# A novel chaotic transient search optimization algorithm for global optimization, real-world engineering problems and feature selection

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**Abstract**

Metaheuristic optimization algorithms manage the search process to explore search domains efficiently and are used efficiently in large-scale, complex problems. Transient Search Algorithm (TSO) is a recently proposed physics-based metaheuristic method inspired by the transient behavior of switched electrical circuits containing storage elements such as inductance and capacitance. TSO is still a new metaheuristic method; it tends to get stuck with local optimal solutions and offers solutions with low precision and a sluggish convergence rate. In order to improve the performance of metaheuristic methods, different approaches can be integrated and methods can be hybridized to achieve faster convergence with high accuracy by balancing the exploitation and exploration stages. Chaotic maps are effectively used to improve the performance of metaheuristic methods by escaping the local optimum and increasing the convergence rate. In this study, chaotic maps are included in the TSO search process to improve performance and accelerate global convergence. In order to prevent the slow convergence rate and the classical TSO algorithm from getting stuck in local solutions, 10 different chaotic maps that generate chaotic values instead of random values in TSO processes are proposed for the first time. Thus, ergodicity and non-repeatability are improved, and convergence speed and accuracy are increased. The performance of Chaotic Transient Search Algorithm (CTSO) in global optimization was investigated using the IEEE Congress on Evolutionary Computation (CEC)'17 benchmarking functions. Its performance in real-world engineering problems was investigated for speed reducer, tension compression spring, welded beam design, pressure vessel, and three-bar truss design problems. In addition, the performance of CTSO as a feature selection method was evaluated on 10 different University of California, Irvine (UCI) standard datasets. The results of the simulation showed that Gaussian and Sinusoidal maps in most of the comparison functions, Sinusoidal map in most of the real-world engineering problems, and finally the generally proposed CTSOs in feature selection outperform standard TSO and other competitive metaheuristic methods. Real application results demonstrate that the suggested approach is more effective than standard TSO.

**Keywords:** Chaotic Transient Search Optimization Algorithm, Chaotic Maps, Benchmark Functions, Real-World Engineering Problems, Feature Selection

## 1. Introduction

Optimization is the process of identifying the most optimal solution to a problem from among all possible alternatives. Given the nature of optimization algorithms, they can be broadly classified into two categories: deterministic algorithms and stochastic intelligent algorithms. Additionally, stochastic algorithms are categorized into two types: heuristic algorithms and metaheuristic algorithms (Yang et al., 2012).

44    Deterministic optimization algorithms are insufficient for large-scale combinatorial and
45    nonlinear problems. Usually, due to the natural solution mechanisms of deterministic algorithms,
46    the problem of interest is modeled in such a way that the algorithm handles it. The solution strategy
47    of deterministic methods usually depends on the types of objectives and constraints and the types
48    of variables used in modeling the problem. The efficacy of these methods is significantly
49    influenced by the solution space, the quantity of decision variables, and the number of constraints
50    involved in problem formulation. An additional noteworthy limitation is the absence of
51    overarching solution approaches that can be implemented for problem formulations featuring
52    diverse decision objectives, variables, and constraints. That is, most algorithms solve models with
53    certain types of objective functions or constraints. However, optimization problems in many
54    different fields such as management science, computing, and engineering simultaneously require
55    different types of decision variables, objective functions, and constraints in their formulation.
56    Therefore, metaheuristic optimization algorithms have been proposed. These have become very
57    popular methods in recent years because they have good computational power and are easy to
58    transform (Bianchi et al., 2009).
59    Recently, metaheuristic algorithms have gained unexpected popularity. This is because they
60    have demonstrated their superiority in tackling several optimization challenges. General-purpose
61    metaheuristic methods are examined in different categories: biology-based, chemistry-based,
62    mathematics-based, music-based, physics-based, plant-based, swarm-based, social-based, sports-
63    based, water-based, and hybrid-based (Altay & Alatas, 2020). The emergence of physics-based
64    algorithms was caused by physics phenomena in nature. The most well-known include Big Bang-
65    Big Crunch (Erol & Eksin, 2006), Electromagnetism-like heuristic (Birbil & Fang, 2003), Central
66    force optimization algorithm (Formato, 2009), Multi Verse Optimization (MVO) (Mirjalili,
67    Mirjalili & Hatamlou, 2016), Galaxy-based search algorithm (Hosseini, 2011), Henry gas
68    solubility optimization (Hashim et al., 2019), Gradient-based optimizer (Ahmadianfar, Bozorg-
69    Haddad & Chu, 2020), Equilibrium optimizer (Faramarzi et al., 2020), Flow Direction Algorithm
70    (Karami et al., 2021), Archimedes Optimization Algorithm (Hashim et al., 2021), Transit search
71    algorithm (Mirrashid & Naderpour, 2022), and Transient Search Algorithm (TSO) (Qais, Hasanien
72    & Alghuwainem, 2020).
73    Metaheuristic methods are being developed thanks to their simplicity, cheap computational
74    cost, gradient-free mechanism, and flexibility, and the interest in the use of these methods is
75    increasing day by day. In this area, there is a theorem called No Free Lunch (NFL), which proves
76    that there is no general algorithm for solving all optimization problems and allows this work area
77    to be used actively. It has been mathematically proven by the NFL theorem that there is no single
78    optimization method that solves all optimization algorithms. Thus, while the optimization
79    algorithm produces good results in solving one problem, it can produce bad results in another. The
80    gap here also encourages researchers working in this field to produce new methods, improve
81    existing methods, or hybridize methods by combining them. Another gap is that, due to the
82    stochastic optimization process, it is very difficult to maintain a balance between exploration and
83    exploitation in the creation of any metaheuristic algorithm. TSO is a very new physics-based
84    method inspired by the transient behavior of switched electrical circuits containing storage
85    elements such as inductance and capacitance. As with other metaheuristic methods, TSO faces the
86    challenge of achieving the right balance between exploration and exploitation. This study focuses
87    on finding a solution to this problem, improving the speed of convergence and the ability of TSO
88    to find the global optimum solution, and ultimately improving the performance of TSO in terms
89    of various metrics. To the best of our knowledge, no studies have been done on how to increase

90 global convergence and performance rates while avoiding trapping TSOs in local solutions. For
91 the first time, chaos theory has been applied to TSO in this work to eliminate the drawbacks of the
92 method. In this work, chaotic maps are embedded inside TSO to create novel algorithms known
93 as chaotic TSO algorithms.

94     In this study, 10 different chaotic maps were integrated into the TSO algorithm. The main
95 motivation for the study is to use the number sequences obtained from different chaotic maps
96 instead of the critical parameters produced by random numbers in the TSO algorithm. By using
97 chaotic maps with ergodic, irregular, and stochastic features in chaotic map TSO, it is aimed at
98 avoiding local solutions more easily compared to the TSO method. In this way, it is aimed to
99 increase global convergence and obtain a better curve by improving the exploration and
100 exploitation stages of the TSO algorithm. The proposed method has been applied to the accepted
101 CEC'17 benchmark functions, real-world engineering design problems, and feature selection in
102 the literature.

103     The remainder of the article is organized as follows: In the second section, the working principle
104 of the TSO algorithm is given. In the third section, chaotic maps are examined, and their equations
105 are given. In the fourth section, the proposed chaotic TSO method is explained in detail. In the
106 fifth section, the experimental results are given. This part consists of three separate stages. First of
107 all, the proposed method was tested on the CEC'17 benchmark functions, and statistical analyses
108 were made and supported by figures. Then the proposed method is adapted to five real-world
109 engineering problems and the performance analyses of the proposed method and the standard TSO
110 method are examined. Finally, the proposed methods and the TSO method on feature selection
111 were adapted and performance analyses were carried out on classification problems. In the last
112 section, section 6, the conclusion part is included.

113

114 **2. Literature Review**

115

116     Various optimization methods have been proposed in the literature that can be used in
117 optimization problems. These methods have become very popular not only in computer science
118 but also in other research areas (Altay, 2022a). Complex reliability allocation problems (Negi et
119 al., 2021), traveling salesman problem (Mzili, Riffi & Mzili, 2022), association rule mining (Altay
120 & Alatas, 2021), dynamic ship routing and scheduling problem (Das et al., 2022), laser cutting
121 process (Madić et al., 2022), machine learning (Altay & Varol, 2023), and process synthesis
122 problem (Altay, 2022b) are some of them. There are studies comparing the performance of
123 metaheuristic methods (Chakrabarty & Lahiri, 2023).

124     There is no best optimization algorithm to solve all problems. While an optimization algorithm
125 may solve one problem very well, it may not achieve successful results in another. This encourages
126 researchers to propose new methods and improve existing ones. When the literature is examined,
127 it is seen that many optimization algorithms have been proposed. Some of those; Group teaching
128 optimization algorithm (Zhang & Jin, 2020), Dwarf Mongoose optimization algorithm (Agushaka,
129 Ezugwu & Abualigah, 2022), Chimp optimization algorithm (Khishe & Mosavi, 2020), Material
130 generation algorithm (Talatahari, Azizi & Gandomi, 2021), Social mimic optimization algorithm
131 (Balochian & Baloochian, 2019), Arithmetic trigonometric optimization algorithm (Devan et al.,
132 2022), Fertilization optimization algorithm (Devan et al., 2022), African vultures optimization
133 algorithm (Abdollahzadeh, Gharehchopogh & Mirjalili, 2021), Aquila optimizer (Abualigah et al.,
134 2021b), Circle search algorithm (Qais et al., 2022), The water optimization algorithm (Daliri &
135 Asghari, 2022), Gold rush optimizer (Zolfi, 2023). The transient search algorithm is one of the

136   physics-based metaheuristic methods that have emerged recently. This method has been tested on
137   some problems, but as far as we know, there is no study related to the development of the method.
138   The fact that it is a new method and that no improvement has been made in this area yet has been
139   our source of motivation.
140       It is seen that the Transient Search algorithm has been successfully used in PEM fuel cell
141   modeling (Hasanien et al., 2022), IoT intrusion detection system (Fatani et al., 2021), optimum
142   allocation of more than one distributed generator in the Radial electricity distribution network
143   (Bhadoriya & Gupta, 2022), and improving the voltage ride-through capability of the wind turbine
144   (Qais & Hasanien, 2020).
145       Chaos theory has been extensively employed to enhance exploration and exploitation as
146   nonlinear theory has undergone constant study and improvement. Numerous metaheuristic
147   algorithms' premature convergence issues have been effectively resolved using chaos theory.
148   Many researchers have added chaotic mapping mechanisms to various metaheuristic algorithms to
149   augment the algorithm's capacity to find optimum solutions, improve random diversification, and
150   obtain optimal or sub-optimal answers in complicated multi-modal circumstances (Arora &
151   Anand, 2019). The application of chaos theory to different metaheuristic methods can be
152   summarized in Table 1.

### 3.   TSO algorithm

156       In this section, the background of the TSO algorithm and the operation of the method are
157   discussed. Pseudo code of TSO algorithm is given.

### 3.1.      Background of Transient Search Optimization Algorithm

161       The transient performance of electrical circuits has been the inspiration for the metaheuristic
162   optimization method TSO, which has been proposed in recent years. Electrical circuits contain
163   different elements that store energy. These can be capacitors (C), inductors (L), or a combination
164   of both (LC). Generally, an electrical circuit containing a resistor (R), C, or L has a transient
165   response and a steady-state response. This situation is shown in Eq. (1). If the electrical circuit
166   contains an energy storage element together with the resistor, these circuits are classified as first-
167   order circuits. If there are two energy storage elements next to the resistor in the electrical circuit,
168   they are called second-order circuits. The switching of such circuits cannot be changed until the
169   steady-state values of R and L are reached. The transient response of the first-order circuit is
170   calculated by the differential equation in Eq. (2). Eq. (2) can be solved to find the solution of $x(t)$
171   shown in Eq. (3).

173   $Complete\ response = Transient\ response + Final\ response$                    (1)

174   $\frac{d}{dt}x(t) + \frac{x(t)}{\tau} = K$                    (2)

175   $x(t) = x(\infty) + (x(0) - x(\infty))e^{\frac{-t}{\tau}}$                    (3)

176   where time $t$, $x(t)$  can be called the capacitor voltage $v(t)$  of the RC circuit or the inductor
177   current $i(t)$ of the RL circuit. $\tau$ is called the time constant of the circuit. $\tau = RC$ and $\tau = L/R$ are
178   for circuit RC and RL, respectively, and $K$ is a constant based on the initial value of $x(0)$.  $x(\infty)$

179     is the final response value. The transient response of a quadratic circuit is calculated using the
180     differential equation shown in Eq. (4). The solution of the quadratic differential equation is shown
181     in Eq. (5). Here, the response of the RLC circuit is considered a low-damped response.
182

183     $\frac{d^2}{dt^2}x(t) + 2\alpha\frac{d}{dt}x(t) + w_0^2 x(t) = f(t)$          (4)

184     $x(t) = e^{-\alpha t}\big(B_1\cos\big((2\pi f_d t)\big) + B_2\big((2\pi f_d t)\big)\big) + x(\infty)$          (5)

185     where $\alpha$ is the damping coefficient, $w_0$ is the resonant frequency, $f_d$ is the damped resonance
186     frequency, and $B_1$ ve $B_2$ are fixed values. The low damped response occurs when $\alpha < w_0$ causes
187     damped oscillations of the transient response of the RLC circuit.
188

189     **3.2. Transient Optimization Algorithm**
190

191       The working logic of the TSO algorithm is similar to the working logic of other metaheuristic
192     optimization algorithms and consists of three steps. In the first of these steps, the initial population
193     is created by creating search agents within the lower and upper limits of the exploration area. The
194     second step is called the exploration phase. In this step, the best solution is sought. The third and
195     final step is called the exploitation stage, and it is aimed at reaching a steady state or the best
196     solution. Search agents in the initial population are randomly generated as in Eq. (6).
197

198     $Y = lb + rand \times (ub - lb)$          (6)

199       where the $lb$ value represents the lower limit of the search area, and the $ub$ value the upper limit
200     of the search area. The $rand$ value represents a uniformly distributed random sequence of
201     numbers. The second step, the discovery phase, is designed by inspired by the oscillations of the
202     second order RLC circuits around zero. However, the use of TSO here is inspired by the
203     exponential decay of the first order discharge. The $r_1$ value, which is a random number, is used to
204     balance the exploration $(r_1 \geq 0.5)$ and exploitation $(r_1 < 0.5)$ phases. The use and mathematical
205     model of TSO is shown in Eq. (7), inspired by Eq. (3) and Eq. (5). TSO's best solution $Y_l^*$,
206     simulates the steady state or final value $(x(\infty))$ of the electrical circuit, also $B_1 = B_2 =$
207     $\left|Y_l - C_1.Y_l^*\right|$.
208

209     $f(x) = \begin{cases} Y_l^* + \big(Y_l - C_1.Y_1^*\big)e^{-T}, & r_1 < 0.5 \\ Y_l^* + e^{-T}[\cos(2\pi T) + \sin(2\pi T)]\left|Y_l - C_1.Y_l^*\right|, & r_1 \geq 0.5 \end{cases}$          (7)

210     $T = 2 \times z \times r_2 - z$          (8)

211     $C_1 = k \times z \times r_3 + 1$          (9)

212     $z = 2 - 2\left(l/L_{max}\right)$          (10)

213    In Eq. (10), $z$ represents a value ranging from 2 to 0 as understood from the equation. The value
214    $l$ represents the number of iterations, $T$ and $C_1$ are random coefficients, $r_1$, $r_2$ and $r_3$ are evenly
215    distributed random numbers and take values between 0 and 1. $Y_l^*$ indicates the best position, $k$ is
216    a constant and $L_{max}$ is the maximum number of iterations. The balance between exploration and
217    exploitation processes is achieved with a $T$ coefficient ranging from -2 to 2. Exploitation phase is
218    obtained when $T > 0$ (to the smallest value) and exploration process is obtained when $T < 0$ (to
219    the highest value). The pseudo-code of TSO is given in Algorithm 1.
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**Algorithm 1. Pseudo-code of TSO algorithm**

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Initialize the population and the best positions $Y_l$, $Y_l^*$
Evaluate the cost function of the population
**while** $l < L_{max}$
    Update the values of $T$ and $C_1$ using Eq. (8) and Eq. (9)
    **do** all populations $Y_l$
        Update the population place by Eq. (7)
    **end do**
    Calculate the cost function of all new population
    Update the best value if the recent cost function is less than the previous best cost function
    $l = l + 1$
**end while**
Output the best value $Y_l^*$

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221    ## 4.   Chaotic Maps
222
223    In a nonlinear, dynamical system that is non-periodic, non-convergent, and bounded, chaos is
224    a deterministic, random-like technique. Chaos is the randomness of a straightforward deterministic
225    dynamical system in mathematics, and chaotic systems can be thought of as sources of
226    randomness. Although chaos appears random and unpredictable, it also has a certain degree of
227    pattern. Instead of using random variables, chaos uses chaotic variables (Arora & Anand, 2019).
228    Numbers generated by chaotic maps have been used successfully in a variety of applications.
229    In general, chaotic maps have three basic qualities: ergodicity, beginning conditions, and semi-
230    stochastic properties. Researchers in a variety of domains, including ecology, medicine,
231    economics, and engineering applications, are drawn to chaotic processes. It is also extensively
232    utilized to improve the performance of optimization algorithms (Altay, 2022c).
233    The effectiveness of swarm intelligence algorithms in tackling a variety of challenging
234    optimization issues has been demonstrated. As a result, further developing these algorithms
235    became a hotly debated scientific subject. The use of chaotic maps in place of random values in
236    swarm intelligence algorithms is one of the more frequently employed innovations. Improved
237    search is anticipated because chaotic maps produce numbers that are non-repeatable and ergodic
238    (Tuba et al., 2018).
239    Chaotic map applications have proven successful in enhancing the stochastic structure of the
240    optimization techniques in a variety of research. The standard TSO's global convergence time was
241    accelerated in this work by the introduction of ten separate chaotic maps, which also prevented the
242    standard TSO from being stuck in local solutions. There are many types of chaotic maps that are
243    employed, including the gauss, circle, sine, logistic, piecewise, iterative, singer, tent, and
244    sinusoidal maps. Table 2 presents an explanation of the variables and equations linked to these
245    chaotic maps. The graphics of the ten chaotic maps are shown in Fig. 1.

