |\{v \in V | Int(v,t)=Int(u,t)=1\}|}{|\{v \in V | \exists t \in T, Int(v,t)=Int(u,t)=1\}|}$

156    Additionally, the diameter of a graph is the length of the shortest path between the two nodes which
157 are farthest from each other.

## PROPOSED ALGORITHM

159 Most of the approaches used for designing scalable overlay networks failed to achieve an appropriate
160 trade-off between the maximum node degree and the diameter of the overlay network. On the one hand,
161 solutions for decreasing the number of connections exist. Chockler et al. presented the problem of
162 constructing overlay networks with the least possible links. They considered this issue as an NP-Complete
163 problem and proposed the GM algorithm to solve it Chockler et al. (2007a). The GM algorithm begins
164 with the overlay network $G = (N, \emptyset)$. There are $\sum m \in M | n : Int(n,m) = 1|$ individual topic-connected
165 components of G for each topic $m \in M$. $Int(x,m)$ indicates whether or not node x is interested in topic m.
166 The algorithm continues by connecting two nodes at each repetition until the resulting overlay comprises
167 maximally one topic-connected component for each $m \in M$. The two nodes connected during each
168 repetition are those with the greatest number of topics in common.

169    On the other hand, a number of solutions presented in the previous section provide overlays with a
170 low diameter, but the maximum node degree of these overlays is considerably high Carvalho et al. (2005)
171 Onus and Richa (2011). We propose herein an algorithm, called Low-diameter Topic-based Pub/Sub
172 Overlay Network with Minimum–Maximum Node Degree (CD-MAX), to simultaneously address both
173 issues. Our proposed method improves the CD-MAX algorithm based on building an overlay using a star
174 topology to avoid a high node degree. The proposed CD-MAX algorithm (Algorithm 1) creates an initial
175 overlay network by selecting the nodes with a minimum interest group at each iteration.

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**Algorithm 1** Constant Diameter Maximum Algorithm (CD-MAX)

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1: $T \Leftarrow Set\ of\ topics$
2: $V \Leftarrow Set\ of\ nodes$
3: $Max_n \Leftarrow Number\ of\ Nodes - 1$            $\triangleright Max_n : maximum\ number\ of\ neighbors\ for\ a\ node$
4: $Max_n\_list \Leftarrow Empty\ List$            $\triangleright Max_n\_list : collection\ of\ nodes\ with\ the\ same\ Max_n\ value$
5: **while** T is NOT empty **do**
6:     **for each** $u \in V$ **do**
7:         $Calculate\ interest\ group\ (n_u)$
8:         **if** $n_u < Max_n$ **then**
9:             $Max_n \Leftarrow n_u$
10:             $Clean\ Max_n\_list$
11:             $Append\ node\ u\ to\ Max_n\_list$
12:         **if** $n_u == Max_n$ **then**
13:             $Max_n \Leftarrow n_u$
14:             $Append\ node\ u\ to\ Max_n\_list$
15:     **if** $Length\ of Max_n\_list > 1$ **then**
16:         $u \Leftarrow w \in Max_n\_list\ AND\ w\ has\ the\ largest\ density_u.$
17:         **if** $Length\ of\ u > 1$ **then**
18:             $u \Leftarrow newest\ node\ in\ u$
19:     **else**
20:         $u \Leftarrow w \in Max_n\_list$
21:     **for each** $t \in T$ **do**
22:         **if** $\exists v, Int(u,t) = Int(v,t)$ **then**
23:             $Connect\ nodes\ u\ and\ v\ through\ an\ edge$
24:     $Remove\ all\ topics\ t\ from\ T\ where\ Int(u,t) = 1$

---

176  This initial overlay network is refined to further improve the maximum node degree by the CD-MAX-
177  Ref algorithm (Algorithm 2). A topic-connected overlay is defined as; for each topic $t$, all possible node n
178  interested in that topic are connected. If there is a node interested in topic $t$ which is not subscribed by
179  other nodes throughout the network, the overlay for that topic is connected. In constructing a network,
180  the proposed CD-MAX algorithm selects the node with the smallest interest group ($n_u$) and connects
181  it to every node in its interest group. If more than two nodes with an equal smallest interest group are
182  found, the algorithm will select the node with the highest connection density. Each selected node takes
183  the responsibility of topics to which it has subscribed to. These topics are removed from the list of topics,
184  and the algorithm iterates until the list of topics becomes empty. Algorithm 1 presents this procedure. In
185  the last step of Algorithm 1, CD-MAX algorithm finds the node with the smallest $n_u$ that is interested in
186  the subsets of topics $t$. The selected node u will be connected to its neighbors and the topics subscribed
187  by node u will be removed from the original topic set. In order to proceed, a proof that the CD-MAX
188  algorithm terminates in $O(|V|^2 * |T|^2)$ is provided below.

189  **Lemma 1:** The running time of CD-MAX algorithm is $O(|V|^2 * |T|^2)$.

190  **Proof:** In Algorithm 1, between lines 5 and 24, the outer loop (the `while` loop on line 5) iterates T
191  times. In addition, the inner loop (the `for` loop on line 6) iterates V times. In the worst case, calculation
192  of interest group takes V times. Thus, finding a node with minimum node degree takes $O(|V|^2 * |T|)$.
193  When node u with minimum node degree has been selected, all the topics to which node u is subscribed
194  is removed. At each iteration, one topic is removed from the original topic set. Thus algorithm takes
195  $O(|V|^2 * |T|) * |T| = O(|V|^2 * |T|^2)$ time steps to terminate.

196  **Lemma 2:** CD-MAX algorithm guarantees the diameter of at most 2 for each topic.

197  **Proof:** Since CD-MAX diameter provides a star topology for each topic, each node requires at most 2
198  edges to receive and send any topic it is interested in.

199  **Lemma 3:** The space complexity of CD-MAX algorithm is $O(|T| * |V|)$.

200  **Proof:** CD-MAX uses a star topology for each topic, each node requires at most 2 edges to receive
201  and send any topic it is interested in. Hence, it gives us a space use of $2 * |T * V|$ which gives a space
202  complexity of $O(|T| * |V|)$.
203  As a justification of the algorithm; at each iteration, at least one node is connected to its neighbors (one
204  edge is required), meaning that finding a node with minimum node degree is achieved.

