Review for the manuscript
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Discrete Two Dimensional Fourier Transform in Polar Coordinates Part II: Numerical Computation and approximation of the continuous transform

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1 Mathematical Content

This is a follow-up on an earlier paper, providing illustrations and MATLAB code for the theoretical description given in part I, thus completing the two-paper series.

2 Evaluation and Comments

The paper is certainly a useful complement to the first part providing good information about the ideas of the authors and how they think the method can be applied. At the same time the presentation (also of the first part) is very direct (i.e. based on explicit computations and less on concept) and contains some problematic aspects (especially when it comes to the question of band-limitedness in various ways). On the other hand the examples make more clear how to use it so that the way for further analysis is opened.

So in principle the manuscript should be published, but in a revised from, because it appears a bit long (compared to the content). On the other hand it is not clear why the authors restrict their attention to functions which allow separation in polar coordinates. Especially if they are fairly smooth at appears that the used polar grid may miss a lot of information on the FFT2-side.

3 Comment on the MATLAB package for DHT

The manuscript points to the available MATLAB code for the DHT, the discrete Hankel transform, developed by the authors. It is highly appreciated that this code is available, thus making it a contribution to “reproducible research” and providing a basis for a valid scientific discussion.

Unfortunately there are some critical comment to be made about the MATLAB code.

1. The implementation should be vectorized, because multiple “for loops” make things slower than they could be;

2. instead of just individual examples there should be ready to go M-files where the user specifies a strong (describing the input), and the relevant parameters, as described in the paper and then provides the result;

3. At least there should be M-files which establish the matrix (independent of the input function or matrix f) for any pair of parameters; this would allow the user also to find out what the timing difference is between the different implementations.
4. To call the "true" one-sided exponential function a "continuous" function is a bit disturbing, but OK.

4 Bibliography

The bibliography is rather meager. In particular one has to say that Fourier data on polar coordinates are at the heart of the Fourier based reconstruction method in tomography, where it is pointed out that it is important to be able to convert the given data, interpreted as the values of the two-dimensional FT of the image to be reconstructed as samples on a Polar grid. There are many papers, I would like to point to [3] which has 108 citations (as of know) in GOOGLE scholar. Especially the follow up [4] discusses some issues with polar grids which appear to be relevant also for the material under discussion. Another good survey is [1], which is also highly cited.

There is also a recent book ([2]) which could provide a useful hint for the readers, for modern mathematical methods in this area.

5 Conclusion and Summary

The current submission is an extensive (in the sense of broad and long) illustration of the method presented in the first part of the manuscript. This is positive, because it allows to check the validity of the approach and presents the chance for a critical discussion.

On the other hand it provokes some critical comments, which partically also concern the first part (but thus cannot be changed).

Nevertheless I would suggest to use the opportunity to improve the presentation and the value of the paper for the reader and potential users of this method, while at the same time shortening it (but this is perhaps more a decision of the editors).

In any case the following short-comings have to be addressed and explained to the reader:

1. Aside from the Gauss function the examples are of separable type, i.e. have the form $f(r, \theta) = f_1(r) \cdot f_2(\theta)$, with f_1 being a smooth function on $\mathbb{R}$ (restricted to $\mathbb{R}_+$) and f_2 being a smooth periodic function (in fact just a simple trigonometric polynomial). There is no evidence (but it is implicitely suggested) that such a function is band-limited as a function on $\mathbb{R}^2$. Of course one could think of $\mathbb{C} \setminus \{0\}$ (with polar coordinates) as a product of two Abelian groups, but then one would have to take band-limitedness in the sense of the multiplicative group $\mathbb{R}_+$, and regular samping would correspond to sampling at points $\rho^k, k \in \mathbb{Z}$. But again, in such a situation the function f_1 would have to be taken band-limited in a different sense. In fact, one would have to assume that $f_1^*(x) = f_1(log(x))$ is band-limited as a function on $\mathbb{R}$.
2. Figures 1-4 are very illustrative, but also disappointing because they clearly show that it is not the user how can decide about the polar coordinates to be used. The "whole in the center" is not a simple circle, and this it is questionable whether it is appropriate to talk about "polar grids". It would be fair to speak of a "pseudo-polar grid", but **at least one should mention in the text that it is not a (part of a) true polar grid that is showing up in reality**. In fact, one may think of conversion routines (by simple smooth interpolation methods) which do the conversion between the system of sampling points used and a true polar grid. After all,

what kind of benefit does not have to obtain the values of the FT on a set of points which are not the required ones (or why is such an argument invalid, because there ARE cases, where it is useful as showing up in the paper).

3. Notation has to be simplified. The expression

$$\frac{J_n\left(j_{nk}j_{nm}\right)}{j_{nN}J_{n+1}^2(j_{nk})}$$

appears in each of the formulas (3) - (9) (and later) and should be compactified by giving it a name (a separate symbol).

4. Why is the title only partially capitalized?

5. The parameters are chosen in a reasonable way, but why not explain in the beginning (instead of several times throughout the paper) that(like line 1 of page 3), one has natural numbers $p, k, q, m, n...$ with $1 \leq m, k\text{no comma!} \leq N$ and $-M \leq n, p, q \leq M$.

6. The number of tables (and figures) appears to be excessive and some of them are not really helpful at first sight.

7. The unexperienced reader may want to see one or the other Bessel function (also to understand the sampling pattern and formula (21), including the dependence on the parameter n;

8. **Definitely the authors should provide MATLAB code (for further testing and interaction with potential users) that establishes the four transform matrices as described in the paper, so that users can apply them to other functions and further analyse the behavior.** Since the arguments concerning band-limitedness and consequently the choice of a "Nyquist rate" (which are in fact quite vague such further analysis could prove to be useful.

9. The authors might want to comment on remark 3 (p.11, line 302) and suggest practical ways to overcome the problem of the "hole" in the dataset, by e.g. splitting the function into a small part very much concentrated near zero (and thus with a very smooth and easier to compute FT) and the rest.

10. In the explanation, isn't it just "fftshift" which is used (which is more standard, e.g. around p.13).

11. MATLAB allows to define a function in the following way:

```
gau = @(r)  exp(- pi * (r).^2);
```

thus providing a plot by

```
bas = linspace(-4,4,301); plot(bas,gau(bas)); grid;
```

Hence the matrix f_{nk} could be easily obtained by just first computing the distance of the pseudo-polar grid from zero in the coordinate center.

12. Using the symbol ZERO in a MATLAB code is a bit dangerous (it would be better to call to bzero, for Bessel Zeros), just a hint;

13. A user would definitely be interest in the computational costs (in particular timing), compared to the quality of the output. The report provides some good indications of highly qualitative nature. Maybe there could be a little bit more about the times needed to form the matrix and the time to do the execution. I think, that by avoiding for-loops a speed-up could be obtained easily. Just an example in A-3 (similar in A-1): Why not use for the inner loop

```
psi(ii,:) = q * ones(1,N1-1);
```

References

[1] R. M. Lewitt and S. Matej. Overview of methods for image reconstruction from projections in emission computed tomography. *Proc. IEEE*, 91(10):1588–1611, October 2003.

[2] G. Plonka, D. Potts, G. Steidl, and M. Tasche. *Numerical Fourier Analysis.* Springer, 2018.

[3] H. Stark. Sampling theorems in polar coordinates. *JOSA*, 69(11):1519–1525, 1979.

[4] H. Stark and M. Wengrovitz. Comments and corrections on the use of polar sampling theorems in CT. *IEEE transactions on acoustics, speech, and signal processing*, 31(5):1329–1331, 1983.