246
247    ## 5.  Chaotic TSO Algorithm
248
249    Metaheuristic optimization algorithms suffer from population diversity and early convergence.
250    In addition, the metaheuristic optimization algorithm needs a balance between exploration and
251    exploitation in order to produce an effective solution. Chaotic maps are one of the most effective
252    ways to increase population diversity and the quality of metaheuristic optimization methods. Thus,
253    both the search sensitivity and convergence speed of the metaheuristic optimization method, which
254    are improved with the chaotic map, are improved. Most metaheuristic optimization methods have
255    an exploration and exploitation phase. The exploration phase is used to search the search area as
256    wide as possible, regardless of whether the method is stuck at the local optimum. In the exploitation
257    phase, the method aims to find the best possible solution in a limited area of the search space by
258    finding the most promising area. While the method spends too much time on the exploitation
259    process, which only leads to a local search, spending too much time on the exploration process
260    results in a random search.
261    Although the newly proposed TSO algorithm has been successfully applied in different
262    applications in the literature, it has some disadvantages, as we mentioned above. Therefore, in
263    order to improve the search space of the TSO algorithm, chaotic maps, which are accepted in the
264    literature, have been added to the dynamic behavior and optimization algorithms to make the
265    search space stronger. Eq. (7) is used for exploration and exploitation in the TSO algorithm, whose
266    working principle is very clear. Random variables affecting Eq. (7) are regular random variable
267    values $r_2$ and $r_3$ in Eqs. (8) and (9), respectively. In this study, instead of the $r_3$ value, which is
268    the stochastic component of the TSO algorithm in Eq. (9), chaotic maps with different
269    mathematical equations listed in Table 2 were applied. The mathematical equation is shown in the
270    equation below.
271
272    $$C_1 = k \times z \times CMV + 1 \tag{11}$$

273    Here, $CMV$, that is, sequentially generated chaotic map value is used instead of $r_3$ value.
274    Proposed algorithms using 10 different chaotic map; chebyshev map (CTSO-1), circle map
275    (CTSO-2), gauss map (CTSO-3), iterative map (CTSO-4), logistic map (CTSO-5), piecewise map
276    (CTSO-6), sine map (CTSO-7), singer map (CTSO-8), sinusoidal map (CTSO-9) and tent map
277    (CTSO-10). The flowchart of the CTSO is shown in Fig. 2.
278    Also, the complexity of the proposed CTSO algorithms can be expressed using big-oh notation.
279    The process of the CTSO algorithm starts with the random generation of search agents in the first
280    step, evaluates the search agents using the cost function in the second step, and updates the search
281    agents to the function evaluation value in the third step. Here, the first step is denoted by $O(N)$,
282    where $N$ is the number of search agents. In the second step, the search agents enter the while loop,
283    which has the maximum iteration ($L_{max}$). The complexity of function evaluations of all search
284    agents is expressed as $O(N * L_{max})$. And finally, in the third step, the complexity of updating all
285    search agents with a size ($D$) for total iterations is expressed as $O(N * L_{max} * D)$.
286
287    ## 6. Results and Discussion
288
289    In order to evaluate the performance of the methods proposed in the study, CEC'17 test
290    functions, 5 different real-world problems, and feature selection problems consisting of 10

291   different real-world datasets were applied. The results obtained were compared in detail under this
292   section and the performance analyses of the methods were carried out. TSO and CTSOs are taken
293   as constant parameters with $k$ value of 1 and $z$ value of [0,2]. The experimental tests are performed
294   using MATLAB R2021a and the whole test is executed on a PC (Intel (R) Core (TM) i9–10900k
295   CPU @ 3.70 GHz (10 CPUs), 32 GB, Windows 10–64 bits).

296

297   **6.1. Benchmark Function**

298

299       The performance evaluation of ten distinct CTSO and TSO methods was conducted using the
300   IEEE Congress on Evolutionary Computation (CEC) test functions. The CEC'17 test suite
301   comprises a total of 29 test functions, encompassing a diverse range of function types such as
302   unimodal, multimodal, hybrid, and composition functions. The algorithm's convergence
303   performance is assessed using unimodal functions, namely f1 and f3, while the presence of early
304   convergence and local fixation issues is evaluated using multimodal functions, specifically f4
305   through f10. The assessment of the capacity to evade local optima, which are characterized by
306   numerous local optima, and the equilibrium between exploration and exploitation is carried out
307   through the utilization of hybrid and composition functions (f11-f20 and f21-f30).
308       The lower and upper bounds of all functions included in the CEC'17 test suite are between −100
309   and 100. In order to make a fair evaluation under equal conditions, the number of evaluations was
310   chosen as 1000 and the population as 30. Algorithms were run 30 times in all experiments, and the
311   results of mean (AVG), standard deviation (STD), minimum (MIN), and Friedman mean rank
312   (MR) values are presented in Tables 3A, 3B and 3C in a comparative manner. According to the
313   MR value, CTSO-9 and CTSO-5 showed the best performance in unimodal benchmark functions.
314   Of the multimodal functions, CTSO-3 showed the best performance in 3, CTSO-9 in 2, CTSO-3
315   and CTSO-9 in 1, and CTSO-5 in 1 of them. Of the hybrid and composition functions, CTSO-3
316   showed the best performance in 11, CTSO-9 in 6, CTSO-3 and CTSO-9 in 2, and CTSO-10 in 1
317   of them. In Table 4, the average Friedman mean rank values based on the MR values of all
318   benchmark functions are given. When Table 4 is examined according to the statistical analysis
319   results, CTSO-3 gives the best performance, followed by CTSO-9 with a close value. CTSO-8
320   performed worse than the original TSO. The convergence performance of the algorithms on the
321   CEC'17 test functions is also given in Fig. 3 according to the best values of the algorithms. Fig. 4
322   presents a boxplot of CEC'17 test functions.
323       The results of the fitness functions obtained using the Wilcoxon rank-sum test are given in
324   Table 5. The results obtained were obtained at a significance level of 5 percent. The results
325   obtained must be less than 0.05 to show a significant advantage. The $\triangle$, $\nabla$, and $\approx$ signs in the table
326   indicate the superiority of one of the proposed CTSOs, the TSO is superior, and there is no
327   significant difference between the methods, respectively. When the Table 5 is examined, it is seen
328   that CTSO-3 and CTSO-9 show the best performances by providing superiority to TSO in 26
329   functions. It can be said that Gaussian and sinusoidal chaotic maps are superior to other chaotic
330   maps and standard TSO in benchmark functions.

331

332   **6.2.   Real-World Engineering Problems**

333

334       In this section, five real-world engineering design problems are used to validate the search
335   performance of each implemented algorithm. These problems are the speed reducer problem,
336   tension compression spring design problem, welded beam design problem, pressure vessel, and

337  three-bar truss design problem. All these problems are limited in nature, and therefore an external
338  penalty approach mechanism was used to solve the design constraints. The maximum number of
339  iterations for all problems was determined to be 1000 and the population number was 30. The
340  algorithm parameters are the values used in CEC'17. To visualize and compare the convergence
341  behavior of the examined algorithms, the best fitness values obtained for each problem, which are
342  usually called convergence curves, are drawn. All experiments corresponding to each algorithm
343  were run independently 30 times. The best, mean, worst, and standard deviation values were
344  examined comparatively, and the best solution obtained among the algorithms was highlighted in
345  bold for ease of readability.
346
347  ### 6.2.1.       Speed reducer problem
348
349  This is essentially a gearbox issue, allowing the aircraft engine to spin at its most efficient speed
350  (Dhiman, 2021). Finding the face width $b$ $(x_1)$, the tooth modulus $m$ $(x_2)$, the number of teeth on
351  the pinion $z$ $(x_3)$, the length of the first shaft between the bearings $l_1$ $(x_4)$, the length of the second
352  shaft between the bearings $l_2$ $(x_5)$, the diameter of the first shaft $d_1$ $(x_6)$, and the diameter of the
353  second shaft $d_2$ $(x_7)$ will allow you to determine the minimum values of the seven decision
354  variables in this problem. Fig. 5 shows a schematic illustration of the speed reducer concept. The
355  goal of this design challenge is to determine the speed reducer's lightest possible cost. The
356  mathematical representation of this problem is as in the Appendix 1.
357  The comparative values of the performances of the 10 proposed CTSO methods and standard
358  TSO methods on the speed reducer problem are presented in Table 6. The best, average, worst,
359  and standard deviation values of the methods used are shown in Table 6. In addition, the decision
360  variables depending on the best value of the methods used in the results of 30 runs on this problem
361  are given in Table 7. When Table 6 is examined, it is seen that CTSO-3 and CTSO-9 are superior
362  to other methods on the basis of the best value, and CTSO-9 is more successful when analyzed on
363  the basis of mean value. In addition, the convergence graph of the methods used on the speed
364  reducer problem is shown in Fig. 6.
365
366  ### 6.2.2.       Tension compression spring design problem
367
368  The goal of the tension/compression spring design issue, as outlined by Arora (Arora, 2004), is
369  to provide a spring design with the least amount of weight possible. Fig. 7's schematic illustration
370  of this minimization issue shows some of its restrictions, including the cut-off voltage, ripple
371  frequency, and minimal deviation. There are three choice variables in the tension/compression
372  spring dilemma. These are the following: the wire diameter $d$ $(x_1)$, the average coil diameter $D$ $(x_2$
373  $)$, and the number of active coils $N$ $(x_3)$. The mathematical representation of this problem is as in
374  the Appendix 2.
375  The comparative values of the performances of the 10 proposed CTSO methods and standard
376  TSO methods on the tension compression spring design problem are presented in Table 8. The
377  best, average, worst, and standard deviation values of the methods used are shown in Table 8. In
378  addition, the decision variables depending on the best value of the methods used in the results of
379  30 runs on this problem are given in Table 9. When Table 8 is examined, it is seen that CTSO-9 is
380  superior to other methods on the basis of the best value, and CTSO-6 is more successful when

381 analyzed on the basis of mean value. In addition, the convergence graph of the methods used on
382 the tension compression spring design problem is shown in Fig. 8.
383
384 **6.2.3.      Welded beam design problem**
385
386     The primary goal of the welded beam design challenge is to create a beam for the least amount
387 of money while adhering to specific constraints (Varol Altay & Alatas, 2020). Fig. 9 depicts a
388 welded beam construction made up of beam A and the welding process needed to join it to item
389 B. Five nonlinear inequality constraints and four choice variables make up the issue. The weld
390 thickness, weld joint length, element width, and element thickness are represented by the design
391 parameters $h$ $(x_1)$, $l$ $(x_2)$, $t$ $(x_3)$, and $b$ $(x_4)$, respectively. The mathematical representation of this
392 problem is as in the Appendix 3.
393     The comparative values of the performances of the 10 proposed CTSO methods and standard
394 TSO methods on the welded beam design problem are presented in Table 10. The best, average,
395 worst, and standard deviation values of the methods used are shown in Table 10. In addition, the
396 decision variables depending on the best value of the methods used in the results of 30 runs on this
397 problem are given in Table 11. When Table 10 is examined, it is seen that CTSO-1 is superior to
398 other methods on the basis of the best value, and CTSO-4 is more successful when analyzed on
399 the basis of mean value. In addition, the convergence graph of the methods used on the welded
400 beam design problem is shown in Fig. 10.
401
402 **6.2.4.      Pressure vessel problem**
403
404     The major goal of this issue is to optimize vessel formation, material use, and welding costs
405 (He & Zhou, 2018). The objective function is constructed using four variables: shell thickness (
406 $x_1$), head thickness ($x_2$), inner radius ($x_3$), and length ($x_4$), without accounting for vessel height.
407 This issue contains four restrictions that must be met. Fig. 11 depicts the pressure vessel design
408 problem's schematic structure. The mathematical representation of this problem is as in the
409 Appendix 4.
410     The comparative values of the performances of the 10 proposed CTSO methods and standard
411 TSO methods on the pressure vessel design problem are presented in Table 12. The best, average,
412 worst, and standard deviation values of the methods used are shown in Table 12. In addition, the
413 decision variables depending on the best value of the methods used in the results of 30 runs on this
414 problem are given in Table 13. When Table 12 is examined, it is seen that CTSO-2 is superior to
415 other methods on the basis of the best value, and CTSO-5 is more successful when analyzed on
416 the basis of mean value. In addition, the convergence graph of the methods used on the pressure
417 vessel design problem is shown in Fig. 12.
418
419 **6.2.5.      Three bar truss design problem**
420
421     In civil engineering, this issue is referred to as a structural optimization problem. By modifying
422 the cross-sectional areas ($x_1$ and $x_2$) while taking into consideration the stress (σ) on each of the
423 truss members, Nowacki's issue seeks to reduce the volume of the three-bar truss. These values'
424 possible value ranges are $0 \leq x_1, x_2 \leq 1$. The graphical illustration of this problem's mathematical
425 formulation is shown in Fig. 13. The mathematical representation of this problem is as in the
426 Appendix 5.

427  The comparative values of the performances of the 10 proposed CTSO methods and standard
428  TSO methods on the three-bar truss design problem are presented in Table 14. The best, average,
429  worst, and standard deviation values of the methods used are shown in Table 14. In addition, the
430  decision variables depending on the best value of the methods used in the results of 30 runs on this
431  problem are given in Table 15. When Table 14 is examined, it is seen that CTSO-9 is superior to
432  other methods on the basis of the best value, and CTSO-10 is more successful when analyzed on
433  the basis of mean value. In addition, the convergence graph of the methods used on the three- bar
434  truss design problem is shown in Fig. 14.
435
436  **6.2.6.  Analysis of CTSOs in real-world engineering problems with other metaheuristic**
437  **optimization algorithms**
438
439  The speed reducer problem has been solved by many different researchers in the literature with
440  different metaheuristic optimization methods. The classical TSO and the proposed CTSO method
441  were compared with the Moth Flame Optimization Algorithm (MFO) (Mirjalili, 2015), Weighted
442  Superposition Attraction (WSA) (Baykaso, 2015), Grey Wolf Optimization (GWO) (Mirjalili,
443  Mirjalili & Lewis, 2014), Artificial Acari Optimization (AAO) (Czerniak, Zarzycki & Ewald,
444  2017), Sine Cosine Algorithm (SCA) (Mirjalili, 2016), and Arithmetic Optimization Algorithm
445  (AOA) (Abualigah et al., 2021a)methods in the literature. The best cost and related decision
446  variables obtained by the proposed method and the methods proposed by other researchers for the
447  speed reducer problem are presented in Table 16. When the results obtained were examined, it was
448  observed that the proposed CTSO-3 and CTSO-9 method gave a much better result than the
449  classical TSO and other competitive methods.
450  There are many methods in the literature to solve the tension compression spring design
451  problem. The classical TSO and the proposed CTSO method were compared with the HGSO,
452  AAO, Whale Optimization Algorithm (WOA) (Mirjalili & Lewis, 2016), Harris Hawk
453  Optimization (HHO) (Heidari et al., 2019), Chaotic Bird Swarm Algorithm (CMBSA) (Varol
454  Altay & Alatas, 2020), Salp Swarm Algorithm (SSA) (Mirjalili et al., 2017), Cumulative Binomial
455  Probability Particle Swarm Optimization (CBPPSO) (Agrawal & Tripathi, 2018), and Competitive
456  Bird Swarm Algorithm (CBSA) (Wang et al.) methods in the literature. The results obtained from
457  this comparison are given in Table 17. Table 17 presents the best cost and relevant decision
458  variables for the tension compression spring design problem. When the literature and the results
459  of the experiments are examined, it is concluded that CTSO-9 gives better results than classical
460  TSO and competitive methods in the literature.
461  Welded beam design problem has been solved by many different researchers in the literature
462  with different metaheuristic optimization methods. The classical TSO and the proposed CTSO
463  method were compared with the HGSO, MAOA, HHO, CMBSA, CBBSA, CBPPSO, Chaotic
464  Grey Wolf Optimization (CGWO) (Kohli & Arora, 2018), Differential Big Bang- Big Crunch
465  Algorithm (DBCA) (Prayogo et al., 2018), Sonar Inspired Optimization (SIO) (Tzanetos &
466  Dounias, 2020), and Hybrid Genetic Algorithm-Ant Colony Optimization-Particle Swarm
467  Optimization (H-GA-ACO-PSO) (Tam et al., 2019) methods in the literature. The best cost and
468  relevant decision variables obtained by the proposed method and the methods proposed by other
469  researchers for the welded beam design problem are presented in Table 18. When the results
470  obtained were examined, it was observed that the proposed CTSO-1 method gave a much better
471  result than the classical TSO and other competitive methods.

472   The pressure vessel problem has been solved by many different researchers in the literature
473   with different metaheuristic optimization methods. The classical TSO and the proposed CTSO
474   method were compared with the MAOA, CSMA, AAO, HHO, CMBSA, SSA, CBPPSO, H-GA-
475   ACO-PSO, and Adaptive Reinforcement Learning based Bat Algorithm (ARLBAT) (Meng, Li &
476   Gao, 2019) methods in the literature. The best cost of the proposed method and the methods
477   suggested by other researchers for the pressure vessel problem and the relevant decision variables
478   are shown in Table 19. When the results are examined, it is seen that the CSMA method gives a
479   better result than the proposed CTSOs, classical TSO and other competitive methods.
480   The three bar truss problem has been solved by many different researchers in the literature with
481   different metaheuristic optimization methods. The classical TSO and the proposed CTSO method
482   were compared with the AOA, SSA, CBPPSO, Cuckoo Search (CS) (Gandomi, Yang & Alavi,
483   2013),  and Mine blast algorithm (MBA) (Sadollah et al., 2013) methods in the literature. The best
484   cost and relevant decision variables obtained by the proposed method and the methods proposed
485   by other researchers for the three bar truss problem are shown in Table 20. When the literature and
486   the results of the experiments are examined, it is concluded that CTSO-9 gives better results than
487   classical TSO and competitive methods in the literature.