205  After the CD-MAX implementation, a refinement process, called CD-MAX-Ref, is applied to the
206  resulting overlay network to further improve the maximum node degree. CD-MAX-Ref checks if any
207  node with a lower node degree $d_u$ exists for each topic. The topic overlay center becomes the newly found
208  alternative node if CD-MAX-Ref finds an alternative node. The edges connecting the current center node
209  of the topic with the subscribers to that topic will be removed, and the alternative node is connected to all
210  the subscribers of that topic. Consequently, the new node becomes the center of the topic(s) subscribed by
211  the current node. The center of the overlay topics subscribed by the current node may be at more than one
212  node. CD-MAX-Ref can be used independently with any overlay network from CD-MAX. Algorithm 2
213  formally describes the CD-MAX-Ref algorithm.

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**Algorithm 2** Constant Diameter Maximum Refinement Algorithm (CD-MAX-Ref)

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1: $T \Leftarrow Set\ of\ topics,\ N \Leftarrow Set\ of\ nodes$
2: **while** $Max_n\ improves$ **do**
3:  **for each** $topic\ t \in T$ **do**
4:   $d_t \Leftarrow degree\ of\ topic\ t,\ u \Leftarrow center\ node\ for\ topic\ t$
5:   **for each** $q \in N$ **do**
6:    **if** $d_q + d_t < d_u - d_t\ and\ Int(q,t) = 1$ **then**
7:     $Remove\ all\ edges\ of\ topic\ t\ connected\ to\ u$
8:     $Connect\ q\ to\ every\ node\ n\ if\ Int(n,t) = 1$
9:     $Update\ node\ degrees$
10:    $Update\ Max_n$

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214  The CD-MAX-Ref algorithm (Algorithm 2) takes over when CD-MAX terminates. It examines all
215  topics and finds the center node for each topic. The center node is a node interested in topic $t \in T$ which is
216  selected to connect all nodes interested in topic $t \in T$. It then searches the overlay for an alternative node,

which will have a lower node degree if it becomes the center node for that topic. If CD-MAX-Ref manages to locate such a node, the edges corresponding to that topic are removed from the center node of the topic, and the newly discovered nodes are connected to the subscribers of that topic. CD-MAX-Ref can decrease the maximum node degree obtained from the CD-MAX algorithm. To prove the improvement made by the CD-MAX-Ref algorithm, let us assume that u has a node degree of $d_u$, and is the center for the k topics. If a node, such as v, is found satisfying $d_v + d_t < d_u - d_t$, the algorithm eliminates all the edges connecting u to the other nodes, except v, and adds edges from v to those nodes. If the m edges are eliminated in this stage, the same number of new edges should be added to connect the neighbors of u to v. However, v already has some edges reflected in $d_v$; hence, the probability of having an overlap between the added and existing edges in v exists. In some cases, the number of added edges will be less than the number of removed edges. In the worst case, CD-MAX-Ref will add the same number of edges to v as was deleted from u. Even in this case, the node degree of v will be smaller (according to the assumption), thereby resulting in a lower maximum node degree. Considering that the node with the highest node degree is processed in the same manner, the algorithm will reduce its node degree and the maximum node degree of the network. Additionally, the CD-MAX-Ref algorithm is not affected on how many topics are associated with a removed edge. In any case, CD-MAX-Ref finds an alternative node for each topic that is related to the removed edge. Before moving forward we prove that the CD-MAX-Ref algorithm terminates in $O(|T|^2 * |N|^3)$.

**Lemma 4:** The running time of CD-MAX-Ref algorithm is $O(|T|^2 * |N|^3)$.

**Proof:** CD-MAX-Ref takes over where CD-MAX terminates which runs in $O(|T|^2 * |N|^2)$. It then examines all topics and finds the center nodes for each topic in $O(|N|)$ in worst case. Thus algorithm takes $O(|T|^2 * |N|^3)$ time steps to be terminated.

**Lemma 5:** The space complexity of CD-MAX-Ref algorithm is $O(|T| * |N|)$.

**Proof:** Since CD-MAX-Ref builds over CD-MAX, it uses similar data structure with more iteration on the same space which does not extend the use of space. Hence, it gives us a space use of in the order of $|T * N|$ giving a space complexity of $O(|T| * |N|)$.

The complexity of a constructed pub/sub overlay network can be determined through the cost of broadcast consideration. The total time required to send a chunk of information from a publisher to specific group of nodes that subscribe certain type of information is a crucial factor that truly depends on the resulted diameter between publishers and subscribers. Meanwhile, due to bandwidth and memory constraints, it is also necessary to keep the maximum node degree of an overlay low. For example, a star topology would be the best option in terms of overlay diameter. But, on the other hand, in terms of memory, bandwidth and energy consumption, it will become a big problem in this scenario when number of nodes grows in the network. Hence, providing a trade-off between diameter and maximum node degree of a pub/sub overlay is an ideal way in pub/ sub overlay network design.

## Evaluation of CD-MAX with Examples:

The examples 1 and 2 are presented to clarify the steps used by the proposed CD-MAX and CD-MAX-Ref algorithms. Following examples 3-5 additionally compare CD-MAX and CD-MAX-Ref algorithms over other constant diameter (CD) overlay design algorithms (CD-ODA, CD-ODA I, CD-ODA II, and 2D-ODA).

### Example 1:

In this example, we assume that $(n-1)/4$ nodes are interested in each topic enumerated as $\{10, 20\}, \{20, 30\}$, $\{30, 40\}$, and $\{40, 50\}$ (Figure 1). In addition, node u has subscribed to topics $\{10, 20, 30, 40, 50, 60\}$.

According to the CD-ODA algorithms, node u is the center of all topics, and it will be connected to all the other nodes. Figure 1 shows that the CD-MAX algorithm constructs an overlay with a maximum node degree of $(2 * (n-1)/4)$. In this example, the CD-MAX algorithm improved the maximum degree of the overlay by 50%. In this case, the CD-MAX-Ref did not improve the node degree.

### Example 2

In this example, it is assumed to have 4 different collections interested in 4 different topics as listed in Table 1. In order to construct the overlay network all CD algorithms (including CD-MAX) need 6n-2 edges. The overlay network could be generated by the CD-MAX algorithm at a maximum node degree at a rate of 3n-1, whereas previous constant diameter algorithms have the maximum node degree at a rate of 4n-1.
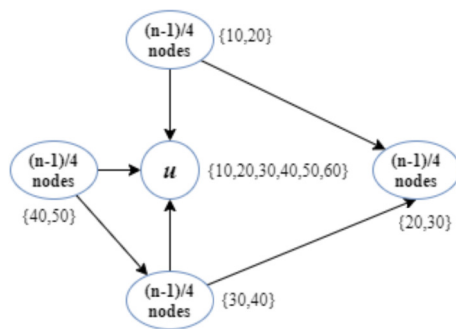
**Figure 1.** Implementation of the CD-MAX Algorithm over Example 1
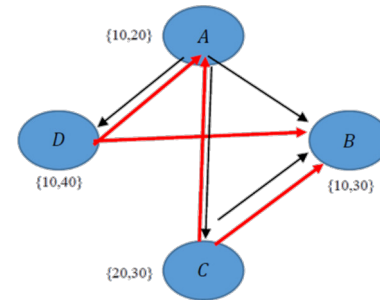
**Figure 2.** Implementation of CD-MAX Algorithm over Example 2

The red and black arrows demonstrate the implementation of CD-MAX and other existing algorithms respectively (Figure 2). This case is not valid for CD-MAX-Ref, which does not have a lower maximum node degree for this example.