488
489   **6.3. Feature Selection**
490

491   This section shows a comparison of performance between the suggested 10 distinct CTSO
492   approaches and the state-of-the-art over ten different benchmark datasets using standard TSO and
493   the state-of-the-art (Australian, ionosphere, spectheart, sonar, wine, heart, thyroid, tic-tac-toe,
494   vehicle, krvskp). These datasets, whose descriptions are provided in Table 21, include a broad
495   variety of attributes and come in various shapes and sizes. The purpose of this experiment is to
496   assess the applicability of the suggested CTSO approaches for feature selection by analyzing their
497   impact on benchmark datasets and comparing their performance to existing techniques. The
498   experiment will explicitly look at how much better the performance of different datasets is when
499   compared to the other suggested approaches. The efficiency of the recommended CTSO
500   approaches is assessed using the accuracy measure, the standard deviation, and the number of
501   features. The minimum, average, and maximum fitness values are also taken into account.
502   The population size for this experiment was set at 10, and the maximum number of iterations
503   was 100. The suggested 10 different CTSO results were contrasted with standard TSO. Tables 22
504   and 23 for each of the 10 benchmark datasets show the proposed CTSO strategy and all other
505   techniques investigated, together with their minimum and average fitness values. When Table 22
506   is examined, the CTSO-9 method gave the best results in 5 of the 10 data sets and came first. It
507   was followed by the CTSO-3 method, which gave the best results in 4 out of 10 datasets. When
508   Table 23 is examined, CTSO-1, CTSO-3, CTSO-5, and CTSO-9 methods achieved the best results
509   in 2 out of 10 data sets. CTSO-2 and CTSO-8 did not find good results on any dataset.
510   Tables 24 and 25 for each of the 10 benchmark datasets show the proposed CTSO strategy and
511   all other techniques investigated, together with their average and maximum accuracy measures.
512   When Table 24 is examined, WOA method achieved the best results in 4 out of 10 data sets and
513   were ranked first according to the average accuracy measure value. The CTSO-1, CTSO-2, CTSO-
514   3, CTSO-4, CTSO-5, CTSO-8, TSO, and AO methods did not produce a high value in any of the
515   10 datasets. When Table 25 is examined, the CTSO-9 method obtained the highest value in 5 out
516   of 10 data sets according to the maximum accuracy metric. It is followed by CTSO-6 and CTSO-

517    7, producing the highest value in 4 out of 10 datasets. The CTSO-2, CTSO-6, TSO, AO, SMA,
518    and WOA methods could not gain superiority in any data set.
519      Table 26 presents a comparison of CTSO approaches and competitive metaheuristic methods
520    according to selected features. When table 26 is examined, it is seen that the SMA method chooses
521    the least feature in 5 out of 10 data sets. In addition, it was seen that CTSO-9, which obtained the
522    highest accuracy value in the majority of data sets, obtained the highest results by choosing an
523    average number of features compared to other methods.

525    **7. Conclusion**

527      TSO is a recently developed physics-based method that draws inspiration from the transient
528    behavior of switched electrical circuits with storage components like inductance and capacitance.
529    Our motivation was that this method is very new, tends to be stuck with local optimal solutions,
530    has difficulties maintaining the balance between exploration and exploitation, and is not working
531    to improve the performance of TSO. In this paper, a number of chaotic maps have been
532    incorporated in order to customize the TSO settings. By using chaotic maps with ergodic, irregular,
533    and stochastic features in chaotic map TSO, it is aimed at avoiding local solutions more easily
534    compared to the TSO method. In this way, the balance between exploration and exploitation has
535    been achieved, and global convergence has increased. Ten distinct chaotic maps have been studied
536    on twenty-nine different benchmark test functions, five different real-world engineering problems,
537    and 10 different UCI standard datasets for feature selection. The findings demonstrated that the
538    use of chaotic maps is capable of greatly enhancing TSO's overall performance in a general sense.
539    The approach that employs the Gauss and sinusoidal maps for supplying the values of parameter
540    $c_1$ has been determined to be the best chaotic TSO after being put through a series of comparisons
541    with other chaotic TSOs. Gaussian and sinusoidal maps create a better balance between
542    exploration and exploitation, which prevents the algorithm from being stuck in a local optimal
543    state while it is in the process of optimizing its parameters. This is the reason for the enhanced
544    performance. The results of the simulation demonstrated that Gaussian and Sinusoidal maps
545    performed better than the standard TSO for the majority of the benchmark functions, that
546    Sinusoidal maps performed better than the standard TSO for the majority of the real-world
547    engineering problems, and that CTSOs generally suggested in the feature selection performed
548    better than the standard TSO overall. According to the findings of the statistical analysis,
549    customized algorithms clearly improve both the dependability of the global optimality and the
550    quality of the outcomes.
551      Due to the fact that chaotic TSOs are relatively new, one potentially fruitful direction for future
552    study would be to investigate how these algorithms perform in a parallel or distributed setting. The
553    proposed CTSO method can also be applied promisingly in many different fields. Among them,
554    many different fields such as machine learning, artificial intelligence, image processing, different
555    real-world engineering problems, data mining, and big data are suitable for the application of
556    algorithms. Like other proposed metaheuristic optimization methods, CTSO cannot guarantee that
557    its solution will always equal the global optimum, due to its stochastic nature.

559    **Appendix 1: Speed reducer problem**

560    **Minimize:**

562 $f(x) = 0.7854x_2^2x_1(14.9334x_3 - 43.0934 + 3.3333x_3^2) + 0.7854(x_5x_7^2 + x_4x_6^2) - 1.508x_1(x_7^2 + x_6^2) + 7.477(x_7^3$

563 $+ x_6^3)$

565 **Subject to:**

567 $g_1(x) = -x_1x_2^2x_3 + 27 \leq 0$

569 $g_2(x) = -x_1x_2^2x_3^2 + 397.5 \leq 0$

571 $g_3(x) = -x_2x_6^4x_3x_4^{-3} + 1.93 \leq 0$

573 $g_4(x) = -x_2x_7^4x_3x_5^{-3} + 1.93 \leq 0$

574 $g_5(x) = 10x_6^{-3}\sqrt{16.91 \times 10^6 + (745x_4x_2^{-1}x_3^{-1})^2} - 1100 \leq 0$

576 $g_6(x) = 10x_7^{-3}\sqrt{157.5 \times 10^6 + (745x_5x_2^{-1}x_3^{-1})^2} - 850 \leq 0$

577 $g_7(x) = x_2x_3 - 40 \leq 0$

579 $g_8(x) = -x_1x_2^{-1} + 5 \leq 0$

581 $g_9(x) = x_1x_2^{-1} - 12 \leq 0$

583 $g_{10}(x) = 1.5x_6 - x_4 + 1.9 \leq 0$

584 $g_{11}(x) = 1.1x_7 - x_5 + 1.9 \leq 0$

586 **with bounds:**

588 $0.7 \leq x_2 \leq 0.8, 17 \leq x_3 \leq 28, 2.6 \leq x_1 \leq 3.6, 5 \leq x_7 \leq 5.5, 7.3 \leq x_5, x_4 \leq 8.3, 2.9 \leq x_6 \leq 3.9$

590 **Appendix 2: Tension compression spring design problem**

591 **Minimize:**

593 $f(x) = (x_3 + 2)x_1^2x_2$

595 **Subject to:**

597 $g_1(x) = 1 - \frac{x_2^3x_3}{71785x_1^4} \leq 0$

599 $g_2(x) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} \leq 0$

601 $g_3(x) = 1 - \frac{140.45x_1}{x_2^3x_3} \leq 0$

603 $g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \leq 0$

605 **with bounds:**

607 $0.05 \leq x_1 \leq 2.00, 0.25 \leq x_2 \leq 1.30, 2.00 \leq x_3 \leq 15.0$

608     **Appendix 3: Welded beam design problem**

609     **Minimize:**
610
611     $f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2)$
612
613     **Subject to:**
614
615     $g_1(x) = \tau(x) - \tau_{max} \le 0$
616
617     $g_2(x) = \sigma(x) - \sigma_{max} \le 0$
618
619     $g_3(x) = x_1 - x_4 \le 0$
620
621     $g_4(x) = 1.10471x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \le 0$
622
623     $g_5(x) = 0.125 - x_1 \le 0$
624
625     $g_6(x) = \delta(x) - \delta_{max} \le 0$
626
627      $g_7(x) = P - P_c(x) \le 0$
628
629     where;

630     $\tau(x) = \sqrt{(\tau')^2 + (2\tau'\tau'')\frac{x_2}{2R} + (\tau'')^2}, \tau' = \frac{6000}{\sqrt{2}x_1x_2}, \tau'' = \frac{MR}{J}, M = 6000(14.0 + \frac{x_2}{2}), R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$

631

632     $J = \left\{x_1x_2\sqrt{2}\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}, \sigma(x) = \frac{504000}{x_4x_3^2}, \delta(x) = \frac{2.1952}{x_4x_3^3}, P_c(x) = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{196}\left(1 - \frac{x_3\sqrt{\frac{E}{4G}}}{28}\right)$

633

634     $P_c(x) = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{196}\left(1 - \frac{x_3\sqrt{\frac{E}{4G}}}{28}\right)$

635     $\tau_{max} = 13600psi, \sigma_{max} = 30000psi, \delta_{max} = 0.25in, E = 30 \times 10^6psi, G = 12 \times 10^6psi$
636
637     **with bounds:**
638
639      $0.125 \le x_1 \le 2, 0.1 \le x_4 \le 2$ ve $0.1 \le x_2, x_3 \le 10$
640
641     **Appendix 4: Pressure vessel problem**

642     **Minimize:**
643
644     $f(x) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3$
645
646     **Subject to:**
647
648     $g_1(x) = -x_1 + 0.0193x_3 \le 0$
649     $g_2(x) = -x_2 + 0.00954x_3 \le 0$

650     $g_3(x) = -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0$
651
652     $g_4(x) = x_4 - 240 \le 0$
653

654 **with bounds:**

655

656 $0.51 \leq x_1, x_2 \leq 99.49, 10 \leq x_3, x_4 \leq 200$

**Appendix 5: Three bar truss design problem**

658 **Minimize:**

659 $f(x) = l(x_2 + 2\sqrt{2x_1})$

660

661 **Subject to:**

662 $g_1(x) = \dfrac{x_2}{2x_2 x_1 + \sqrt{2x_1^2}} P - \sigma \leq 0$

663 $g_2(x) = \dfrac{x_2 + \sqrt{2x_1}}{2x_2 x_1 + \sqrt{2x_1^2}} P - \sigma \leq 0$

664 $g_3(x) = \dfrac{1}{x_1 + \sqrt{2x_2}} P - \sigma \leq 0$

665 **where**

666 $l = 100, P = 2, and \ \sigma = 2$

667 **with bounds:**

668

669 $0 \leq x_1, x_2 \leq 1$

670

671 **References**

672 **Abdollahzadeh B, Gharehchopogh FS, Mirjalili S**. 2021. African vultures optimization algorithm: A new nature-inspired metaheuristic
673     algorithm for global optimization problems. *Computers and Industrial Engineering* **158**:107408. DOI: 10.1016/j.cie.2021.107408.

674 **Abualigah L, Diabat A**. 2022. Chaotic binary Group Search Optimizer for feature selection. *Expert Systems with Applications* **192**:116368. DOI:
675     10.1016/j.eswa.2021.116368.

676 **Abualigah L, Diabat A, Mirjalili S, Abd Elaziz M, Gandomi AH**. 2021a. The Arithmetic Optimization Algorithm. *Computer Methods in*
677     *Applied Mechanics and Engineering* **376**:113609. DOI: 10.1016/j.cma.2020.113609.

678 **Abualigah L, Yousri D, Abd Elaziz M, Ewees AA, Al-qaness MAA, Gandomi AH**. 2021b. Aquila Optimizer: A novel meta-heuristic
679     optimization algorithm. *Computers and Industrial Engineering* **157**:107250. DOI: 10.1016/j.cie.2021.107250.

680 **Agrawal A, Tripathi S**. 2021. Particle swarm optimization with adaptive inertia weight based on cumulative binomial probability. *Evolutionary*
681     *Intelligence*. **14**:305-313. DOI: 10.1007/s12065-018-0188-7.

682 **Agushaka JO, Ezugwu AE, Abualigah L**. 2022. Dwarf Mongoose Optimization Algorithm. *Computer Methods in Applied Mechanics and*
683     *Engineering* **391**:114570. DOI: 10.1016/j.cma.2022.114570.

684 **Ahmadianfar I, Bozorg-Haddad O, Chu X**. 2020. Gradient-based optimizer: A new metaheuristic optimization algorithm. *Information Sciences*
685     **540**:131–159. DOI: 10.1016/j.ins.2020.06.037.

686 **Altay EV**. 2022a. Hybrid Archimedes optimization algorithm enhanced with mutualism scheme for global optimization problems. *Artificial*
687     *Intelligence Review* 1-62. DOI: 10.1007/s10462-022-10340-z.

688 **Altay EV**. 2022b. Solution of Process Synthesis Problem Using Metaheuristic Optimization Algorithms. In: 2022 Innovations in Intelligent
689     Systems and Applications Conference (ASYU). IEEE, 1-5.

690 **Altay O**. 2022c. Chaotic slime mould optimization algorithm for global optimization. *Artificial Intelligence Review* **55(5)**:3979-4040. DOI:
691     10.1007/s10462-021-10100-5.

692 **Altay EV, Alatas B**. 2020. Randomness as source for inspiring solution search methods: Music based approaches. *Physica A: Statistical*
693     *Mechanics and its Applications* **537**:122650. DOI: 10.1016/j.physa.2019.122650.

694 **Altay EV, Alatas B**. 2021. Differential evolution and sine cosine algorithm based novel hybrid multi-objective approaches for numerical
695     association rule mining. *Information Sciences* **554**:198–221. DOI: 10.1016/j.ins.2020.12.055.

696 **Altay O, Varol E**. 2023. A novel hybrid multilayer perceptron neural network with improved grey wolf optimizer. *Neural Computing and*
697   *Applications* **35**:529–556. DOI: 10.1007/s00521-022-07775-4.

698 **Arora J**. 2004. Introduction to optimum design.Elsevier.

699 **Arora S, Anand P**. 2019. Chaotic grasshopper optimization algorithm for global optimization. *Neural Computing and Applications* **31**:4385–
700   4405. DOI: 10.1007/s00521-018-3343-2.

701 **Balochian S, Baloochian H**. 2019. Social mimic optimization algorithm and engineering applications. *Expert Systems with Applications*
702   **134**:178–191. DOI: 10.1016/j.eswa.2019.05.035.

703 **Baykaso A**. 2015. Weighted Superposition Attraction ( WSA ): A swarm intelligence algorithm for optimization problems – Part 2 : Constrained
704   optimization. *Applied Soft Computing* **37**:396–415. DOI: 10.1016/j.asoc.2015.08.052.

705 **Bhadoriya JS, Gupta AR**. 2022. A novel transient search optimization for optimal allocation of multiple distributed generator in the radial
706   electrical distribution network. *International Journal of Emerging Electric Power Systems* **23**:23–45.

707 **Bianchi L, Dorigo M, Gambardella LM, Gutjahr WJ**. 2009. A survey on metaheuristics for stochastic combinatorial optimization. *Natural*
708   *Computing* **8**:239–287. DOI: 10.1007/s11047-008-9098-4.

709 **Bingol H, Alatas B**. 2020. Chaos based optics inspired optimization algorithms as global solution search approach. *Chaos, Solitons and Fractals*
710   **141**:110434. DOI: 10.1016/j.chaos.2020.110434.

711 **Birbil ŞI, Fang SC**. 2003. An electromagnetism-like mechanism for global optimization. *Journal of Global Optimization* **25**:263–282. DOI:
712   10.1023/A:1022452626305.

713 **Chakrabarty J, Lahiri SK**. 2023. A comparative study of metaheuristics algorithms based on their performance of complex benchmark
714   problems. *Decision Making: Applications in Management and Engineering* **6(1)**:341–364.

715 **Czerniak JM, Zarzycki H, Ewald D**. 2017. Simulation Modelling Practice and Theory AAO as a new strategy in modeling and simulation of
716   constructional problems optimization. *Simulation Modelling Practice and Theory* **76**:22–33. DOI: 10.1016/j.simpat.2017.04.001.

717 **Daliri A, Asghari A**. 2022. The water optimization algorithm : a novel metaheuristic for solving optimization problems. *Applied Intelligence*
718   **52**:17990–18029.

719 **Das M, Roy A, Maity S, Kar S, Sengupta S**. 2022. Solving fuzzy dynamic ship routing and scheduling problem through modified genetic
720   algorithm. *Decision Making: Applications in Management and Engineering* **5(2)**:329–361.

721 **Demir FB, Tuncer T, Kocamaz AF**. 2020. A chaotic optimization method based on logistic-sine map for numerical function optimization.
722   *Neural Computing and Applications* **32**:14227–14239. DOI: 10.1007/s00521-020-04815-9.

723 **Devan PAM, Hussin FA, Ibrahim RB, Bingi K, Nagarajapandian M, Assaad M**. 2022. An Arithmetic-Trigonometric Optimization
724   Algorithm with Application for Control of Real-Time Pressure Process Plant. *Sensors* **22**:1–26. DOI: 10.3390/s22020617.

725 **Dhiman G**. 2021. ESA: a hybrid bio-inspired metaheuristic optimization approach for engineering problems. *Engineering with Computers*
726   **37**:323-353.DOI: 10.1007/s00366-019-00826-w.

727 **Erol OK, Eksin I**. 2006. A new optimization method: Big Bang-Big Crunch. *Advances in Engineering Software* **37**:106–111. DOI:
728   10.1016/j.advengsoft.2005.04.005.

729 **Farah A, Belazi A**. 2018. A novel chaotic Jaya algorithm for unconstrained numerical optimization. *Nonlinear Dynamics* **93**:1451–1480. DOI:
730   10.1007/s11071-018-4271-5.

731 **Faramarzi A, Heidarinejad M, Stephens B, Mirjalili S**. 2020. Equilibrium optimizer: A novel optimization algorithm. *Knowledge-Based*
732   *Systems* **191**:105190. DOI: 10.1016/j.knosys.2019.105190.

733 **Fatani A, Elaziz MABD, Dahou A, Al-qaness MAA, Lu S**. 2021. IoT Intrusion Detection System Using Deep Learning and Enhanced
734   Transient Search Optimization. *IEEE Access* **9**:123448–123464. DOI: 10.1109/ACCESS.2021.3109081.

735 **Formato RA**. 2009. Central force optimization: A new deterministic gradient-like optimization metaheuristic. *Opsearch* **46**:25–51. DOI:
736   10.1007/s12597-009-0003-4.

737 **Gandomi AH, Yang XS, Alavi AH**. 2013. Cuckoo search algorithm: A metaheuristic approach to solve structural optimization problems.
738   *Engineering with Computers* **29**:17–35. DOI: 10.1007/s00366-011-0241-y.

739 **Gharehchopogh FS, Maleki I, Dizaji ZA**. 2021. *Chaotic vortex search algorithm: metaheuristic algorithm for feature selection. Evolutionary*
740   *Intelligence* **15(3)**:1777-1808. DOI: 10.1007/s12065-021-00590-1.

741 **Hasanien HM, Shaheen MAM, Turky RA, Qais MH, Jurado F, Alghuwainem S, Kamel S, Tostado-v M**. 2022. Precise modeling of PEM
742   fuel cell using a novel Enhanced Transient Search Optimization algorithm. *Energy* **247**:123530. DOI: 10.1016/j.energy.2022.123530.

743 **Hashim FA, Houssein EH, Mabrouk MS, Al-Atabany W, Mirjalili S**. 2019. Henry gas solubility optimization: A novel physics-based
744   algorithm. *Future Generation Computer Systems* **101**:646–667. DOI: 10.1016/j.future.2019.07.015.

745  **Hashim FA, Hussain K, Houssein EH, Mabrouk MS, Al-Atabany W**. 2021. Archimedes optimization algorithm: a new metaheuristic
746      algorithm for solving optimization problems. *Applied Intelligence* **51**:1531–1551. DOI: 10.1007/s10489-020-01893-z.

747  **He X, Zhou Y**. 2018. Enhancing the performance of differential evolution with covariance matrix self-adaptation. *Applied Soft Computing*
748      *Journal* **64**:227–243. DOI: 10.1016/j.asoc.2017.11.050.

749  **Heidari AA, Mirjalili S, Faris H, Aljarah I, Mafarja M, Chen H**. 2019. Harris hawks optimization: Algorithm and applications. *Future*
750      *Generation Computer Systems* **97**:849–872. DOI: 10.1016/j.future.2019.02.028.

751  **Hosseini HS**. 2011. Principal components analysis by the galaxy-based search algorithm: a novel metaheuristic for continuous optimisation.
752      *International Journal of Computational Science and Engineering* **6**:132. DOI: 10.1504/ijcse.2011.041221.

753  **Karami H, Anaraki MV, Farzin S, Mirjalili S.** 2021. Flow Direction Algorithm (FDA): A Novel Optimization Approach for Solving
754      Optimization Problems. *Computers and Industrial Engineering* **156**:107224. DOI: 10.1016/j.cie.2021.107224.

755  **Kaur G, Arora S**. 2018. Chaotic whale optimization algorithm. *Journal of Computational Design and Engineering* **5**:275–284. DOI:
756      10.1016/j.jcde.2017.12.006.

757  **Khishe M, Mosavi MR**. 2020. Chimp optimization algorithm. *Expert Systems with Applications* **149**:113338. DOI: 10.1016/j.eswa.2020.113338.

758  **Kohli M, Arora S**. 2018. Journal of Computational Design and Engineering Chaotic grey wolf optimization algorithm for constrained
759      optimization problems. *Journal of Computational Design and Engineering* **5**:458–472. DOI: 10.1016/j.jcde.2017.02.005.

760  **Kutlu Onay F, Aydemïr SB**. 2022. Chaotic hunger games search optimization algorithm for global optimization and engineering problems.
761      *Mathematics and Computers in Simulation* **192**:514–536. DOI: 10.1016/j.matcom.2021.09.014.

762  **Li H, Liu J, Chen L, Bai J, Sun Y, Lu K**. 2019. Chaos-enhanced moth-flame optimization algorithm for global optimization. *Journal of Systems*
763      *Engineering and Electronics* **30**:1144–1159. DOI: 10.21629/JSEE.2019.06.10.

764  **Li X-D, Wang J-S, Hao W-K, Zhang M, Wang M**. 2022. Chaotic arithmetic optimization algorithm. *Applied Intelligence* **52**:16718-16757.
765      DOI: 10.1007/s10489-021-03037-3.

766  **Madić M, Gostimirović M, Rodić D, Radovanović M, Coteaţă M**. 2022. Mathematical modelling of the CO2 laser cutting process using
767      genetic programming. *Facta Universitatis, Series: Mechanical Engineering* **20(3)**:665–676.

768  **Meng XB, Li HX, Gao XZ**. 2019. An adaptive reinforcement learning-based bat algorithm for structural design problems. *International Journal*
769      *of Bio-Inspired Computation* **14**:114–124. DOI: 10.1504/IJBIC.2019.101639.

770  **Mirjalili S**. 2015. Moth-flame optimization algorithm: A novel nature-inspired heuristic paradigm. *Knowledge-Based Systems* **89**:228–249. DOI:
771      10.1016/j.knosys.2015.07.006.

772  **Mirjalili S**. 2016. SCA: A Sine Cosine Algorithm for solving optimization problems. *Knowledge-Based Systems* **96**:120–133. DOI:
773      10.1016/j.knosys.2015.12.022.

774  **Mirjalili S, Gandomi AH, Mirjalili SZ, Saremi S, Faris H, Mirjalili SM**. 2017. Salp Swarm Algorithm: A bio-inspired optimizer for
775      engineering design problems. *Advances in Engineering Software* **114**:163–191. DOI: 10.1016/j.advengsoft.2017.07.002.

776  **Mirjalili S, Lewis A**. 2016. The Whale Optimization Algorithm. *Advances in Engineering Software* **95**:51–67. DOI:
777      10.1016/j.advengsoft.2016.01.008.

778  **Mirjalili S, Mirjalili SM, Hatamlou A**. 2016. Multi-Verse Optimizer: a nature-inspired algorithm for global optimization. *Neural Computing*
779      *and Applications* **27**:495–513. DOI: 10.1007/s00521-015-1870-7.

780  **Mirjalili S, Mirjalili SM, Lewis A**. 2014. Grey Wolf Optimizer. *Advances in Engineering Software* **69**:46–61. DOI:
781      10.1016/j.advengsoft.2013.12.007.

782  **Mirrashid M, Naderpour H**. 2022. Transit search: An optimization algorithm based on exoplanet exploration. *Results in Control and*
783      *Optimization* **7**:100127. DOI: 10.1016/j.rico.2022.100127.

784  **Mohammed HM, Rashid TA**. 2021. Chaotic fitness-dependent optimizer for planning and engineering design. *Soft Computing* **25**:14281–14295.
785      DOI: 10.1007/s00500-021-06135-z.

786  **Mzili T, Riffi ME, Mzili I**. 2022. A novel dıscrete rat swarm optımızatıon ( drso ) algorıthm for solvıng the travelıng salesman problem.
787      *Decision Making: Applications in Management and Engineering* **5**:287–299.

788  **Negi G, Kumar A, Pant S, Ram M**. 2021. Optimization of complex system reliability using hybrid grey wolf optimizer. *Decision Making:*
789      *Applications in Management and Engineering* **4(2)**:241–256.

790  **Pierezan J, dos Santos Coelho L, Cocco Mariani V, Hochsteiner de Vasconcelos Segundo E, Prayogo D**. 2021. Chaotic coyote algorithm
791      applied to truss optimization problems. *Computers and Structures* **242**:106353. DOI: 10.1016/j.compstruc.2020.106353.

792  **Prayogo D, Cheng MY, Wu YW, Herdany AA, Prayogo H**. 2018. Differential Big Bang - Big Crunch algorithm for construction-engineering
793      design optimization. *Automation in Construction* **85**:290–304. DOI: 10.1016/j.autcon.2017.10.019.

794  **Qais MH, Hasanien HM**. 2020. Optimal Transient Search Algorithm-Based PI Controllers for Enhancing Low Voltage Ride-Through Ability of
795  Grid-Linked PMSG-Based Wind Turbine. *Electronics* **9(11)**:1807.

796  **Qais MH, Hasanien HM, Alghuwainem S**. 2020. Transient search optimization: a new meta-heuristic optimization algorithm. *Applied*
797  *Intelligence* **50**:3926–3941. DOI: 10.1007/s10489-020-01727-y.

798  **Qais MH, Hasanien HM, Turky RA, Alghuwainem S, Tostado-v M, Jurado F**. 2022. Circle Search Algorithm : A Geometry-Based
799  Metaheuristic Optimization Algorithm. *Mathematics* **10(10)**:1626.

800  **Rizk-Allah RM, Hassanien AE, Bhattacharyya S**. 2018. Chaotic crow search algorithm for fractional optimization problems. *Applied Soft*
801  *Computing Journal* **71**:1161–1175. DOI: 10.1016/j.asoc.2018.03.019.

802  **Sadollah A, Bahreininejad A, Eskandar H, Hamdi M**. 2013. Mine blast algorithm : A new population based algorithm for solving constrained
803  engineering optimization problems. *Applied Soft Computing Journal* **13**:2592–2612. DOI: 10.1016/j.asoc.2012.11.026.

804  **Sayed GI, Darwish A, Hassanien AE**. 2018. A new chaotic multi-verse optimization algorithm for solving engineering optimization problems.
805  *Journal of Experimental and Theoretical Artificial Intelligence* **30**:293–317. DOI: 10.1080/0952813X.2018.1430858.

806  **Sayed GI, Khoriba G, Haggag MH**. 2018. A novel chaotic salp swarm algorithm for global optimization and feature selection. *Applied*
807  *Intelligence* **48**:3462–3481. DOI: 10.1007/s10489-018-1158-6.

808  **Sayed GI, Tharwat A, Hassanien AE**. 2019. Chaotic dragonfly algorithm: an improved metaheuristic algorithm for feature selection. *Applied*
809  *Intelligence* **49**:188–205. DOI: 10.1007/s10489-018-1261-8.

810  **Talatahari S, Azizi M, Gandomi AH**. 2021. Material generation algorithm: A novel metaheuristic algorithm for optimization of engineering
811  problems. *Processes* **9**:1–35. DOI: 10.3390/pr9050859.

812  **Tam JH, Chao Ong Z, Ismail Z, Chin B, Shin A, Khoo Y, Ang BC, Khoo SY**. 2019. International Journal of Computer Mathematics A new
813  hybrid GA−ACO−PSO algorithm for solving various engineering design problems A new hybrid GA−ACO−PSO algorithm for solving
814  various engineering design problems. *International Journal Of Computer Mathematics* **96**:883–919. DOI:
815  10.1080/00207160.2018.1463438.

816  **Tuba E, Capor-Hrosik R, Alihodzic A, Jovanovic R, Tuba M**. 2018. Chaotic elephant herding optimization algorithm. *SAMI 2018 - IEEE 16th*
817  *World Symposium on Applied Machine Intelligence and Informatics Dedicated to the Memory of Pioneer of Robotics Antal (Tony) K.*
818  *Bejczy, Proceedings* 2018-Febru:213–216. DOI: 10.1109/SAMI.2018.8324842.

819  **Tzanetos A, Dounias G**. 2020. Sonar inspired optimization (SIO) in engineering applications. *Evolving Systems* **11**:531–539. DOI:
820  10.1007/s12530-018-9250-z.

821  **Varol Altay E, Alatas B**. 2020. Bird swarm algorithms with chaotic mapping. *Artificial Intelligence Review* **53**:1373–1414. DOI:
822  10.1007/s10462-019-09704-9.

823  **Wang X, Deng Y, Duan H**. 2018. Edge-based target detection for unmanned aerial vehicles using competitive Bird Swarm Algorithm.
824  *Aerospace Science and Technology* **78**:708-720.

825  **Yang X, Gandomi A, Talatahari S, Alavi A**. 2012. Metaheuristics in water, geotechnical and transport engineering.Newnes.

826  **Yang L, Gao S, Yang H, Cai Z, Lei Z, Todo Y**. 2021. Adaptive chaotic spherical evolution algorithm. *Memetic Computing* **13**:383–411. DOI:
827  10.1007/s12293-021-00341-w.

828  **Zhang Q, Chen H, Luo J, Xu Y, Wu C, Li C**. 2018. Chaos enhanced bacterial foraging optimization for global optimization. *IEEE Access*
829  **6**:64905–64919. DOI: 10.1109/ACCESS.2018.2876996.

830  **Zhang C, Ding S**. 2021. A stochastic configuration network based on chaotic sparrow search algorithm. *Knowledge-Based Systems* **220**:106924.
831  DOI: 10.1016/j.knosys.2021.106924.

832  **Zhang Y, Jin Z**. 2020. Group teaching optimization algorithm: A novel metaheuristic method for solving global optimization problems. *Expert*
833  *Systems with Applications* **148**:113246. DOI: 10.1016/j.eswa.2020.113246.

834  **Zolfi K**. 2023. Gold rush optimizer. A new population-based metaheuristic algorithm. *Operations Research and Decisions* **33**(1):113-150. DOI:
835  10.37190/ord230108.

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Literature review on chaos theorem of metaheuristic methods

| Ref. | Year | Proposed Model | Chaotic Maps | Application |
|---|---|---|---|---|
| (Farah & Belazi, 2018) | 2018 | Jaya algorithm | 2D cross chaotic map | Benchmark function |
| (Zhang et al., 2018) | 2018 | Bacterial Foraging Optimization | Logistic map | Benchmark function |
| (Sayed, Khoriba & Haggag, 2018) | 2018 | Salp swarm algorithm | Ten different chaotic maps | Benchmark function and feature selection |
| (Tuba et al., 2018) | 2018 | Elephant Herding Optimization | Two different chaotic maps | Benchmark function |
| (Rizk-Allah, Hassanien & Bhattacharyya, 2018) | 2018 | Crow search algorithm | Ten different chaotic maps | Benchmark function and real-world engineering design problem |
| (Kaur & Arora, 2018) | 2018 | Whale optimization algorithm | Ten different chaotic maps | Benchmark function |
| (Sayed, Darwish & Hassanien, 2018) | 2018 | Multi-verse optimization algorithm | Ten different chaotic maps | Real-world engineering design problem |
| (Arora & Anand, 2019) | 2018 | Grasshopper optimization algorithm | Ten different chaotic maps | Benchmark function |
| (Li et al., 2019) | 2019 | Moth-flame optimization | Ten different chaotic maps | Benchmark function and real-world engineering design problem |
| (Sayed, Tharwat & Hassanien, 2019) | 2019 | Dragonfly algorithm | Ten different chaotic maps | Feature selection |
| (Demir, Tuncer & Kocamaz, 2020) | 2020 | Chaotic Optimization Algorithm | Logistic-sine chaotic map | Benchmark function and real-world engineering design problem |
| (Bingol & Alatas, 2020) | 2020 | Optics inspired optimization | Five different chaotic maps | Benchmark function and real-world engineering design problem |
| (Varol Altay & Alatas, 2020) | 2020 | Bird Swarm Algorithm | Ten different chaotic maps | Benchmark function and real-world engineering design problems |
| (Pierezan et al., 2021) | 2020 | Coyote Optimization Algorithm | Tinkerbell chaotic map | Truss optimization problems |
| (Gharehchopogh, Maleki & Dizaji, 2021) | 2021 | Vortex search algorithm | Ten different chaotic maps | Feature selection |
| (Yang et al., 2021) | 2021 | Spherical evolution algorithm | Twelve different chaotic maps | Benchmark function |
| (Mohammed & Rashid, 2021) | 2021 | Fitness-dependent optimizer | Ten different chaotic maps | Benchmark function and real-world engineering design problems |
| (Zhang & Ding, 2021) | 2021 | Sparrow search algorithm | Logistic map | Benchmark function and stochastic configuration network |
| (Li et al., 2022) | 2021 | Arithmetic optimization algorithm | Ten different chaotic maps | Benchmark function and real-world engineering design problems |
| (Kutlu Onay & Aydemir, 2022) | 2021 | Hunger games search optimization | Ten different chaotic maps | Benchmark function and real-world engineering design problems |
| (Altay, 2022c) | 2022 | Slime mould optimization | Ten different chaotic maps | Benchmark function and real-world engineering design problems |
| (Abualigah & Diabat, 2022) | 2022 | Group Search Optimizer | Five different chaotic maps | Feature selection |

Variables and equations of chaotic maps

| CM No | CM Name | CM Equation |
|-------|---------|-------------|
| 1 | Chebyshev map | $X_{n+1} = \cos(k\cos^{-1} x_n)$ |
| 2 | Circle map | $X_{n+1} = X_n + b - \left(\frac{a}{2\pi}\right)\sin(2\pi X_n) \bmod(1)\ a = 0.5$ and $b = 0.2$ |
| 3 | Gauss map | $X_{n+1} = \begin{cases} 0, & X_n = 0 \\ \frac{1}{X_n \bmod(1)}, & X_n \in (0,1) \end{cases}$, $\frac{1}{X_n \bmod(1)} = \frac{1}{X_n} - \left\lfloor \frac{1}{X_n} \right\rfloor$ |
| 4 | Iterative map | $X_{n+1} = \sin\left(\frac{a\pi}{x_n}\right), a = 0.7$ |
| 5 | Logistic map | $X_{n+1} = aX_n(1 - X_n), a = 4$ |
| 6 | Piecewise map | $x_{n+1} = \begin{cases} \frac{x_n}{P}, & 0 \le x_n < P \\ \frac{x_n - P}{0.5 - P}, & P \le x_n < 0.5 \\ \frac{1 - P - x_n}{0.5 - P}, & 0.5 \le x_n < 1 - P \\ \frac{1 - x_n}{P}, & 1 - P \le x_n < 1 \end{cases}$, $P = 0.4$ |
| 7 | Sine map | $X_{n+1} = \frac{a}{4}\sin(\pi x_n), 0 < a \le 4$ |
| 8 | Singer map | $X_{n+1} = \mu(7.86x_n - 23.31x_n^2 + 28.75x_n^3 - 13.302875x_n^4), \mu=1.07$ |
| 9 | Sinusoidal map | $X_{n+1} = ax_n^2\sin(\pi X_n), a = 2.3$ and $X_0 = 0.7$ |
| 10 | Tent map | $f(x) = \begin{cases} X_n/0.7, & X_n < 0 \\ 10/3\ X_n(1 - X_n), & otherwise \end{cases}$ |