**Table 1.** Example 2 - Topic Assignments

| Nodes | Topics |
| --- | --- |
| Collection A | 10,20 |
| Collection B | 10,30 |
| Collection C | 20,30 |
| Collection D | 10,40 |

**Table 2.** Example 3 - Topic Assignments

| Nodes | Topics | Degree |
| --- | --- | --- |
| 0 | $\{1,2,3,5,7,8\}$ | 7 |
| 1 | $\{0,1,3,5,7,8,9\}$ | 6 |
| 2 | $\{1,4,5\}$ | 6 |
| 3 | $\{2,4,6\}$ | 6 |
| 4 | $\{0,2,3,4,9\}$ | 6 |
| 5 | $\{2,3,6\}$ | 6 |
| 6 | $\{2,5\}$ | 6 |
| 7 | $\{1,6\}$ | 5 |

**Table 3.** Example 4 - Topic Assignments

| Nodes | Topics |
| --- | --- |
| $N_i$ | $\{x_i\}$ |
| $N_{(i,j)}$ | $\{x_{(i,j)}\}$ |
| $N_{(1,2,3,\ldots,n)}$ | $\{x_{(1,2,3,\ldots,n)}\}$ |

**Evaluation of CD-MAX-Ref with Examples:**

*Example 3*

Table 2 presents eight different nodes located throughout the network. The nodes with their respective topics deployed over the network and their degrees are shown. Node number 7 has the lowest node degree; hence, it is selected as the first node to be connected to its interest group. The node becomes the center of topics 1,6. Therefore, topics 1 and 6 are removed from the original topic list, and the topics list after step 1 becomes $\{0,2,3,4,5,7,8,9\}$.

As previously explained, when more than two nodes have an equal lowest node degree, the node with the higher node density will be selected as a topic center. In the second step, nodes 1 to 6 have the lowest node degrees. However, node 1 has the highest node density (Figure 3); hence, this node is selected to be connected to its interest group (Figure 4). As a result, node 1 becomes the center of topics $\{0,1,3,5,7,8,9\}$. The remaining topic list will be $\{2,4\}$.

In the following steps, as shown in Figs. 5 and 6, nodes 2 and 6 are selected to become the center of topics 4 and 2, respectively. Consequently, topics 4 and 2 are removed from the topic list.

Figure 6 demonstrates the result of the CD-MAX implementation over Example 3. For this overlay, CD-MAX provides an overlay with a maximum node degree of 6. The CD-MAX-Ref algorithm is applied to the resulting overlay to further reduce the node degree. The node with the highest degree (i.e., node 1) is checked, and CD-MAX-Ref finds other nodes, which are interested in topics $\{0,3,5,7,8,9\}$, and have lower node degrees. Therefore, all the edges connecting node 1 to its interest group are removed. Nodes 0 and 4 are then selected to be the center of topic sets $\{5,7,8\}$ and $\{0,3,9\}$, respectively (Figure 7). As visualized in Figure 3, since node 7 has the lowest node degree, it is chosen as the first node to be connected to its interest group and it becomes the center of topics 1,6. As visualized in Figure 4, Nodes 1 to 6 have the lowest node degrees. But, node 1 is the node which has the highest node density; hence, this
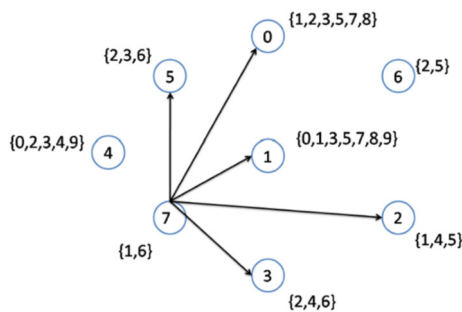
**Figure 3.** Implementation of the CD-MAX Algorithm over Example 3 Part 1
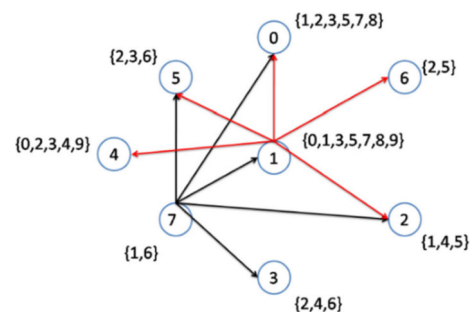
**Figure 4.** Implementation of the CD-MAX Algorithm over Example 3 Part 2

node is selected to be connected to its interest group and it becomes the center of topics 0,1,3,5,7,8,9.

The next node to be considered is node number 7, which maintains the maximum node degree of the overlay at 5. As visualized in Figure 5, Node 2 is the best option as it has the lowest node degree and it
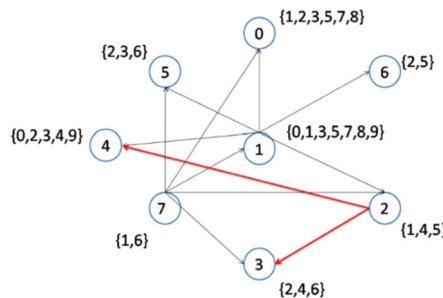
**Figure 5.** Implementation of the CD-MAX Algorithm over Example 3 Part 3
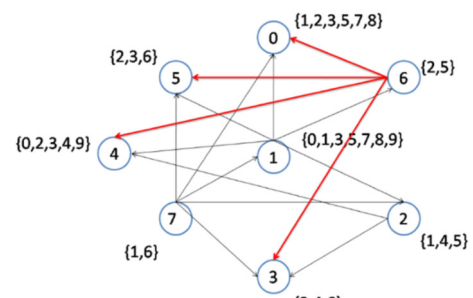
**Figure 6.** Implementation of the CD-MAX Algorithm over Example 3 Part 4

becomes the center of topic 4. After that, in the next step, Node 6 is the best option as it has the lowest node degree and it becomes the center of topic 2 (see Figure 6).