Experimental results on CEC'17 test functions

1  Table 3-A

| | f1 | | | | | f3 | | | |
|---|---|---|---|---|---|---|---|---|---|
| Algorithm | AVG | STD | MIN | MR | Algorithm | AVG | STD | MIN | MR |
| TSO | 4.95E+10 | 8.21E+09 | 2.48E+10 | 7.20 | TSO | 9.11E+04 | 3.47E+03 | 8.10E+04 | 5.47 |
| CTSO-1 | 4.92E+10 | 8.39E+09 | 3.13E+10 | 7.13 | CTSO-1 | 9.14E+04 | 3.08E+03 | 8.21E+04 | 5.13 |
| CTSO-2 | 4.99E+10 | 8.71E+09 | 3.24E+10 | 7.33 | CTSO-2 | 9.09E+04 | 3.33E+03 | 8.32E+04 | 5.10 |
| CTSO-3 | **1.57E+08** | 1.45E+08 | 1.86E+07 | 1.53 | CTSO-3 | 1.55E+05 | 6.93E+04 | 6.08E+04 | 9.30 |
| CTSO-4 | 4.79E+10 | 6.77E+09 | 3.44E+10 | 6.33 | CTSO-4 | 9.13E+04 | 5.09E+03 | 7.18E+04 | 6.27 |
| CTSO-5 | 4.97E+10 | 7.32E+09 | 3.54E+10 | 6.87 | CTSO-5 | 9.02E+04 | 5.42E+03 | 6.97E+04 | **4.77** |
| CTSO-6 | 4.96E+10 | 7.20E+09 | 3.52E+10 | 6.90 | CTSO-6 | 9.19E+04 | 2.38E+03 | 8.70E+04 | 5.53 |
| CTSO-7 | 4.98E+10 | 9.49E+09 | 2.63E+10 | 7.20 | CTSO-7 | 9.06E+04 | 4.41E+03 | 7.63E+04 | 5.03 |
| CTSO-8 | 5.01E+10 | 9.07E+09 | 3.54E+10 | 6.80 | CTSO-8 | **9.04E+04** | 6.25E+03 | 6.91E+04 | 5.53 |
| CTSO-9 | 1.83E+08 | 2.95E+08 | 3.75E+07 | **1.47** | CTSO-9 | 1.36E+05 | 4.90E+04 | 6.55E+04 | 8.90 |
| CTSO-10 | 4.95E+10 | 8.31E+09 | 3.34E+10 | 7.23 | CTSO-10 | 9.13E+04 | 3.25E+03 | 8.17E+04 | 4.97 |

| | f4 | | | | | f5 | | | |
|---|---|---|---|---|---|---|---|---|---|
| Algorithm | AVG | STD | MIN | MR | Algorithm | AVG | STD | MIN | MR |
| TSO | 1.23E+04 | 3.21E+03 | 4.76E+03 | 7.17 | TSO | 9.12E+02 | 3.32E+01 | 8.37E+02 | 6.57 |
| CTSO-1 | 1.19E+04 | 2.29E+03 | 7.99E+03 | 6.67 | CTSO-1 | 9.19E+02 | 3.13E+01 | 8.51E+02 | 7.77 |
| CTSO-2 | 1.20E+04 | 2.21E+03 | 7.98E+03 | 6.83 | CTSO-2 | 9.06E+02 | 4.49E+01 | 8.16E+02 | 6.30 |
| CTSO-3 | 6.16E+02 | 6.91E+01 | 5.26E+02 | **1.50** | CTSO-3 | 7.94E+02 | 4.53E+01 | 7.10E+02 | 1.77 |
| CTSO-4 | 1.25E+04 | 2.95E+03 | 8.86E+03 | 7.30 | CTSO-4 | 9.16E+02 | 3.51E+01 | 8.30E+02 | 7.23 |
| CTSO-5 | 1.13E+04 | 3.14E+03 | 5.83E+03 | 6.20 | CTSO-5 | 9.09E+02 | 3.83E+01 | 7.97E+02 | 6.77 |
| CTSO-6 | 1.23E+04 | 2.99E+03 | 7.30E+03 | 7.47 | CTSO-6 | 9.17E+02 | 3.78E+01 | 8.13E+02 | 7.63 |
| CTSO-7 | 1.24E+04 | 3.15E+03 | 7.15E+03 | 7.13 | CTSO-7 | 9.07E+02 | 3.14E+01 | 8.41E+02 | 6.23 |
| CTSO-8 | 1.31E+04 | 2.72E+03 | 8.01E+03 | 7.83 | CTSO-8 | 9.20E+02 | 2.67E+01 | 8.76E+02 | 7.43 |
| CTSO-9 | **6.07E+02** | 4.59E+01 | 4.94E+02 | 1.50 | CTSO-9 | **7.83E+02** | 5.52E+01 | 6.87E+02 | **1.73** |
| CTSO-10 | 1.17E+04 | 3.39E+03 | 6.65E+03 | 6.40 | CTSO-10 | 9.10E+02 | 2.71E+01 | 8.34E+02 | 6.57 |

| | f6 | | | | | f7 | | | |
|---|---|---|---|---|---|---|---|---|---|
| Algorithm | AVG | STD | MIN | MR | Algorithm | AVG | STD | MIN | MR |
| TSO | 6.85E+02 | 8.78E+00 | 6.59E+02 | 7.23 | TSO | 1.44E+03 | 5.86E+01 | 1.31E+03 | 6.73 |
| CTSO-1 | 6.83E+02 | 8.22E+00 | 6.64E+02 | 6.17 | CTSO-1 | 1.43E+03 | 5.71E+01 | 1.31E+03 | 5.97 |
| CTSO-2 | 6.82E+02 | 7.18E+00 | 6.67E+02 | 5.97 | CTSO-2 | 1.43E+03 | 6.92E+01 | 1.24E+03 | 6.03 |
| CTSO-3 | **6.67E+02** | 1.08E+01 | 6.47E+02 | **2.17** | CTSO-3 | 1.46E+03 | 1.27E+02 | 1.25E+03 | 6.70 |
| CTSO-4 | 6.83E+02 | 7.81E+00 | 6.66E+02 | 6.07 | CTSO-4 | 1.43E+03 | 5.07E+01 | 1.26E+03 | 5.87 |
| CTSO-5 | 6.86E+02 | 8.16E+00 | 6.64E+02 | 7.17 | CTSO-5 | **1.41E+03** | 7.34E+01 | 1.25E+03 | **4.70** |
| CTSO-6 | 6.84E+02 | 1.02E+01 | 6.64E+02 | 6.37 | CTSO-6 | 1.43E+03 | 5.70E+01 | 1.30E+03 | 6.23 |
| CTSO-7 | 6.87E+02 | 8.70E+00 | 6.61E+02 | 7.73 | CTSO-7 | 1.42E+03 | 6.65E+01 | 1.22E+03 | 5.30 |
| CTSO-8 | 6.86E+02 | 6.16E+00 | 6.75E+02 | 7.20 | CTSO-8 | 1.43E+03 | 5.03E+01 | 1.31E+03 | 5.80 |
| CTSO-9 | 6.70E+02 | 1.04E+01 | 6.52E+02 | 2.73 | CTSO-9 | 1.44E+03 | 1.36E+02 | 1.18E+03 | 6.10 |
| CTSO-10 | 6.86E+02 | 7.40E+00 | 6.69E+02 | 7.20 | CTSO-10 | 1.45E+03 | 5.05E+01 | 1.30E+03 | 6.57 |

| | f8 | | | | | f9 | | | |
|---|---|---|---|---|---|---|---|---|---|
| Algorithm | AVG | STD | MIN | MR | Algorithm | AVG | STD | MIN | MR |
| TSO | 1.12E+03 | 2.52E+01 | 1.08E+03 | 6.63 | TSO | 1.05E+04 | 1.34E+03 | 8.17E+03 | 7.13 |
| CTSO-1 | 1.12E+03 | 3.33E+01 | 1.04E+03 | 6.50 | CTSO-1 | 1.03E+04 | 1.32E+03 | 7.94E+03 | 6.70 |
| CTSO-2 | 1.13E+03 | 2.10E+01 | 1.09E+03 | 7.93 | CTSO-2 | 1.04E+04 | 1.07E+03 | 8.43E+03 | 6.87 |
| CTSO-3 | **1.01E+03** | 4.72E+01 | 9.17E+02 | 1.73 | CTSO-3 | **7.25E+03** | 1.57E+03 | 4.58E+03 | **2.00** |
| CTSO-4 | 1.12E+03 | 2.95E+01 | 1.07E+03 | 6.43 | CTSO-4 | 1.01E+04 | 1.31E+03 | 6.02E+03 | 6.30 |
| CTSO-5 | 1.11E+03 | 3.19E+01 | 1.02E+03 | 5.63 | CTSO-5 | 1.03E+04 | 1.23E+03 | 7.82E+03 | 7.07 |
| CTSO-6 | 1.13E+03 | 2.50E+01 | 1.07E+03 | 7.37 | CTSO-6 | 1.03E+04 | 1.19E+03 | 7.59E+03 | 7.10 |
| CTSO-7 | 1.13E+03 | 2.54E+01 | 1.05E+03 | 6.97 | CTSO-7 | 1.05E+04 | 1.44E+03 | 7.58E+03 | 6.70 |
| CTSO-8 | 1.13E+03 | 2.69E+01 | 1.08E+03 | 7.73 | CTSO-8 | 1.08E+04 | 1.46E+03 | 7.90E+03 | 7.73 |
| CTSO-9 | **1.01E+03** | 4.35E+01 | 9.09E+02 | **1.50** | CTSO-9 | 7.43E+03 | 1.65E+03 | 4.89E+03 | 2.40 |
| CTSO-10 | 1.13E+03 | 1.82E+01 | 1.10E+03 | 7.57 | CTSO-10 | 9.85E+03 | 1.47E+03 | 7.05E+03 | 6.00 |

| | f10 | | | | | f11 | | | |
|---|---|---|---|---|---|---|---|---|---|
| Algorithm | AVG | STD | MIN | MR | Algorithm | AVG | STD | MIN | MR |
| TSO | 8.69E+03 | 5.78E+02 | 7.64E+03 | 5.73 | TSO | 9.85E+03 | 3.09E+03 | 4.06E+03 | 6.40 |
| CTSO-1 | 8.95E+03 | 6.98E+02 | 7.58E+03 | 6.83 | CTSO-1 | 1.05E+04 | 2.80E+03 | 5.00E+03 | 7.07 |
| CTSO-2 | 8.83E+03 | 8.47E+02 | 7.30E+03 | 6.77 | CTSO-2 | 9.79E+03 | 2.15E+03 | 6.53E+03 | 6.77 |
| CTSO-3 | **6.73E+03** | 1.33E+03 | 4.70E+03 | **2.37** | CTSO-3 | **1.68E+03** | 3.27E+02 | 1.24E+03 | **1.40** |
| CTSO-4 | 8.86E+03 | 5.27E+02 | 8.05E+03 | 6.43 | CTSO-4 | 1.04E+04 | 2.00E+03 | 6.26E+03 | 7.10 |
| CTSO-5 | 9.01E+03 | 6.78E+02 | 7.64E+03 | 7.40 | CTSO-5 | 1.01E+04 | 2.33E+03 | 3.00E+03 | 7.20 |
| CTSO-6 | 8.99E+03 | 7.03E+02 | 7.82E+03 | 7.33 | CTSO-6 | 9.57E+03 | 2.20E+03 | 4.63E+03 | 6.60 |
| CTSO-7 | 8.49E+03 | 5.62E+02 | 7.30E+03 | 5.40 | CTSO-7 | 1.01E+04 | 2.73E+03 | 4.57E+03 | 6.90 |
| CTSO-8 | 8.93E+03 | 7.24E+02 | 7.33E+03 | 7.27 | CTSO-8 | 1.04E+04 | 2.16E+03 | 5.38E+03 | 7.50 |
| CTSO-9 | 7.66E+03 | 1.41E+03 | 4.85E+03 | 4.03 | CTSO-9 | 2.52E+03 | 2.84E+03 | 1.33E+03 | 1.90 |
| CTSO-10 | 8.81E+03 | 6.64E+02 | 7.56E+03 | 6.43 | CTSO-10 | 1.01E+04 | 1.77E+03 | 7.43E+03 | 7.17 |

4 Table 3-B

| | f12 | | | | | f13 | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Algorithm | AVG | STD | MIN | MR | Algorithm | AVG | STD | MIN | MR |
| TSO | 1.01E+10 | 4.02E+09 | 3.60E+09 | 7.70 | TSO | 2.70E+09 | 3.49E+09 | 2.27E+08 | 6.57 |
| CTSO-1 | 7.78E+09 | 3.34E+09 | 2.90E+09 | 6.20 | CTSO-1 | 2.53E+09 | 3.16E+09 | 4.33E+08 | 7.50 |
| CTSO-2 | 8.51E+09 | 3.52E+09 | 2.90E+09 | 6.70 | CTSO-2 | 2.84E+09 | 4.16E+09 | 6.69E+07 | 7.00 |
| CTSO-3 | **2.04E+07** | 2.31E+07 | 6.18E+05 | **1.53** | CTSO-3 | **5.68E+04** | 4.66E+04 | 1.14E+04 | **1.33** |
| CTSO-4 | 8.93E+09 | 3.95E+09 | 2.87E+09 | 6.90 | CTSO-4 | 2.46E+09 | 3.43E+09 | 8.13E+07 | 6.57 |
| CTSO-5 | 8.59E+09 | 3.59E+09 | 3.45E+09 | 6.57 | CTSO-5 | 2.92E+09 | 3.56E+09 | 7.57E+08 | 7.53 |
| CTSO-6 | 9.35E+09 | 3.31E+09 | 3.23E+09 | 7.30 | CTSO-6 | 1.89E+09 | 2.52E+09 | 1.55E+08 | 6.40 |
| CTSO-7 | 9.48E+09 | 4.68E+09 | 3.75E+09 | 7.20 | CTSO-7 | 2.27E+09 | 2.67E+09 | 1.58E+08 | 6.70 |
| CTSO-8 | 9.97E+09 | 4.17E+09 | 3.87E+09 | 7.43 | CTSO-8 | 2.98E+09 | 4.32E+09 | 1.60E+08 | 7.20 |
| CTSO-9 | 2.47E+07 | 3.62E+07 | 1.01E+06 | **1.47** | CTSO-9 | 4.67E+07 | 2.36E+08 | 1.40E+04 | 1.80 |
| CTSO-10 | 9.12E+09 | 3.70E+09 | 3.15E+09 | 7.00 | CTSO-10 | 3.02E+09 | 3.45E+09 | 2.00E+08 | 7.40 |

| | f14 | | | | | f15 | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Algorithm | AVG | STD | MIN | MR | Algorithm | AVG | STD | MIN | MR |
| TSO | 7.27E+06 | 5.82E+06 | 3.40E+05 | 7.27 | TSO | 4.34E+08 | 6.96E+08 | 7.91E+06 | 6.07 |
| CTSO-1 | 1.01E+07 | 2.04E+07 | 1.53E+05 | 6.33 | CTSO-1 | 5.47E+08 | 5.45E+08 | 1.82E+06 | 7.47 |
| CTSO-2 | 7.56E+06 | 1.02E+07 | 2.16E+05 | 7.27 | CTSO-2 | 4.20E+08 | 4.48E+08 | 2.70E+07 | 6.97 |
| CTSO-3 | 1.45E+06 | 3.77E+06 | 3.86E+05 | 2.43 | CTSO-3 | **1.64E+04** | 1.28E+04 | 3.64E+03 | **1.50** |
| CTSO-4 | 5.56E+06 | 5.02E+06 | 4.65E+05 | 6.57 | CTSO-4 | 4.58E+08 | 5.59E+08 | 1.23E+07 | 7.20 |
| CTSO-5 | 6.28E+06 | 5.64E+06 | 2.45E+05 | 6.73 | CTSO-5 | 5.61E+08 | 7.55E+08 | 6.00E+06 | 7.17 |
| CTSO-6 | 8.11E+06 | 7.55E+06 | 2.68E+05 | 7.50 | CTSO-6 | 3.62E+08 | 5.47E+08 | 1.49E+07 | 6.33 |
| CTSO-7 | 4.99E+06 | 3.97E+06 | 3.18E+05 | 6.07 | CTSO-7 | 4.26E+08 | 3.91E+08 | 1.32E+07 | 7.40 |
| CTSO-8 | 7.43E+06 | 7.47E+06 | 8.20E+04 | 6.57 | CTSO-8 | 5.19E+08 | 5.05E+08 | 2.04E+07 | 7.80 |
| CTSO-9 | **9.26E+05** | 1.87E+06 | 5.91E+03 | **2.33** | CTSO-9 | 1.69E+04 | 1.30E+04 | 2.28E+03 | **1.50** |
| CTSO-10 | 1.21E+07 | 3.24E+07 | 2.11E+05 | 6.93 | CTSO-10 | 3.68E+08 | 4.55E+08 | 4.16E+06 | 6.60 |

| | f16 | | | | | f17 | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Algorithm | AVG | STD | MIN | MR | Algorithm | AVG | STD | MIN | MR |
| TSO | 5.48E+03 | 9.37E+02 | 8.03E+03 | 7.17 | TSO | 3.45E+03 | 7.68E+02 | 2.30E+03 | 7.00 |
| CTSO-1 | 5.74E+03 | 1.07E+03 | 8.56E+03 | 7.43 | CTSO-1 | 4.19E+03 | 4.53E+03 | 2.48E+03 | 6.43 |
| CTSO-2 | 5.17E+03 | 9.01E+02 | 7.32E+03 | 6.30 | CTSO-2 | 3.50E+03 | 1.28E+03 | 2.64E+03 | 5.47 |
| CTSO-3 | 3.55E+03 | 7.81E+02 | 5.57E+03 | 2.07 | CTSO-3 | **2.74E+03** | 3.60E+02 | 2.14E+03 | **3.00** |
| CTSO-4 | 5.58E+03 | 1.08E+03 | 7.68E+03 | 6.97 | CTSO-4 | 4.32E+03 | 3.29E+03 | 2.02E+03 | 6.93 |
| CTSO-5 | 5.16E+03 | 6.95E+02 | 6.30E+03 | 6.37 | CTSO-5 | 3.35E+03 | 8.24E+02 | 2.63E+03 | 6.23 |
| CTSO-6 | 5.53E+03 | 9.81E+02 | 8.44E+03 | 7.13 | CTSO-6 | 3.56E+03 | 1.03E+03 | 2.42E+03 | 6.20 |
| CTSO-7 | 5.71E+03 | 1.05E+03 | 7.73E+03 | 7.37 | CTSO-7 | 5.03E+03 | 6.07E+03 | 2.42E+03 | 6.20 |
| CTSO-8 | 5.44E+03 | 9.72E+02 | 8.31E+03 | 6.57 | CTSO-8 | 4.64E+03 | 4.14E+03 | 2.47E+03 | 7.37 |
| CTSO-9 | **3.49E+03** | 6.43E+02 | 5.33E+03 | **1.93** | CTSO-9 | 2.94E+03 | 3.69E+02 | 2.29E+03 | 4.53 |
| CTSO-10 | 5.29E+03 | 1.03E+03 | 7.54E+03 | 6.70 | CTSO-10 | 4.58E+03 | 4.16E+03 | 2.58E+03 | 6.63 |

| | f18 | | | | | f19 | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Algorithm | AVG | STD | MIN | MR | Algorithm | AVG | STD | MIN | MR |
| TSO | 9.48E+07 | 1.03E+08 | 5.85E+06 | 7.57 | TSO | 4.94E+08 | 4.74E+08 | 1.90E+07 | 6.77 |
| CTSO-1 | 6.32E+07 | 6.84E+07 | 1.73E+06 | 6.67 | CTSO-1 | 3.97E+08 | 3.36E+08 | 2.65E+07 | 6.60 |
| CTSO-2 | 7.71E+07 | 9.13E+07 | 1.58E+06 | 6.90 | CTSO-2 | 3.78E+08 | 3.98E+08 | 2.84E+07 | 5.97 |
| CTSO-3 | **1.20E+06** | 3.65E+06 | 9.33E+04 | **1.53** | CTSO-3 | 1.68E+05 | 5.67E+05 | 2.92E+03 | **1.43** |
| CTSO-4 | 1.08E+08 | 1.26E+08 | 5.89E+06 | 7.73 | CTSO-4 | 5.81E+08 | 4.94E+08 | 8.05E+07 | 7.23 |
| CTSO-5 | 6.37E+07 | 7.02E+07 | 1.02E+06 | 6.30 | CTSO-5 | 5.74E+08 | 4.75E+08 | 2.91E+07 | 7.37 |
| CTSO-6 | 1.36E+08 | 2.52E+08 | 2.39E+06 | 7.60 | CTSO-6 | 5.88E+08 | 5.07E+08 | 2.55E+07 | 7.10 |
| CTSO-7 | 5.56E+07 | 6.10E+07 | 2.13E+06 | 6.27 | CTSO-7 | 5.15E+08 | 4.67E+08 | 1.45E+07 | 6.93 |
| CTSO-8 | 9.00E+07 | 1.17E+08 | 1.06E+06 | 7.27 | CTSO-8 | 5.74E+08 | 4.58E+08 | 3.09E+07 | 7.33 |
| CTSO-9 | 2.68E+06 | 6.58E+06 | 3.96E+04 | 1.87 | CTSO-9 | **6.94E+04** | 1.61E+05 | 2.21E+03 | 1.57 |
| CTSO-10 | 5.50E+07 | 5.24E+07 | 9.39E+05 | 6.30 | CTSO-10 | 6.64E+08 | 6.30E+08 | 2.52E+07 | 7.70 |

| | f20 | | | | | f21 | | | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Algorithm | AVG | STD | MIN | MR | Algorithm | AVG | STD | MIN | MR |
| TSO | 2.97E+03 | 1.91E+02 | 2.56E+03 | 5.43 | TSO | 2.72E+03 | 4.50E+01 | 2.63E+03 | 7.07 |
| CTSO-1 | 2.97E+03 | 1.91E+02 | 2.52E+03 | 6.50 | CTSO-1 | 2.72E+03 | 5.31E+01 | 2.62E+03 | 7.20 |
| CTSO-2 | 2.97E+03 | 1.59E+02 | 2.60E+03 | 5.80 | CTSO-2 | 2.71E+03 | 4.52E+01 | 2.62E+03 | 6.80 |
| CTSO-3 | 2.99E+03 | 2.50E+02 | 2.40E+03 | 6.23 | CTSO-3 | **2.60E+03** | 5.54E+01 | 2.49E+03 | **2.13** |
| CTSO-4 | 3.03E+03 | 2.44E+02 | 2.53E+03 | 6.53 | CTSO-4 | 2.71E+03 | 5.90E+01 | 2.63E+03 | 6.43 |
| CTSO-5 | 3.01E+03 | 1.41E+02 | 2.74E+03 | 6.60 | CTSO-5 | 2.71E+03 | 5.16E+01 | 2.60E+03 | 6.20 |

| CTSO-6 | 2.96E+03 | 1.78E+02 | 2.59E+03 | 5.73 | CTSO-6 | 2.74E+03 | 4.94E+01 | 2.64E+03 | 7.83 |
|--------|----------|----------|----------|------|--------|----------|----------|----------|------|
| CTSO-7 | 2.96E+03 | 1.84E+02 | 2.66E+03 | 5.53 | CTSO-7 | 2.72E+03 | 6.25E+01 | 2.61E+03 | 6.97 |
| CTSO-8 | 2.99E+03 | 1.60E+02 | 2.65E+03 | 6.17 | CTSO-8 | 2.71E+03 | 6.74E+01 | 2.60E+03 | 5.93 |
| CTSO-9 | 3.06E+03 | 2.49E+02 | 2.56E+03 | 6.67 | CTSO-9 | **2.60E+03** | 5.86E+01 | 2.51E+03 | 2.30 |
| CTSO-10 | **2.92E+03** | 2.16E+02 | 2.54E+03 | **4.80** | CTSO-10 | 2.72E+03 | 4.95E+01 | 2.61E+03 | 7.13 |

7  Table 3-C

| | f22 | | | | | f23 | | | |
|-----------|----------|----------|----------|------|-----------|----------|----------|----------|------|
| Algorithm | AVG | STD | MIN | MR | Algorithm | AVG | STD | MIN | MR |
| TSO | 9.47E+03 | 9.40E+02 | 6.59E+03 | 6.33 | TSO | 3.46E+03 | 1.44E+02 | 3.19E+03 | 5.80 |
| CTSO-1 | 9.66E+03 | 8.69E+02 | 6.58E+03 | 7.13 | CTSO-1 | 3.54E+03 | 1.60E+02 | 3.29E+03 | 7.43 |
| CTSO-2 | 9.86E+03 | 7.73E+02 | 7.16E+03 | 7.73 | CTSO-2 | 3.49E+03 | 1.85E+02 | 3.22E+03 | 6.07 |
| CTSO-3 | 8.50E+03 | 1.45E+03 | 5.29E+03 | 3.93 | CTSO-3 | **3.36E+03** | 1.62E+02 | 3.01E+03 | **3.83** |
| CTSO-4 | 9.36E+03 | 9.72E+02 | 6.94E+03 | 6.20 | CTSO-4 | 3.49E+03 | 1.93E+02 | 3.05E+03 | 6.20 |
| CTSO-5 | 9.34E+03 | 9.78E+02 | 6.16E+03 | 5.77 | CTSO-5 | 3.47E+03 | 1.95E+02 | 3.16E+03 | 5.47 |
| CTSO-6 | 9.52E+03 | 8.58E+02 | 7.13E+03 | 6.57 | CTSO-6 | 3.53E+03 | 1.80E+02 | 3.28E+03 | 6.87 |
| CTSO-7 | 9.56E+03 | 6.84E+02 | 7.67E+03 | 6.57 | CTSO-7 | 3.52E+03 | 1.73E+02 | 3.20E+03 | 7.03 |
| CTSO-8 | 9.50E+03 | 1.12E+03 | 6.35E+03 | 6.47 | CTSO-8 | 3.55E+03 | 2.67E+02 | 3.21E+03 | 6.77 |
| CTSO-9 | **8.06E+03** | 1.86E+03 | 2.84E+03 | **3.03** | CTSO-9 | **3.36E+03** | 1.61E+02 | 2.98E+03 | 4.10 |
| CTSO-10 | 9.58E+03 | 5.84E+02 | 8.57E+03 | 6.27 | CTSO-10 | 3.48E+03 | 1.47E+02 | 3.18E+03 | 6.43 |

| | f24 | | | | | f25 | | | |
|-----------|----------|----------|----------|------|-----------|----------|----------|----------|------|
| Algorithm | AVG | STD | MIN | MR | Algorithm | AVG | STD | MIN | MR |
| TSO | 3.71E+03 | 2.08E+02 | 3.37E+03 | 6.80 | TSO | 4.68E+03 | 4.06E+02 | 3.79E+03 | 8.03 |
| CTSO-1 | 3.63E+03 | 1.90E+02 | 3.31E+03 | 5.30 | CTSO-1 | 4.48E+03 | 4.97E+02 | 3.56E+03 | 6.23 |
| CTSO-2 | 3.75E+03 | 2.98E+02 | 3.36E+03 | 6.70 | CTSO-2 | 4.50E+03 | 3.61E+02 | 3.72E+03 | 6.60 |
| CTSO-3 | 3.53E+03 | 2.47E+02 | 3.04E+03 | 4.33 | CTSO-3 | 3.02E+03 | 4.51E+01 | 2.93E+03 | 1.57 |
| CTSO-4 | 3.72E+03 | 2.39E+02 | 3.38E+03 | 6.40 | CTSO-4 | 4.58E+03 | 4.62E+02 | 3.87E+03 | 7.27 |
| CTSO-5 | 3.76E+03 | 2.26E+02 | 3.31E+03 | 7.17 | CTSO-5 | 4.59E+03 | 3.97E+02 | 3.82E+03 | 7.50 |
| CTSO-6 | 3.74E+03 | 2.66E+02 | 3.35E+03 | 6.60 | CTSO-6 | 4.65E+03 | 4.67E+02 | 3.79E+03 | 7.43 |
| CTSO-7 | 3.72E+03 | 3.32E+02 | 3.38E+03 | 5.93 | CTSO-7 | 4.54E+03 | 4.32E+02 | 3.64E+03 | 6.70 |
| CTSO-8 | 3.76E+03 | 3.23E+02 | 3.42E+03 | 6.50 | CTSO-8 | 4.54E+03 | 3.53E+02 | 4.00E+03 | 7.17 |
| CTSO-9 | **3.50E+03** | 1.88E+02 | 3.21E+03 | **3.43** | CTSO-9 | **3.01E+03** | 3.92E+01 | 2.93E+03 | **1.43** |
| CTSO-10 | 3.74E+03 | 2.92E+02 | 3.35E+03 | 6.83 | CTSO-10 | 4.45E+03 | 3.90E+02 | 3.79E+03 | 6.07 |

| | f26 | | | | | f27 | | | |
|-----------|----------|----------|----------|------|-----------|----------|----------|----------|------|
| Algorithm | AVG | STD | MIN | MR | Algorithm | AVG | STD | MIN | MR |
| TSO | 1.13E+04 | 1.15E+03 | 8.76E+03 | 7.67 | TSO | 4.26E+03 | 4.63E+02 | 3.57E+03 | 7.30 |
| CTSO-1 | 1.10E+04 | 1.06E+03 | 8.23E+03 | 6.90 | CTSO-1 | 4.21E+03 | 4.56E+02 | 3.49E+03 | 6.40 |
| CTSO-2 | 1.13E+04 | 1.02E+03 | 9.57E+03 | 7.73 | CTSO-2 | 4.12E+03 | 3.83E+02 | 3.56E+03 | 6.23 |
| CTSO-3 | **8.45E+03** | 1.44E+03 | 4.32E+03 | **1.90** | CTSO-3 | **3.55E+03** | 2.19E+02 | 3.23E+03 | **1.93** |
| CTSO-4 | 1.13E+04 | 1.18E+03 | 9.20E+03 | 7.50 | CTSO-4 | 4.16E+03 | 3.06E+02 | 3.45E+03 | 6.93 |
| CTSO-5 | 1.07E+04 | 1.22E+03 | 8.24E+03 | 6.00 | CTSO-5 | 4.18E+03 | 3.88E+02 | 3.45E+03 | 7.20 |
| CTSO-6 | 1.06E+04 | 9.86E+02 | 9.22E+03 | 5.80 | CTSO-6 | 4.13E+03 | 3.67E+02 | 3.64E+03 | 6.50 |
| CTSO-7 | 1.13E+04 | 1.34E+03 | 8.39E+03 | 7.60 | CTSO-7 | 4.09E+03 | 4.01E+02 | 3.58E+03 | 5.97 |
| CTSO-8 | 1.06E+04 | 1.25E+03 | 8.48E+03 | 5.63 | CTSO-8 | 4.26E+03 | 3.71E+02 | 3.55E+03 | 7.73 |
| CTSO-9 | 9.01E+03 | 1.50E+03 | 3.63E+03 | 2.77 | CTSO-9 | 3.66E+03 | 2.77E+02 | 3.28E+03 | 2.93 |
| CTSO-10 | 1.10E+04 | 1.37E+03 | 8.98E+03 | 6.50 | CTSO-10 | 4.23E+03 | 4.74E+02 | 3.61E+03 | 6.87 |

| | f28 | | | | | f29 | | | |
|-----------|----------|----------|----------|------|-----------|----------|----------|----------|------|
| Algorithm | AVG | STD | MIN | MR | Algorithm | AVG | STD | MIN | MR |
| TSO | 6.95E+03 | 7.63E+02 | 5.11E+03 | 7.80 | TSO | 7.09E+03 | 1.32E+03 | 5.34E+03 | 7.30 |
| CTSO-1 | 6.78E+03 | 8.33E+02 | 5.49E+03 | 7.23 | CTSO-1 | 6.60E+03 | 1.07E+03 | 4.83E+03 | 5.93 |
| CTSO-2 | 6.52E+03 | 6.75E+02 | 5.35E+03 | 6.03 | CTSO-2 | 6.82E+03 | 1.39E+03 | 4.71E+03 | 6.57 |
| CTSO-3 | **3.36E+03** | 4.68E+01 | 3.28E+03 | **1.43** | CTSO-3 | **5.59E+03** | 6.93E+02 | 4.80E+03 | **3.00** |
| CTSO-4 | 6.59E+03 | 8.56E+02 | 5.32E+03 | 6.63 | CTSO-4 | 6.64E+03 | 1.65E+03 | 4.67E+03 | 5.53 |
| CTSO-5 | 6.71E+03 | 8.21E+02 | 5.50E+03 | 6.83 | CTSO-5 | 7.58E+03 | 3.68E+03 | 5.25E+03 | 6.73 |
| CTSO-6 | 6.59E+03 | 9.89E+02 | 4.39E+03 | 6.80 | CTSO-6 | 6.87E+03 | 1.25E+03 | 5.42E+03 | 6.70 |
| CTSO-7 | 6.78E+03 | 8.11E+02 | 5.23E+03 | 7.17 | CTSO-7 | 6.62E+03 | 1.13E+03 | 5.10E+03 | 6.27 |
| CTSO-8 | 6.74E+03 | 8.69E+02 | 5.12E+03 | 6.93 | CTSO-8 | 7.11E+03 | 1.40E+03 | 5.01E+03 | 7.27 |
| CTSO-9 | 3.38E+03 | 8.88E+01 | 3.26E+03 | 1.57 | CTSO-9 | 5.73E+03 | 8.07E+02 | 4.46E+03 | 3.53 |
| CTSO-10 | 6.90E+03 | 6.42E+02 | 5.73E+03 | 7.57 | CTSO-10 | 7.06E+03 | 1.55E+03 | 4.58E+03 | 7.17 |

| | f30 | | | |
|-----------|----------|----------|----------|------|
| Algorithm | AVG | STD | MIN | MR |
| TSO | 8.51E+08 | 9.03E+08 | 1.91E+08 | 7.43 |

| | | | | |
|---|---|---|---|---|
| CTSO-1 | 6.09E+08 | 5.87E+08 | 7.08E+07 | 6.37 |
| CTSO-2 | 5.23E+08 | 5.57E+08 | 8.79E+07 | 5.87 |
| CTSO-3 | **1.87E+06** | 2.13E+06 | 5.87E+04 | **1.50** |
| CTSO-4 | 6.15E+08 | 5.41E+08 | 3.57E+07 | 6.57 |
| CTSO-5 | 7.89E+08 | 5.65E+08 | 4.65E+07 | 7.57 |
| CTSO-6 | 8.63E+08 | 7.59E+08 | 1.21E+08 | 7.20 |
| CTSO-7 | 8.42E+08 | 7.37E+08 | 8.26E+07 | 7.43 |
| CTSO-8 | 9.36E+08 | 7.41E+08 | 1.48E+08 | 7.93 |
| CTSO-9 | 3.02E+06 | 6.92E+06 | 3.88E+04 | **1.50** |
| CTSO-10 | 6.63E+08 | 5.55E+08 | 8.49E+07 | 6.63 |

Experimental results on CEC'17 test functions (continued)

| TSO | CTSO-1 | CTSO-2 | CTSO-3 | CTSO-4 | CTSO-5 | CTSO-6 | CTSO-7 | CTSO-8 | CTSO-9 | CTSO-10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 6.87 | 6.66 | 6.57 | 2.66 | 6.68 | 6.59 | 6.83 | 6.62 | 7.00 | 2.85 | 6.68 |