Therefore, CD-MAX-Ref should find alternative nodes for topics 1 and 6. All the edges joined to node number 7 are removed as node numbers 1 and 5 become the center of topics 1 and 6, respectively (Figure 8). Consequently, the maximum node degree of the overlay decreases by 2.
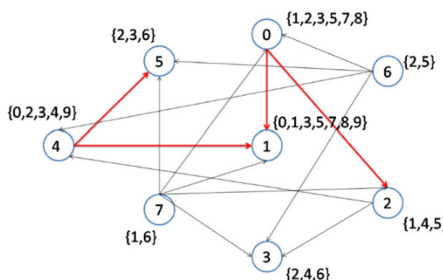
**Figure 7.** Implementation of CD-MAX-Ref Algorithm over Example 3 Part 5
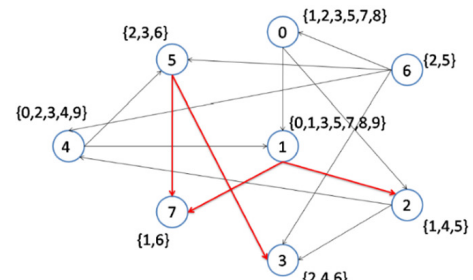
**Figure 8.** Implementation of CD-MAX-Ref Algorithm over Example 3 Part 6

***Example 4***

In this example, there are $(3n/2) + 1$ nodes placed over the network. Each node subscribes to specific topics which are listed in Table 3. In order to construct the overlay network, all three existing CD algorithms (CD-ODA, CD-ODA I, CD-ODA II, and 2D-ODA), the node interested in topics $\{x_1, x_2, x_3, \ldots, x_n\}$ acts as a center of the overlay which would be connected to all other nodes participating in the network.

309       However, the CD-MAX functions in a different manner. For example, the node which subscribes to $x_1$
310   is connected to the nodes which are interested in $x_{1,2}$ and $\{x_1, x_2, x_3, \ldots, x_n\}$. This algorithm provides the
311   overlay with a maximum node degree n related to the node interested in topics $\{x_1, x_2, x_3, \ldots, x_n\}$. After
312   this step, CD-MAX-Ref re-constructs the overlay with a lower maximum node degree provided by the
313   normal CD-MAX. To illustrate, the node that is interested in $\{x_1, x_2, x_3, \ldots, x_n\}$ has the maximum node
314   degree. All edges which are connected to this node are removed (Grey Arrows) and CD-MAX-Ref then
    finds other nodes with lower node degree (Figure 9). These new nodes $\left(N_{(i,j)}\right)$ take over the responsibility
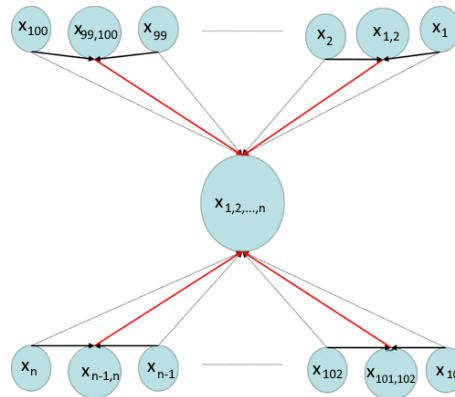
**Figure 9.** Implementation of CD-MAX Algorithm over Example 4

315
316   of the topics, which are already subscribed by node u. Note that nodes $N_{(i,j)}$ have priority over nodes $N_i$
317   and $N_j$ through the CD-MAX-Ref implementation. The red arrows imply the new edges that are added by
318   the CD-MAX-Ref algorithm. In this example, CD-MAX-Ref plays a key role in decreasing the maximum
319   node degree down to $(n)/2$.
320      As a summary of all examples, in Table 4, maximum node degrees for overlay networks are listed for
    the existing CD algorithms in comparison with the CD-MAX algorithms.

|  | Examples |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| **Algorithm Name** | **I** | **II** | **III** | **IV** |
| CD-ODA | n-1 | 4n-1 | 8n-1 | 3n/2 |
| CD-ODA I | n-1 | 4n-1 | 8n-1 | 3n/2 |
| CD-ODA II | n-1 | 4n-1 | 7n-1 | 3n/2 |
| 2D-ODA | n-1 | 4n-1 | 7n-1 | 3n/2 |
| CD-MAX & CD-MAX-Ref | (2*(n-1)/4)+1 | 3n-1 | 5n-1 | n/2 |

**Table 4.** Maximum Node Degree of Overlay Networks Built by CD Algorithms

321

## EXPERIMENTAL RESULTS

322
323   This section presents a comparative evaluation of our proposed method using different overlay networks.
324   The algorithm comparisons were conducted based on the average and maximum node degree over the
325   resulting overlays. Both the number of topics and the number of nodes varied throughout the simulation.
326   As noted earlier, each node has a specific subscription size and subscribes to 10 topics because of the
327   memory restriction in the experiments. Note that only in the last simulation, this number was increased to
328   values between 15 and 35. Each node $n \in N$ can be interested in each topic $t \in T$ with a probability of $p_i$,
329   in which $\sum_i p_i = 1$. The topic distribution probabilities $p_i$ have a Zipf distribution with $\alpha = 0.5$ as used in
330   similar studies by Carvalho et al. (2005) and Liu et al. (2005). During the experiments, we considered
331   the impact of varying the number of nodes, topics, and topic popularity distribution on the average and
332   maximum node degrees.
     Throughout the experiments, we presented the results of the CD-ODA II algorithm as the representative
    of the CD-ODA algorithm series because of its better performance. The Tables 5-12 have confidence
    values added to the tables. Since the values shown are the average of the multiple executions with smooth

differences, the confidence intervals are calculated. The calculated confidence intervals are also depicted in the aforementioned tables. The confidence interval of the results are calculated as:

$$confidence(\alpha, std\_dev, n) = norms\_inv(1 - \alpha/2).std\_dev/\sqrt{n}$$

333 where:

334      $norms\_inv$ =Inverse of the standard normal cumulative distribution with the given probability

335      $std\_dev$ = Standard deviation of the given values

336      $n$ = number of values (in this study, number of runs, i.e. 10)

337      $\alpha$ = significance level, which is a probability between 0 and 1

338 **Average and Maximum Node Degree Values for Varying Number Of Nodes**

339 In this experiment, the number of nodes varied between 200 to 5000, while the number of topics was kept
340 constant (100). The subscription size was fixed at 10. Each node randomly subscribed to different topics.
341 The average node degrees (see Table 5 and Figure 10) provided by each algorithm slightly decreased
342 as the number of nodes increased. This result indicates that because of the increased probability of
343 having overlaid edges in sub-graphs, a smaller number of edges connected a larger number of nodes,
344 and the average node degree of the overlay will decrease. However, the maximum node degree of the
345 constant-diameter algorithms increased with the increasing number of nodes (see Table 6 and Figure 11).
346 This observation is also valid in the case of the 2D-ODA algorithm.