Wilcoxon rank-sum test on CEC'17 test functions

| | CTSO-1 | CTSO-2 | CTSO-3 | CTSO-4 | CTSO-5 | CTSO-6 | CTSO-7 | CTSO-8 | CTSO-9 | CTSO-10 |
|---|---|---|---|---|---|---|---|---|---|---|
| f1 | 8.53E-01 ≈ | 8.65E-01 ≈ | 3.02E-11 Δ | 1.91E-01 ≈ | 5.89E-01 ≈ | 7.96E-01 ≈ | 9.71E-01 ≈ | 7.28E-01 ≈ | 3.02E-11 Δ | 7.06E-01 ≈ |
| f3 | 9.82E-01 ≈ | 5.30E-01 ≈ | 1.86E-06 ▽ | 2.97E-01 ≈ | 6.00E-01 ≈ | 4.83E-01 ≈ | 6.20E-01 ≈ | 5.20E-01 ≈ | 4.74E-06 ▽ | 9.00E-01 ≈ |
| f4 | 5.69E-01 ≈ | 6.84E-01 ≈ | 3.02E-11 Δ | 9.00E-01 ≈ | 1.62E-01 ≈ | 9.47E-01 ≈ | 1.00E+00 ≈ | 3.40E-01 ≈ | 3.02E-11 Δ | 3.04E-01 ≈ |
| f5 | 4.55E-01 ≈ | 6.00E-01 ≈ | 1.96E-10 Δ | 4.92E-01 ≈ | 9.23E-01 ≈ | 2.64E-01 ≈ | 1.81E-01 ≈ | 4.83E-01 ≈ | 6.72E-10 Δ | 6.00E-01 ≈ |
| f6 | 3.48E-01 ≈ | 9.93E-02 ≈ | 6.53E-08 Δ | 1.41E-01 ≈ | 9.71E-01 ≈ | 3.04E-01 ≈ | 4.29E-01 ≈ | 9.00E-01 ≈ | 1.39E-06 Δ | 8.19E-01 ≈ |
| f7 | 2.64E-01 ≈ | 5.40E-01 ≈ | 5.01E-01 ≈ | 1.09E-01 ≈ | 3.03E-02 Δ | 6.10E-01 ≈ | 7.01E-02 Δ | 2.97E-01 ≈ | 3.40E-01 ≈ | 8.53E-01 ≈ |
| f8 | 5.30E-01 ≈ | 1.15E-01 ≈ | 2.37E-10 Δ | 5.20E-01 ≈ | 5.55E-02 ≈ | 5.59E-01 ≈ | 5.40E-01 ≈ | 3.95E-01 ≈ | 3.34E-11 Δ | 1.81E-01 ≈ |
| f9 | 8.07E-01 ≈ | 8.88E-01 ≈ | 2.67E-09 Δ | 6.73E-01 ≈ | 6.31E-01 ≈ | 8.30E-01 ≈ | 1.00E+00 ≈ | 3.33E-01 ≈ | 3.65E-08 Δ | 1.76E-01 ≈ |
| f10 | 2.28E-01 ≈ | 5.30E-01 ≈ | 4.44E-07 Δ | 3.33E-01 ≈ | 5.37E-02 ≈ | 1.12E-01 ≈ | 2.71E-01 ≈ | 1.54E-01 ≈ | 6.67E-03 Δ | 5.30E-01 ≈ |
| f11 | 2.90E-01 ≈ | 9.23E-01 ≈ | 3.02E-11 Δ | 2.12E-01 ≈ | 3.11E-01 ≈ | 9.94E-01 ≈ | 6.20E-01 ≈ | 2.71E-01 ≈ | 8.10E-10 Δ | 4.73E-01 ≈ |
| f12 | 2.07E-02 Δ | 1.37E-01 ≈ | 3.02E-11 Δ | 2.40E-01 ≈ | 1.12E-01 ≈ | 5.30E-01 ≈ | 5.20E-01 ≈ | 9.23E-01 ≈ | 3.02E-11 Δ | 3.79E-01 ≈ |
| f13 | 7.28E-01 ≈ | 8.30E-01 ≈ | 3.02E-11 Δ | 5.11E-01 ≈ | 4.12E-01 ≈ | 1.91E-01 ≈ | 5.59E-01 ≈ | 8.65E-01 ≈ | 1.21E-10 Δ | 5.01E-01 ≈ |
| f14 | 3.71E-01 ≈ | 6.95E-01 ≈ | 1.60E-07 Δ | 2.71E-01 ≈ | 6.00E-01 ≈ | 9.23E-01 ≈ | 1.67E-01 ≈ | 7.06E-01 ≈ | 2.02E-08 Δ | 6.84E-01 ≈ |
| f15 | 6.35E-02 ≈ | 1.19E-01 ≈ | 3.02E-11 Δ | 1.41E-01 ≈ | 6.57E-02 ≈ | 5.49E-01 ≈ | 1.22E-01 ≈ | 3.15E-02 Δ | 3.02E-11 Δ | 3.87E-01 ≈ |
| f16 | 3.71E-01 ≈ | 2.97E-01 ≈ | 3.20E-09 Δ | 8.30E-01 ≈ | 2.40E-01 ≈ | 9.82E-01 ≈ | 4.38E-01 ≈ | 7.06E-01 ≈ | 6.72E-10 Δ | 3.48E-01 ≈ |
| f17 | 5.59E-01 ≈ | 7.01E-02 ≈ | 2.15E-06 Δ | 6.31E-01 ≈ | 3.79E-01 ≈ | 8.65E-01 ≈ | 6.10E-01 ≈ | 3.79E-01 ≈ | 1.17E-03 Δ | 7.28E-01 ≈ |
| f18 | 1.71E-01 ≈ | 5.40E-01 ≈ | 5.49E-11 Δ | 7.73E-01 ≈ | 1.19E-01 ≈ | 9.59E-01 ≈ | 6.79E-02 ≈ | 6.10E-01 ≈ | 1.33E-10 Δ | 1.22E-01 ≈ |
| f19 | 7.28E-01 ≈ | 2.64E-01 ≈ | 3.02E-11 Δ | 4.20E-01 ≈ | 5.59E-01 ≈ | 4.46E-01 ≈ | 9.94E-01 ≈ | 4.92E-01 ≈ | 3.02E-11 Δ | 3.79E-01 ≈ |
| f20 | 4.83E-01 ≈ | 7.17E-01 ≈ | 3.04E-01 ≈ | 3.33E-01 ≈ | 1.58E-01 ≈ | 6.20E-01 ≈ | 9.00E-01 ≈ | 5.01E-01 ≈ | 9.05E-02 ≈ | 5.89E-01 ≈ |
| f21 | 9.94E-01 ≈ | 5.11E-01 ≈ | 3.82E-09 Δ | 4.29E-01 ≈ | 3.11E-01 ≈ | 1.58E-01 ≈ | 7.39E-01 ≈ | 2.46E-01 ≈ | 3.20E-09 Δ | 9.59E-01 ≈ |
| f22 | 3.71E-01 ≈ | 6.35E-02 ≈ | 2.38E-03 Δ | 6.84E-01 ≈ | 5.79E-01 ≈ | 7.73E-01 ≈ | 8.88E-01 ≈ | 6.31E-01 ≈ | 3.77E-04 Δ | 9.23E-01 ≈ |
| f23 | 4.84E-02 Δ | 6.95E-01 ≈ | 1.44E-02 Δ | 7.62E-01 ≈ | 8.53E-01 ≈ | 1.12E-01 ≈ | 1.58E-01 ≈ | 2.58E-01 ≈ | 1.63E-02 Δ | 5.30E-01 ≈ |
| f24 | 1.49E-01 ≈ | 7.84E-01 ≈ | 4.23E-03 Δ | 7.73E-01 ≈ | 2.90E-01 ≈ | 6.10E-01 ≈ | 3.18E-01 ≈ | 8.65E-01 ≈ | 5.97E-05 Δ | 9.82E-01 ≈ |
| f25 | 1.08E-02 Δ | 4.36E-02 Δ | 3.02E-11 Δ | 3.40E-01 ≈ | 5.69E-01 ≈ | 4.12E-01 ≈ | 1.30E-01 ≈ | 1.19E-01 ≈ | 3.02E-11 Δ | 9.07E-03 Δ |
| f26 | 4.12E-01 ≈ | 1.00E+00 ≈ | 6.72E-10 Δ | 9.59E-01 ≈ | 6.79E-02 ≈ | 1.38E-02 Δ | 7.39E-01 ≈ | 3.39E-02 Δ | 3.08E-08 Δ | 2.06E-01 ≈ |
| f27 | 6.84E-01 ≈ | 2.23E-01 ≈ | 1.55E-09 Δ | 6.00E-01 ≈ | 5.49E-01 ≈ | 3.63E-01 ≈ | 9.63E-02 ≈ | 8.88E-01 ≈ | 6.05E-07 Δ | 6.41E-01 ≈ |
| f28 | 3.11E-01 ≈ | 3.03E-02 Δ | 3.02E-11 Δ | 9.93E-02 ≈ | 1.91E-01 ≈ | 1.81E-01 ≈ | 3.95E-01 ≈ | 2.46E-01 ≈ | 3.02E-11 Δ | 7.51E-01 ≈ |
| f29 | 1.96E-01 ≈ | 2.97E-01 ≈ | 6.53E-07 Δ | 7.24E-02 ≈ | 6.95E-01 ≈ | 5.11E-01 ≈ | 1.26E-01 ≈ | 8.77E-01 ≈ | 5.86E-06 Δ | 9.12E-01 ≈ |
| f30 | 2.46E-01 ≈ | 3.39E-02 Δ | 3.02E-11 Δ | 2.90E-01 ≈ | 5.01E-01 ≈ | 9.59E-01 ≈ | 7.51E-01 ≈ | 3.18E-01 ≈ | 3.02E-11 Δ | 4.64E-01 ≈ |
| (Δ/▽/≈) | 3/0/26 | 3/0/26 | 26/1/2 | 0/0/29 | 1/0/28 | 1/0/28 | 0/0/29 | 2/0/27 | 26/1/2 | 1/0/28 |

Statistical results of the used algorithms for speed reducer problem

| Algorithm | Best | Mean | Worst | SD |
|---|---|---|---|---|
| TSO | 3043.966 | 338765.8 | 2180277 | 634222.4 |
| CTSO1 | 3133.385 | 209405.2 | 1769248 | 470918.1 |
| CTSO2 | 3008.135 | 643475.8 | 2297437 | 856143.2 |
| CTSO3 | **2994.423** | 3805.273 | 5521.033 | 872.899 |
| CTSO4 | 3050.252 | 422165.5 | 2001481 | 630221.1 |
| CTSO5 | 3003.839 | 135976.5 | 1034844 | 289203.5 |
| CTSO6 | 3015.958 | 231801 | 2363101 | 636178.5 |
| CTSO7 | 3014.59 | 210227.5 | 1625269 | 455196.1 |
| CTSO8 | 3050.025 | 594289.3 | 2401982 | 850915.1 |
| CTSO9 | **2994.423** | **3627.353** | 4774.773 | 562.9862 |
| CTSO10 | 3036.987 | 358518.4 | 2078853 | 620159.3 |

Comparison of the best optimum solution for the speed reducer problem

| Algorithm | Parameters values | | | | | | | $f_{min}$ |
|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | |
| TSO | 3.501019 | 0.7 | 17 | 7.3 | 7.917685 | 3.367619 | 5.349237 | 3043.966 |
| CTSO1 | 3.499917 | 0.7 | 17 | 7.994088 | 7.911673 | 3.787018 | 5.28737 | 3133.385 |
| CTSO2 | 3.504471 | 0.7 | 17 | 7.99709 | 7.857388 | 3.35903 | 5.287425 | 3008.135 |
| CTSO3 | 3.49999 | 0.7 | 17 | 7.3 | 7.715319 | 3.350541 | 5.286655 | **2994.423** |
| CTSO4 | 3.499871 | 0.7 | 17 | 8.067661 | 8.067661 | 3.361258 | 5.345938 | 3050.252 |
| CTSO5 | 3.499944 | 0.7 | 17 | 8.006041 | 7.768105 | 3.353007 | 5.288778 | 3003.839 |
| CTSO6 | 3.500284 | 0.7 | 17 | 8.048994 | 8.074678 | 3.35588 | 5.29533 | 3015.958 |
| CTSO7 | 3.509419 | 0.7 | 17 | 7.630128 | 8.036515 | 3.373788 | 5.28744 | 3014.590 |
| CTSO8 | 3.499876 | 0.7 | 17 | 8.014485 | 7.877796 | 3.352811 | 5.356255 | 3050.025 |
| CTSO9 | 3.49999 | 0.7 | 17 | 7.3 | 7.715319 | 3.350541 | 5.286654 | **2994.423** |
| CTSO10 | 3.500698 | 0.7 | 17 | 7.3 | 8.072679 | 3.361987 | 5.335593 | 3036.987 |

Statistical results of the used algorithms for tension-compression spring design problem (Case 1)

| Algorithm | Best | Mean | Worst | SD |
|-----------|------|------|-------|-----|
| TSO | 0.012673 | 0.013698 | 0.017631 | 0.001075 |
| CTSO1 | 0.012715 | 0.013655 | 0.017233 | 0.00081 |
| CTSO2 | 0.012702 | 0.013486 | 0.016016 | 0.000722 |
| CTSO3 | 0.01268 | 0.014229 | 0.020937 | 0.001895 |
| CTSO4 | 0.012784 | 0.013732 | 0.016824 | 0.000896 |
| CTSO5 | 0.012714 | 0.013537 | 0.015575 | 0.000668 |
| CTSO6 | 0.012739 | **0.013413** | 0.015612 | 0.000617 |
| CTSO7 | 0.01268 | 0.013579 | 0.015701 | 0.000774 |
| CTSO8 | 0.012683 | 0.013597 | 0.017302 | 0.000961 |
| CTSO9 | **0.012670** | 0.013483 | 0.017367 | 0.001136 |
| CTSO10 | 0.012714 | 0.013481 | 0.015773 | 0.000701 |

Comparison of the best optimum solution for the tension compression spring design problem

| Algorithm | Parameters values | | | $f_{min}$ |
|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | |
| TSO | 0.05215 | 0.367888 | 10.66628 | 0.012673 |
| CTSO1 | 0.053121 | 0.391907 | 9.497097 | 0.012715 |
| CTSO2 | 0.050521 | 0.329228 | 13.11549 | 0.012702 |
| CTSO3 | 0.050809 | 0.335913 | 12.62168 | 0.01268 |
| CTSO4 | 0.054124 | 0.418139 | 8.436915 | 0.012784 |
| CTSO5 | 0.050561 | 0.330017 | 13.0706 | 0.012714 |
| CTSO6 | 0.053384 | 0.398798 | 9.208447 | 0.012739 |
| CTSO7 | 0.052278 | 0.370952 | 10.50679 | 0.01268 |
| CTSO8 | 0.05217 | 0.368334 | 10.65104 | 0.012683 |
| CTSO9 | 0.05119 | 0.344834 | 12.02127 | **0.012670** |
| CTSO10 | 0.053257 | 0.395514 | 9.333766 | 0.012714 |

Statistical results of the used algorithms for welded beam design problem

| Algorithm | Best | Mean | Worst | SD |
|-----------|------|------|-------|-----|
| TSO | 1.691099 | 2.765785 | 3.713832 | 0.681901 |
| CTSO1 | **1.680985** | 2.706922 | 3.696062 | 0.69335 |
| CTSO2 | 1.692163 | 2.630818 | 3.687721 | 0.754546 |
| CTSO3 | 1.909833 | 3.307382 | 5.11409 | 0.943303 |
| CTSO4 | 1.699917 | **2.416367** | 3.745437 | 0.663697 |
| CTSO5 | 1.760383 | 2.792383 | 3.786065 | 0.544282 |
| CTSO6 | 1.721776 | 2.423815 | 3.684699 | 0.621039 |
| CTSO7 | 1.74493 | 2.67853 | 3.994741 | 0.69217 |
| CTSO8 | 1.764811 | 2.587149 | 3.675218 | 0.590975 |
| CTSO9 | 1.807061 | 3.159026 | 6.243182 | 1.010093 |
| CTSO10 | 1.739581 | 2.665869 | 3.709204 | 0.646727 |

Comparison of the best optimum solution for welded beam design problem

| Algorithm | Parameters values | | | | $f_{min}$ |
|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | |
| TSO | 0.198654 | 3.371947 | 9.112068 | 0.202756 | 1.691099 |
| CTSO1 | 0.200332 | 3.329221 | 9.164297 | 0.200695 | **1.680985** |
| CTSO2 | 0.195439 | 3.539739 | 9.191104 | 0.198922 | 1.692163 |
| CTSO3 | 0.235488 | 3.063879 | 8.008572 | 0.261938 | 1.909833 |
| CTSO4 | 0.202884 | 3.345369 | 9.074833 | 0.204389 | 1.699917 |
| CTSO5 | 0.146154 | 4.743619 | 9.193987 | 0.19883 | 1.760383 |
| CTSO6 | 0.197589 | 3.424587 | 8.970536 | 0.209319 | 1.721776 |
| CTSO7 | 0.161839 | 4.37461 | 9.210771 | 0.198758 | 1.74493 |
| CTSO8 | 0.220815 | 3.104299 | 8.68097 | 0.223645 | 1.764811 |
| CTSO9 | 0.202509 | 3.452743 | 8.545876 | 0.230036 | 1.807061 |
| CTSO10 | 0.200633 | 3.398656 | 8.875607 | 0.213808 | 1.739581 |

Statistical results of the used algorithms for pressure vessel design problem

| Algorithm | Best | Mean | Worst | SD |
|---|---|---|---|---|
| TSO | 6591.539 | 9358.735 | 15198.4 | 2047.207 |
| CTSO1 | 6934.041 | 9290.626 | 12997.45 | 1833.564 |
| CTSO2 | **6325.269** | 8872.022 | 13581.73 | 2116.409 |
| CTSO3 | 6720.31 | 18807.36 | 55057.06 | 10642.19 |
| CTSO4 | 6485.754 | 8708.514 | 13722.88 | 1568.458 |
| CTSO5 | 6479.417 | **8662.113** | 12317.9 | 1380.715 |
| CTSO6 | 6494.753 | 8878.358 | 13742.33 | 1651.033 |
| CTSO7 | 6782.928 | 8763.569 | 12770.58 | 1438.112 |
| CTSO8 | 6722.005 | 8687.164 | 14207.82 | 1804.541 |
| CTSO9 | 7836.811 | 17484.35 | 38598.81 | 8019.801 |
| CTSO10 | 6794.113 | 8275.311 | 11169.49 | 1131.863 |

Comparison of the best optimum solution for pressure vessel design problem

| Algorithm | Parameters values | | | | $f_{min}$ |
|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | |
| TSO | 1 | 0.5 | 49.9536 | 98.7225 | 6591.539 |
| CTSO1 | 1.0625 | 0.5625 | 53.06296 | 75.76345 | 6934.041 |
| CTSO2 | 0.875 | 0.5 | 45.277 | 140.8673 | **6325.269** |
| CTSO3 | 1 | 0.5625 | 51.66779 | 85.64049 | 6720.31 |
| CTSO4 | 1 | 0.5 | 51.01429 | 90.49689 | 6485.754 |
| CTSO5 | 1 | 0.5 | 51.10119 | 89.90082 | 6479.417 |
| CTSO6 | 0.9375 | 0.5 | 47.36856 | 120.7645 | 6494.753 |
| CTSO7 | 1.0625 | 0.5625 | 54.90602 | 63.63404 | 6782.928 |
| CTSO8 | 1 | 0.5625 | 51.64737 | 85.79055 | 6722.005 |
| CTSO9 | 1.25 | 0.5 | 51.69956 | 85.4083 | 7836.811 |
| CTSO10 | 1.0625 | 0.5625 | 54.77942 | 64.46914 | 6794.113 |