| | GM | | CD-ODA-II | | 2D-ODA | | CD-MAX | | CD-MAX-Ref | |
|---|---|---|---|---|---|---|---|---|---|---|
| Nodes | Degree | Conf. | Degree | Conf. | Degree | Conf. | Degree | Conf. | Degree | Conf. |
| 200 | 6,62 | ±0,05 | 13,56 | ±0,07 | 14,41 | ±0,11 | 13,92 | ±0,11 | 18,26 | ±0,09 |
| 300 | 6,25 | ±0,03 | 13,79 | ±0,07 | 14,73 | ±0,11 | 14,07 | ±0,08 | 18,97 | ±0,07 |
| 400 | 5,96 | ±0,03 | 13,83 | ±0,05 | 14,50 | ±0,14 | 14,17 | ±0,09 | 19,26 | ±0,02 |
| 500 | 5,80 | ±0,02 | 13,91 | ±0,07 | 14,67 | ±0,09 | 14,19 | ±0,07 | 19,43 | ±0,01 |
| 750 | 5,52 | ±0,01 | 13,98 | ±0,03 | 14,60 | ±0,11 | 14,23 | ±0,07 | 19,62 | ±0,01 |
| 1000 | 5,38 | ±0,01 | 13,97 | ±0,05 | 14,50 | ±0,11 | 14,27 | ±0,05 | 19,72 | ±0,01 |
| 2500 | 4,94 | ±0,01 | 14,02 | ±0,02 | 14,37 | ±0,08 | 14,31 | ±0,04 | 19,89 | ±0,00 |
| 5000 | 4,93 | ±0,02 | 14,01 | ±0,03 | 14,40 | ±0,10 | 14,37 | ±0,06 | 19,89 | ±0,02 |

**Table 5.** Average Node Degree for Different Number of Nodes (Mean Degree with confidence $\alpha = 0.05$)

| | GM | | CD-ODA-II | | 2D-ODA | | CD-MAX | | CD-MAX-Ref | |
|---|---|---|---|---|---|---|---|---|---|---|
| Nodes | Degree | Conf. | Degree | Conf. | Degree | Conf. | Degree | Conf. | Degree | Conf. |
| 200 | 19,80 | ±1,30 | 137,80 | ±2,77 | 131,40 | ±1,94 | 113,70 | ±1,98 | 38,10 | ±0,90 |
| 300 | 21,50 | ±0,00 | 203,80 | ±3,68 | 199,10 | ±3,26 | 176,80 | ±1,62 | 51,30 | ±1,34 |
| 400 | 22,80 | ±0,00 | 272,60 | ±2,45 | 261,40 | ±4,93 | 237,50 | ±1,99 | 62,60 | ±1,44 |
| 500 | 22,20 | ±0,00 | 337,00 | ±1,98 | 334,70 | ±6,27 | 303,40 | ±2,13 | 75,20 | ±1,84 |
| 750 | 22,60 | ±0,00 | 510,90 | ±3,72 | 498,30 | ±5,00 | 458,70 | ±3,71 | 102,90 | ±2,87 |
| 1000 | 23,90 | ±0,00 | 678,60 | ±4,79 | 677,30 | ±6,91 | 619,10 | ±3,41 | 132,60 | ±3,11 |
| 2500 | 32,60 | ±0,00 | 1686,70 | ±7,61 | 1683,60 | ±7,10 | 1584,20 | ±6,78 | 296,67 | ±2,36 |
| 5000 | 36,00 | ±0,00 | 1692,00 | ±7,12 | 1685,00 | ±9,08 | 1580,00 | ±7,65 | 296,00 | ±3,08 |

**Table 6.** Max. Node Degree for Different Number of Nodes (Mean Degree with confidence $\alpha = 0.05$)

347      Unlike the other constant-diameter algorithms, in which a small number of nodes covered most of the
348 topics, to decrease the node degree of the overlay, the CD-MAX and CD-MAX-Ref algorithms chose
349 nodes with a lower correlation to become the center of the topics. More edges would be needed to connect
350 the nodes, thereby raising the average node degree.
351      Compared with the GM algorithm, CD-MAX and CD-MAX-Ref for 1000 nodes requires 2,65 and
352 3,66 times more edges in average, respectively (see Table 5). Considering the maximum node degree
353 with a growing number of nodes, more nodes should be connected to the overlay center nodes. Thus, the
354 maximum node degree provided by every constant-diameter algorithm sharply increased (see Table 6).

The maximum node degree of the GM algorithm will decrease relatively (does not increase while others are increasing fast enough) because more nodes with a higher correlation distributed the node degree of the overlay. Although the GM algorithm had low maximum and average node degree, it had a higher diameter Chockler et al. (2007a) Onus and Richa (2011) Carvalho et al. (2005). While the CD-MAX required less number of edges to build the overlay network compared to other ODA algorithms, CD-MAX-Ref outperformed all competitors by at least four times. In addition, it became the only algorithm with the ability of approaching the performance of the GM algorithm.
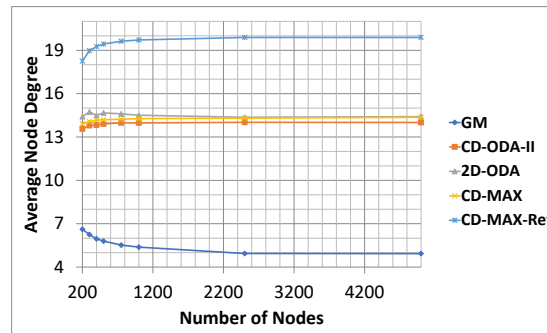
**Figure 10.** Average Node Degree for Different Number of Nodes
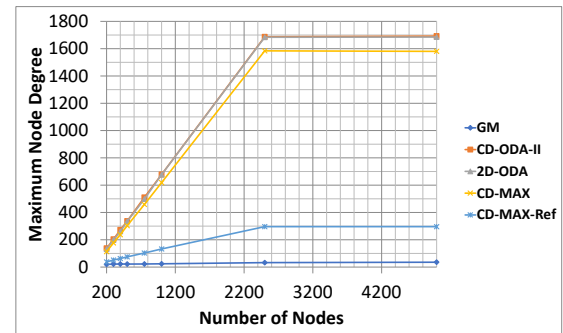
**Figure 11.** Maximum Node Degree for Different Number of Nodes

**Average and Maximum Node Degree Values for Varying Number of Topics**

In this experiment, the number of nodes and the subscription size were fixed at 100 and 10, respectively. Meanwhile, the number of topics varied from 200 to 400. The overlays face two different conditions when the number of topics was increased. First, the correlation between the nodes decreased; thus, more edges were used to connect the nodes. Second, the number of nodes without any connection to a neighbor increased. The average node degree will increase if the first condition dominates. The entry in Table 7, which corresponds to CD-MAX-Ref with a number of topics of 250, is an indication of the first condition. In contrast, the average node degree will decrease if the second condition dominates. The second condition has a greater effect than the first condition (see Table 7 and Figure 12). Hence, the overall average node degree of the overlay for every algorithm reduced when the quantity of topics increased.

The second condition affected the maximum degree of all the algorithms. Table 8 and Figure 13 shows that the maximum node degree of all the algorithms decreased as the set of topics indicated more diversity.

| | GM | | CD-ODA-II | | 2D-ODA | | CD-MAX | | CD-MAX-Ref | |
|---|---|---|---|---|---|---|---|---|---|---|
| Topics | Degree | Conf. | Degree | Conf. | Degree | Conf. | Degree | Conf. | Degree | Conf. |
| 200 | 9,99 | ±0,11 | 13,16 | ±0,07 | 12,80 | ±0,07 | 13,53 | ±0,12 | 14,52 | ±0,11 |
| 250 | 10,38 | ±0,11 | 12,67 | ±0,05 | 12,21 | ±0,04 | 13,06 | ±0,12 | 13,74 | ±0,12 |
| 300 | 10,67 | ±0,12 | 12,27 | ±0,11 | 11,78 | ±0,09 | 12,63 | ±0,08 | 12,99 | ±0,12 |
| 350 | 10,65 | ±0,06 | 11,77 | ±0,06 | 11,36 | ±0,05 | 12,14 | ±0,08 | 12,34 | ±0,07 |
| 400 | 10,37 | ±0,04 | 11,25 | ±0,04 | 10,88 | ±0,06 | 11,59 | ±0,09 | 11,81 | ±0,09 |

**Table 7.** Avg. Node Degree for Different Number of Topics (Mean Degree with confidence $\alpha = 0.05$)

**Average and Maximum Node Degree Values for Varying Subscription Size**

For the final experiment, the numbers of nodes and topics were kept at 200 and 100, respectively. However, the subscription size was varied between 15 and 35. As noted earlier, each node randomly subscribed to different topics using the interest function. Tables 9-10 and Figures 14-15 illustrate the effects of changing the subscription size on the selected algorithms. As the subscription size grows, the nodes can get connected with each other with a higher correlation rate. Subsequently, the rate of the average node degree decreases. Meanwhile, the contribution between the nodes rises when the subscription size grows. Therefore, the GM algorithm can find many node pairs, which dramatically reduces the total number of

| | GM | | CD-ODA-II | | 2D-ODA | | CD-MAX | | CD-MAX-Ref | |
|---|---|---|---|---|---|---|---|---|---|---|
| Topics | Degree | Conf. | Degree | Conf. | Degree | Conf. | Degree | Conf. | Degree | Conf. |
| 200 | 22,10 | ±1,29 | 42,90 | ±1,22 | 30,30 | ±3,43 | 29,90 | ±0,85 | 18,70 | ±0,51 |
| 250 | 21,90 | ±1,22 | 37,00 | ±1,78 | 26,90 | ±1,95 | 23,80 | ±0,49 | 16,90 | ±0,46 |
| 300 | 22,70 | ±1,21 | 31,70 | ±1,13 | 23,80 | ±1,54 | 20,10 | ±0,46 | 15,70 | ±0,51 |
| 350 | 23,00 | ±0,83 | 28,00 | ±1,49 | 22,60 | ±0,98 | 17,50 | ±0,33 | 15,40 | ±0,43 |
| 400 | 20,40 | ±1,25 | 26,10 | ±1,66 | 21,60 | ±2,03 | 15,70 | ±0,30 | 14,30 | ±0,30 |

**Table 8.** Max. Node Degree for Different Number of Topics (Mean Degree with confidence $\alpha = 0.05$)
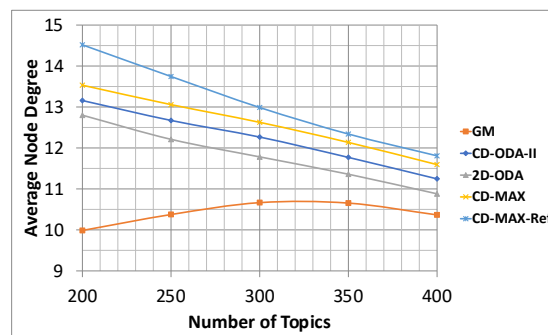
**Figure 12.** Average Node Degree for Different Number of Topics
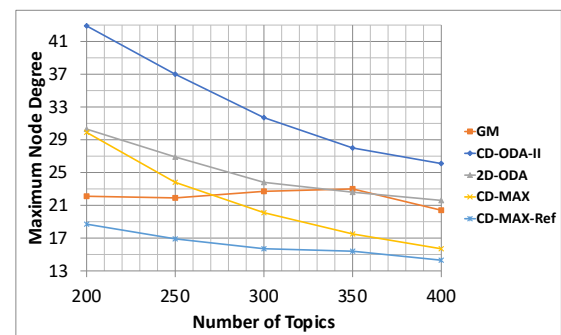
**Figure 13.** Maximum Node Degree for Different Number of Topics

topic-connected components. Hence, the maximum node degree will decrease as the subscription size increases. For all the algorithms with a star topology, the maximum node degree will increase as a single node may be selected as the center of many topics. The results of the final set of experiments showed that CD-MAX required slightly lesser number of edges to build the overlay network when compared with the other algorithms. From the node degree perspective, all constant-diameter algorithms, including the proposed algorithms, showed similar averages. The decrease in the average node degree in the CD-MAX family of algorithms was slightly higher than that in the other competitors, resulting in a higher scalability.

| | GM | | CD-ODA-II | | 2D-ODA | | CD-MAX | | CD-MAX-Ref | |
|---|---|---|---|---|---|---|---|---|---|---|
| Subsc. | Degree | Conf. | Degree | Conf. | Degree | Conf. | Degree | Conf. | Degree | Conf. |
| 15 | 12,86 | ±0,07 | 29,92 | ±0,16 | 32,28 | ±0,43 | 15,09 | ±0,12 | 27,93 | ±0,07 |
| 20 | 12,20 | ±0,06 | 28,21 | ±0,19 | 29,42 | ±0,79 | 14,19 | ±0,14 | 37,08 | ±0,10 |
| 25 | 11,51 | ±0,06 | 25,34 | ±0,41 | 26,31 | ±0,44 | 12,62 | ±0,23 | 46,04 | ±0,10 |
| 30 | 10,83 | ±0,03 | 22,31 | ±0,38 | 23,08 | ±0,43 | 11,20 | ±0,23 | 11,20 | ±0,23 |
| 35 | 10,21 | ±0,04 | 20,93 | ±0,32 | 20,90 | ±0,48 | 10,46 | ±0,16 | 10,46 | ±0,16 |

**Table 9.** Average Node Degree for Different Subscription Size (Mean Degree with confidence $\alpha = 0.05$)

In this experiment, CD-MAX-Ref algorithm tries to find a node with the minimum number of neighbors. In other words, a node with the least connection with others is always an ideal one to be selected. As it is assumed that this algorithm tries to keep the maximum node degree low. Hence, this trend makes the average node degree higher and decrease the maximum node degree of the overlay as the subscription size goes up (see Figure 14). However, as the subscription size increases, the overlay reaches to a threshold at which it is almost impossible to find a node with a lower number of neighbors. Subscription size has been increased and all nodes have higher correlation with each other. Inevitably, the average node degree sharply decreases and the maximum node degree rises more than expected (See Figure 15).

|       | GM    |       | CD-ODA-II |       | 2D-ODA |       | CD-MAX |       | CD-MAX-Ref |       |
|-------|-------|-------|-----------|-------|--------|-------|--------|-------|------------|-------|
| Subsc.| Degree| Conf. | Degree    | Conf. | Degree | Conf. | Degree | Conf. | Degree     | Conf. |
| 15    | 16,10 | ±0,81 | 185,80    | ±1,38 | 183,90 | ±1,97 | 174,10 | ±0,85 | 51,90      | ±0,81 |
| 20    | 14,10 | ±0,94 | 198,00    | ±0,48 | 197,60 | ±0,50 | 193,60 | ±0,50 | 68,70      | ±1,80 |
| 25    | 13,00 | ±1,49 | 199,00    | ±0,00 | 198,90 | ±0,19 | 197,90 | ±0,19 | 82,20      | ±0,91 |
| 30    | 10,60 | ±0,97 | 199,00    | ±0,00 | 199,00 | ±0,00 | 198,80 | ±0,25 | 198,80     | ±0,25 |
| 35    | 9,40  | ±0,84 | 199,00    | ±0,00 | 199,00 | ±0,00 | 199,00 | ±0,00 | 199,00     | ±0,00 |

**Table 10.** Max. Node Degree for Different Subscription Size (Mean Degree with confidence $\alpha = 0.05$)
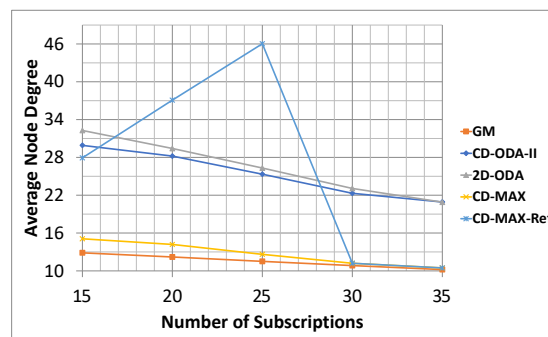
**Figure 14.** Average Node Degree for Different Number of Subscriptions
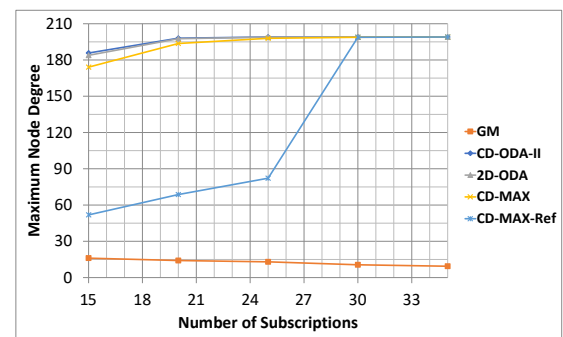
**Figure 15.** Maximum Node Degree for Different Number of Subscriptions

**Comparison of the Running Time Cost of the Algorithms**

An optimized overlay network not only can forward packets with shorter latencies, but also improve the maintenance of connections and provide resiliency against link failures Besta and Hoefler (2014). This optimization is closely dependent on the total degree of nodes. Although designing a topology for optimum resource consumption is not possible, minimizing the total number of links and network diameter and optimizing the algorithm in terms of the time complexity can be considered as distinguishing metrics Voulgaris et al. (2005) Chockler et al. (2007b). Therefore, we included an analytic and experimental run-time analysis of the algorithms for comparison. Tables 11 and 12 include the running times of all the evaluated algorithms and Figures 16 and 17 illustrates the run times in logarithmic scale. The
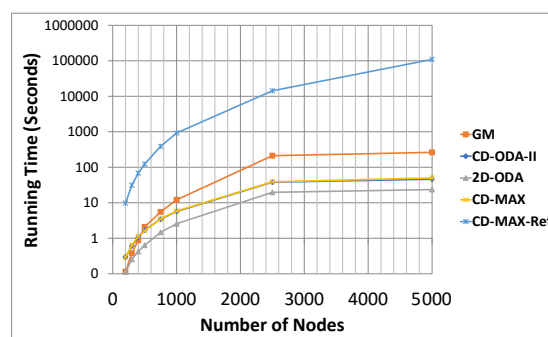
**Figure 16.** Running Time for Different Number of Nodes in Seconds
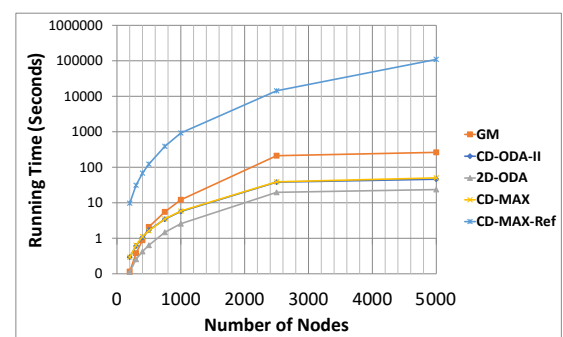
**Figure 17.** Running Time for Different Subscription Size in Seconds

running time of the CD-MAX-Ref algorithm was higher than those of all the other algorithms. Table 11 shows that the growth rate of the running time for the ODA algorithms are similar while the nodes were increasing from 200 to 5000. The growth rate for CD-MAX-Ref and GM are similar, while that for the CD-MAX algorithm is lower. These promising results of CD-MAX illustrate its suitability for the number of nodes that increases beyond a threshold. CD-MAX is the fastest algorithm for building and maintaining the requested topology. Meanwhile, Table 12 shows that the running time for CD-MAX and

415 CD-ODA decreased as the number of subscriptions increased. Considering this, CD-MAX is the fastest
416 algorithm beyond a subscription size of 25. Table 12 indicates that CD-MAX is the fastest algorithm
417 among all algorithms, including the GM algorithm. The relatively higher speed of CD-MAX makes it a
418 more suitable option for dynamic environments, where arrivals and departures are higher.
419     The experiment was carried out under Windows operating system using a computer with an I7-7700HQ
420 processor. As noted in Tables 11 and 12, the confidence intervals may fluctuate among the number of
421 nodes and subscriptions. Due to operating system processor loads in some runs, the running times resulted
422 in higher confidence intervals.

| Nodes | GM Avg. | Conf. | CD-ODA-II Avg. | Conf. | 2D-ODA Avg. | Conf. | CD-MAX Avg. | Conf. | CD-MAX-Ref Avg. | Conf. |
|---|---|---|---|---|---|---|---|---|---|---|
| 200 | 0,12 | ±0,01 | 0,29 | ±0,03 | 0,11 | ±0,01 | 0,29 | ±0,02 | 9,72 | ±0,38 |
| 300 | 0,38 | ±0,02 | 0,61 | ±0,04 | 0,26 | ±0,02 | 0,64 | ±0,03 | 31,13 | ±0,95 |
| 400 | 0,88 | ±0,06 | 1,06 | ±0,08 | 0,42 | ±0,02 | 1,12 | ±0,05 | 68,12 | ±2,08 |
| 500 | 2,09 | ±0,83 | 1,69 | ±0,27 | 0,63 | ±0,04 | 1,62 | ±0,05 | 122,49 | ±1,55 |
| 750 | 5,52 | ±0,17 | 3,44 | ±0,08 | 1,47 | ±0,07 | 3,50 | ±0,10 | 387,86 | ±4,48 |
| 1000 | 12,08 | ±0,04 | 5,67 | ±0,12 | 2,56 | ±0,05 | 5,84 | ±0,15 | 922,76 | ±25,67 |
| 2500 | 211,91 | ±5,28 | 37,95 | ±1,42 | 19,69 | ±1,61 | 38,90 | ±2,26 | 4856,00 | ±150,14 |
| 5000 | 262,86 | ±2,23 | 46,10 | ±1,25 | 23,56 | ±0,95 | 50,20 | ±2,45 | 15049,50 | ±402,14 |

**Table 11.** Running Time for Different Number of Nodes in Seconds (Mean Values with Conf. $\alpha = 0.05$)

| Subsc. | GM Avg. | Conf. | CD-ODA-II Avg. | Conf. | 2D-ODA Avg. | Conf. | CD-MAX Avg. | Conf. | CD-MAX-Ref Avg. | Conf. |
|---|---|---|---|---|---|---|---|---|---|---|
| 15 | 0,16 | ±0,00 | 0,20 | ±0,01 | 0,15 | ±0,00 | 0,19 | ±0,01 | 9,32 | ±0,41 |
| 20 | 0,22 | ±0,01 | 0,15 | ±0,01 | 0,22 | ±0,00 | 0,15 | ±0,01 | 9,18 | ±0,31 |
| 25 | 0,28 | ±0,01 | 0,14 | ±0,01 | 0,31 | ±0,01 | 0,13 | ±0,01 | 9,68 | ±2,45 |
| 30 | 0,35 | ±0,01 | 0,13 | ±0,01 | 0,40 | ±0,01 | 0,12 | ±0,01 | 14,61 | ±2,12 |
| 35 | 0,43 | ±0,01 | 0,12 | ±0,00 | 0,52 | ±0,01 | 0,11 | ±0,01 | 15,39 | ±1,77 |

**Table 12.** Running Time for Different Subs. Size in Seconds (Mean Values with Confidence $\alpha = 0.05$)

## CONCLUSIONS AND FUTURE WORK

424 This study presented a novel algorithm (i.e., CD-MAX) that provides overlay networks with a minimum
425 node degree and a low diameter of 2. The algorithm was implemented in a decentralized manner, and was
426 fast and scalable. The proposed algorithm considerably decreased the maximum node degree, thereby
427 resulting in an overlay network that was more scalable compared to the other algorithms studied. The
428 minimization of the maximum degree plays a key role in a number of networks of very large domains,
429 such as survivable and wireless networks.
430     The study results indicated that the proposed algorithm outperforms the ODA-based algorithms in
431 terms of the decreased diameter and average node degree of the overlay networks and approaches the
432 performance of the GM algorithm.
433     Our analytic and experimental results revealed that the running time of the proposed CD-MAX
434 algorithm is similar to CD-ODA-II and 2D-ODA on average node degree, while outperforming all ODA
435 algorithms excluding GM algorithm on maximum node degree (see Tables 5-6 and Figures 10-11). In
436 addition, CD-MAX builds the network in similar time with ODA algorithms with better maximum node
437 degrees, whereas GM fails to scale out with higher node counts (see Table 11 and Figure 16). As the
438 number of topics increases, the average node degree for CD-MAX and others are similar; however,
439 the CD-MAX and CD-MAX-Ref outperforms on maximum node degree. (See Tables 7-8 and Figures
440 12-13).In terms of different subscription sizes, the CD-MAX algorithm outperforms other ODA algorithms
441 excluding GM on average node degree (see Table 9).
442     By combining the results obtained from the running time experiments and the measurements of the
443 maximum node degree, we can assert that the CD-MAX algorithm is more suitable for networks requiring

high scalability because it simultaneously reduces the communication costs and the running time. In contrast, the CD-MAX-Ref algorithm best suits environments with slow and gradual changes (i.e., having a low churn rate) and those with a large number of topics, and subscription sizes, which are characteristics of typical internet-based systems.

As a future work, studies may concentrate on the simulation of millions of nodes, topics, and subscriptions with a typical churn rate of internet users by employing big data and high-performance computing environments. Further research should consider achieving the best average and low maximum node degrees while optimizing the running time. The proposed algorithms can further be extended to include power consumption optimization Alsultan et al. (2016), and location awareness to build clustered overlay structures to reduce delay, increase bandwidth, improve scalability Bozda and Oztoprak (2008), resilience to node failures, and have load balancing.

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