Statistical results of the used algorithms for three-bar truss design problem

| Algorithm | Best | Mean | Worst | SD |
|---|---|---|---|---|
| TSO | 263.8964 | 264.5518 | 270.4717 | 1.648298 |
| CTSO1 | 263.8987 | 264.8722 | 270.7144 | 2.06893 |
| CTSO2 | 263.9028 | 264.7003 | 270.7078 | 1.696716 |
| CTSO3 | 263.9005 | 266.5551 | 274.3699 | 2.902827 |
| CTSO4 | 263.8962 | 264.4986 | 270.7534 | 1.375953 |
| CTSO5 | 263.8963 | 264.5397 | 270.3816 | 1.46023 |
| CTSO6 | 263.9002 | 264.6418 | 270.7240 | 1.362083 |
| CTSO7 | 263.8966 | 264.6414 | 270.6517 | 1.388371 |
| CTSO8 | 263.9014 | 264.5663 | 270.3992 | 1.670418 |
| CTSO9 | **263.8957** | 266.2048 | 281.5610 | 3.621051 |
| CTSO10 | 263.8962 | **264.3218** | 270.7161 | 1.223462 |

Comparison of the best optimum solution for three-bar truss design problem

| Algorithm | Parameters values | | $f_{min}$ |
| --- | --- | --- | --- |
| | $x_1$ | $x_2$ | |
| TSO | 0.787491 | 0.411599 | 263.8964 |
| CTSO1 | 0.786632 | 0.414043 | 263.8987 |
| CTSO2 | 0.786469 | 0.41449 | 263.9028 |
| CTSO3 | 0.791316 | 0.40082 | 263.9005 |
| CTSO4 | 0.788856 | 0.407739 | 263.8962 |
| CTSO5 | 0.787869 | 0.410515 | 263.8963 |
| CTSO6 | 0.790911 | 0.401968 | 263.9002 |
| CTSO7 | 0.789892 | 0.404812 | 263.8966 |
| CTSO8 | 0.785835 | 0.416332 | 263.9014 |
| CTSO9 | 0.789298 | 0.40648 | **263.8957** |
| CTSO10 | 0.787728 | 0.41092 | 263.8962 |

Comparison of the speed reducer problem

| Algorithm | Parameters values | | | | | | | $f_{min}$ |
|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | |
| MFO (Mirjalili, 2015) | 3.497455 | 0.700 | 17 | 7.82775 | 7.712457 | 3.351787 | 5.286352 | 2998.941 |
| WSA (Baykaso, 2015) | 3.500 | 0.7 | 17 | 7.3 | 7.8 | 3.350215 | 5.286683 | 2996.348 |
| GWO (Mirjalili, Mirjalili & Lewis, 2014) | 3.501 | 0.7 | 17 | 7.3 | 7.811013 | 3.350704 | 5.287411 | 2997.820 |
| AAO (Czerniak, Zarzycki & Ewald, 2017) | 3.4999 | 0.7 | 17 | 7.3 | 7.8 | 3.3502 | 5.2877 | 2997.058 |
| SCA (Mirjalili, 2016) | 3.521 | 0.7 | 17 | 8.3 | 7.923351 | 3.355911 | 5.300734 | 3026.838 |
| AOA (Abualigah et al., 2021a) | 3.50384 | 0.7 | 17 | 7.3 | 7.72933 | 3.35649 | 5.2867 | 2997.916 |
| TSO | 3.501019 | 0.7 | 17 | 7.3 | 7.917685 | 3.367619 | 5.349237 | 3043.966 |
| CTSO-3 | 3.49999 | 0.7 | 17 | 7.3 | 7.715319 | 3.350541 | 5.286655 | **2994.423** |
| CTSO-9 | 3.49999 | 0.7 | 17 | 7.3 | 7.715319 | 3.350541 | 5.286654 | **2994.423** |

Comparison of tension compression spring design problem

| Algorithm | Parameters values | | | $f_{min}$ |
|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | |
| HGSO (Hashim et al., 2019) | 0.0518 | 0.3569 | 11.2023 | 0.0127 |
| AAO (Czerniak, Zarzycki & Ewald, 2017) | 0.0517 | 0.3581 | 11.2015 | 0.0127 |
| WOA (Mirjalili & Lewis, 2016) | 0.0512 | 0.3452 | 12.004 | 0.0127 |
| HHO (Heidari et al., 2019) | 0.0543 | 0.4239 | 8.2187 | 0.0128 |
| CMBSA(Varol Altay & Alatas, 2020) | 0.0519 | 0.3618 | 11.000 | 0.0127 |
| SSA (Mirjalili et al., 2017) | 0.0516 | 0.3547 | 11.4059 | 0.0127 |
| CBPPSO (Agrawal & Tripathi, 2018) | 0.0512 | 0.3465 | 11.9097 | 0.0127 |
| CBSA (Wang et al.) | 0.0516 | 0.3566 | 11.2918 | 0.0127 |
| TSO | 0.05215 | 0.367888 | 10.66628 | 0.012673 |
| CTSO-9 | 0.05119 | 0.344834 | 12.02127 | **0.012670** |

Comparison of welded beam design problem

| Algorithm | Parameters values | | | | $f_{min}$ |
|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | |
| HGSO (Hashim et al., 2019) | 0.2054 | 3.4476 | 9.0269 | 0.2060 | 1.7260 |
| MAOA (Altay, 2022a) | 0.2057 | 3.4705 | 9.0366 | 0.2057 | 1.7246 |
| HHO (Heidari et al., 2019) | 0.1956 | 3.7730 | 9.0307 | 0.2060 | 1.7501 |
| CMBSA (Varol Altay & Alatas, 2020) | 0.2057 | 3.4702 | 9.0377 | 0.2057 | 1.7249 |
| CBPPSO (Agrawal & Tripathi, 2018) | 0.2057 | 3.4704 | 9.0366 | 0.2057 | 1.7249 |
| CGWO (Kohli & Arora, 2018) | 0.3439 | 1.8836 | 9.0313 | 0.2121 | 1.7255 |
| DBCA (Prayogo et al., 2018) | 0.2057 | 3.4705 | 9.0366 | 0.2057 | 1.7249 |
| SIO (Tzanetos & Dounias, 2020) | 0.3314 | 2.0174 | 9.0459 | 0.2088 | 1.7621 |
| H-GA-ACO-PSO (Tam et al., 2019) | 0.2057 | 3.4705 | 9.0366 | 0.2057 | 1.7249 |
| TSO | 0.198654 | 3.371947 | 9.112068 | 0.202756 | 1.6911 |
| CTSO-1 | 0.200332 | 3.329221 | 9.164297 | 0.200695 | **1.6810** |

Comparison of pressure vessel problem

| Algorithm | Parameters values | | | | $f_{min}$ |
|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | |
| MAOA (Altay, 2022a) | 0.7953 | 0.3931 | 41.2274 | 187.7371 | 5914.48511 |
| CSMA (Altay, 2022c) | 0.7778 | 0.3845 | 40.3207 | 199.9850 | **5882.0851** |
| AAO (Czerniak, Zarzycki & Ewald, 2017) | 0.8125 | 0.4375 | 42.0985 | 176.6366 | 6059.7140 |
| HHO (Heidari et al., 2019) | 0.8540 | 0.4329 | 44.0025 | 154.3888 | 6543.4802 |
| CMBSA(Varol Altay & Alatas, 2020) | 0.7780 | 0.3850 | 40.3200 | 200.0000 | 5883.8610 |
| SSA (Mirjalili et al., 2017) | 0.7807 | 0.3859 | 40.4707 | 197.9081 | 6149.0232 |
| CBPPSO (Agrawal & Tripathi, 2018) | 1.125 | 0.6250 | 62.9866 | 20.00000 | 6952.7200 |
| H-GA-ACO-PSO (Tam et al., 2019) | 0.8125 | 0.4375 | 42.0984 | 176.6366 | 6059.7143 |
| ARLBAT (Meng, Li & Gao, 2019) | 0.8125 | 0.4375 | 42.0984 | 176.6366 | 6059.7143 |
| TSO | 1 | 0.5 | 49.9536 | 98.7225 | 6591.5390 |
| CTSO-2 | 0.875 | 0.5 | 45.277 | 140.8673 | 6325.2690 |

Comparison of for three bar truss problem

| Algorithm | Parameters values | | $f_{min}$ |
|---|---|---|---|
| | $x_1$ | $x_2$ | |
| AOA (Abualigah et al., 2021a) | 0.79369 | 0.39426 | 263.9154 |
| SSA (Mirjalili et al., 2017) | 0.78866541 | 0.408275784 | 263.8958 |
| CS (Gandomi, Yang & Alavi, 2013) | 0.78867 | 0.40902 | 263.9716 |
| MBA (Sadollah et al., 2013) | 0.7885650 | 0.4085597 | 263.8959 |
| TSO | 0.787491 | 0.411599 | 263.8964 |
| CTSO-9 | 0.789298 | 0.40648 | **263.8957** |

Description of used benchmark's datasets

| Datasets | Number of features | Number of instances | Classes |
|---|---|---|---|
| Australian | 14 | 690 | 2 |
| Ionosphere | 34 | 351 | 2 |
| Spectheart | 44 | 267 | 2 |
| Sonar | 60 | 208 | 2 |
| wine | 13 | 178 | 3 |
| heart | 13 | 270 | 2 |
| thyroid | 21 | 7200 | 3 |
| Tic-tac-toe | 9 | 958 | 2 |
| vehicle | 18 | 846 | 4 |
| krvskp | 36 | 3196 | 2 |

Comparison of the CTSOs approaches based on the minimum of the fitness values

| | Australian | Ionosphere | Spectheart | Sonar | wine | heart | thyroid | Tic-tac-toe | vehicle | krvskp |
|---|---|---|---|---|---|---|---|---|---|---|
| CTSO-1 | 0.1739 | **0.0286** | 0.1509 | 0.0244 | 0.0286 | 0.1667 | 0.0174 | 0.1832 | 0.2722 | 0.0407 |
| CTSO-2 | **0.1522** | **0.0286** | 0.1509 | 0.0488 | 0.0286 | 0.1667 | 0.0194 | 0.1832 | 0.2663 | 0.0407 |
| CTSO-3 | **0.1522** | 0.0429 | **0.1132** | 0.0488 | 0.0286 | 0.1852 | 0.0174 | **0.1780** | **0.2485** | 0.0313 |
| CTSO-4 | **0.1522** | 0.0429 | 0.1509 | 0.0244 | 0.0286 | 0.1481 | 0.0194 | **0.1780** | 0.2899 | 0.0407 |
| CTSO-5 | 0.1667 | 0.0429 | 0.1321 | 0.0244 | 0.0286 | **0.1296** | 0.0181 | **0.1780** | 0.2663 | 0.0454 |
| CTSO-6 | 0.1667 | 0.0429 | 0.1698 | 0.0488 | **0.0000** | 0.1852 | 0.0194 | **0.1780** | 0.2722 | 0.0438 |
| CTSO-7 | **0.1522** | 0.0429 | 0.1509 | 0.0488 | 0.0286 | 0.1481 | **0.0167** | **0.1780** | 0.2840 | 0.0579 |
| CTSO-8 | 0.1594 | **0.0286** | 0.1698 | 0.0244 | 0.0286 | 0.1481 | 0.0201 | **0.1780** | 0.2840 | 0.0391 |
| CTSO-9 | 0.1594 | 0.0429 | 0.1321 | 0.0244 | **0.0000** | **0.1296** | **0.0167** | **0.1780** | 0.2781 | **0.0266** |
| CTSO-10 | 0.1594 | 0.0429 | 0.1698 | **0.0000** | 0.0286 | 0.1481 | 0.0181 | 0.1832 | 0.2899 | 0.0297 |
| TSO | 0.1884 | 0.0714 | 0.2075 | 0.0976 | 0.0571 | 0.2407 | 0.0264 | 0.2042 | 0.3077 | 0.0704 |
| AO | 0.2174 | 0.0857 | 0.2453 | 0.0976 | 0.0857 | 0.2407 | 0.0319 | 0.2304 | 0.3136 | 0.1017 |
| SMA | 0.1739 | 0.0714 | 0.1887 | 0.0976 | 0.0857 | 0.2407 | 0.0278 | 0.2461 | 0.3254 | 0.0908 |
| WOA | 0.1739 | 0.0571 | 0.1698 | 0.0488 | 0.0571 | 0.2037 | 0.0229 | 0.2042 | 0.2840 | 0.0595 |

Comparison of the CTSOs approaches based on the average of the fitness values

|  | Australian | Ionosphere | Spectheart | Sonar | wine | heart | thyroid | Tic-tac-toe | vehicle | krvskp |
|---|---|---|---|---|---|---|---|---|---|---|
| CTSO-1 | 0.1768 | 0.0629 | 0.1811 | 0.0585 | 0.0486 | 0.2037 | 0.0238 | 0.2042 | **0.2888** | **0.0520** |
| CTSO-2 | 0.1768 | 0.0529 | 0.1736 | 0.0610 | 0.0429 | 0.2056 | 0.0245 | 0.2042 | 0.2970 | 0.0548 |
| CTSO-3 | **0.1674** | **0.0500** | 0.1717 | 0.0561 | 0.0486 | 0.2056 | 0.0238 | 0.1984 | 0.2905 | 0.0601 |
| CTSO-4 | 0.1804 | 0.0557 | 0.1755 | 0.0585 | 0.0514 | 0.1963 | 0.0244 | **0.1848** | 0.3024 | 0.0527 |
| CTSO-5 | 0.1754 | **0.0500** | 0.1792 | 0.0537 | 0.0429 | **0.1815** | 0.0234 | 0.2073 | 0.2911 | 0.0546 |
| CTSO-6 | 0.1848 | 0.0629 | 0.1811 | 0.0659 | **0.0286** | 0.1926 | 0.0235 | 0.2000 | 0.2941 | 0.0576 |
| CTSO-7 | 0.1833 | 0.0714 | 0.1736 | 0.0561 | 0.0343 | 0.2056 | 0.0228 | **0.1848** | 0.2959 | 0.0726 |
| CTSO-8 | 0.1717 | 0.0543 | 0.1849 | 0.0561 | 0.0486 | 0.2093 | 0.0221 | 0.1969 | 0.2994 | 0.0562 |
| CTSO-9 | 0.1696 | 0.0543 | **0.1604** | 0.0561 | 0.0429 | 0.1907 | **0.0214** | 0.1995 | 0.2929 | 0.0538 |
| CTSO-10 | 0.1899 | 0.0643 | 0.1792 | **0.0512** | 0.0486 | 0.1852 | 0.0224 | 0.1990 | 0.2970 | 0.0592 |
| TSO | 0.2268 | 0.0829 | 0.2283 | 0.0976 | 0.0971 | 0.2667 | 0.0332 | 0.2209 | 0.3124 | 0.0889 |
| AO | 0.2638 | 0.0929 | 0.2679 | 0.1073 | 0.1286 | 0.2722 | 0.0382 | 0.2613 | 0.3331 | 0.1668 |
| SMA | 0.1826 | 0.0857 | 0.1943 | 0.1024 | 0.1114 | 0.2574 | 0.0334 | 0.2555 | 0.3385 | 0.1319 |
| WOA | 0.2014 | 0.0643 | 0.1868 | 0.0585 | 0.0600 | 0.2185 | 0.0276 | 0.2105 | 0.2982 | 0.0714 |

Comparison of the CTSOs approaches based on the average of accuracy measure

|  | Australian | Ionosphere | Spectheart | Sonar | wine | heart | thyroid | Tic-tac-toe | vehicle | krvskp |
|---|---|---|---|---|---|---|---|---|---|---|
| CTSO-1 | 78.1884 | 92.1429 | 79.0566 | 92.1951 | 93.7143 | 77.5926 | 96.9792 | 78.1152 | 68.8757 | 92.8326 |
| CTSO-2 | 78.7681 | 91.5714 | 78.1132 | 92.1951 | 95.1429 | 76.6667 | 97.3056 | 77.5916 | 69.6450 | 92.9421 |
| CTSO-3 | 76.5942 | 92.1429 | 78.4906 | 92.6829 | 93.7143 | 76.2963 | 97.5486 | 78.1152 | 70.0592 | 91.1111 |
| CTSO-4 | 77.5362 | 92.2857 | 79.8113 | 93.4146 | 93.1429 | 78.5185 | 97.2292 | 80.0524 | 68.9349 | 92.0344 |
| CTSO-5 | 79.6377 | 92.8571 | 78.8679 | 92.1951 | 95.1429 | 76.6667 | 97.5278 | 77.4346 | 69.4675 | 92.9890 |
| CTSO-6 | 78.9130 | 92.5714 | 80.1887 | 93.4146 | **95.7143** | 77.5926 | 97.6181 | 78.4817 | 69.8225 | 92.9890 |
| CTSO-7 | 79.1304 | 92.7143 | 77.9245 | 91.7073 | 95.4286 | 78.3333 | 97.4167 | **81.3613** | 69.3491 | 91.1894 |
| CTSO-8 | 78.9130 | 92.4286 | 77.7358 | 93.6585 | 94.5714 | 76.6667 | 97.5417 | 79.5812 | 68.9349 | 92.5509 |
| CTSO-9 | 79.8551 | 92.2857 | 79.4340 | 91.9512 | 94.5714 | 77.4074 | **97.6458** | 78.1152 | 69.2308 | **93.0673** |
| CTSO-10 | 79.3478 | 92.4286 | 80.1887 | 93.4146 | 94.5714 | **79.4444** | 97.4583 | 79.0052 | 69.1124 | 91.0798 |
| TSO | 70.3623 | 89.5714 | 74.3396 | 89.5122 | 86.8571 | 71.8519 | 96.2778 | 73.9267 | 66.0947 | 86.0094 |
| AO | 73.6232 | 90.7143 | 73.2075 | 89.2683 | 87.1429 | 72.7778 | 96.1806 | 73.8743 | 66.6864 | 83.3177 |
| SMA | **81.7391** | 91.4286 | 80.5660 | 89.7561 | 88.8571 | 74.2593 | 96.6597 | 74.4503 | 66.1538 | 86.8075 |
| WOA | 79.8551 | **93.5714** | **81.3208** | **94.1463** | 94.0000 | 78.1481 | 97.2361 | 78.9529 | **70.1775** | 92.8638 |

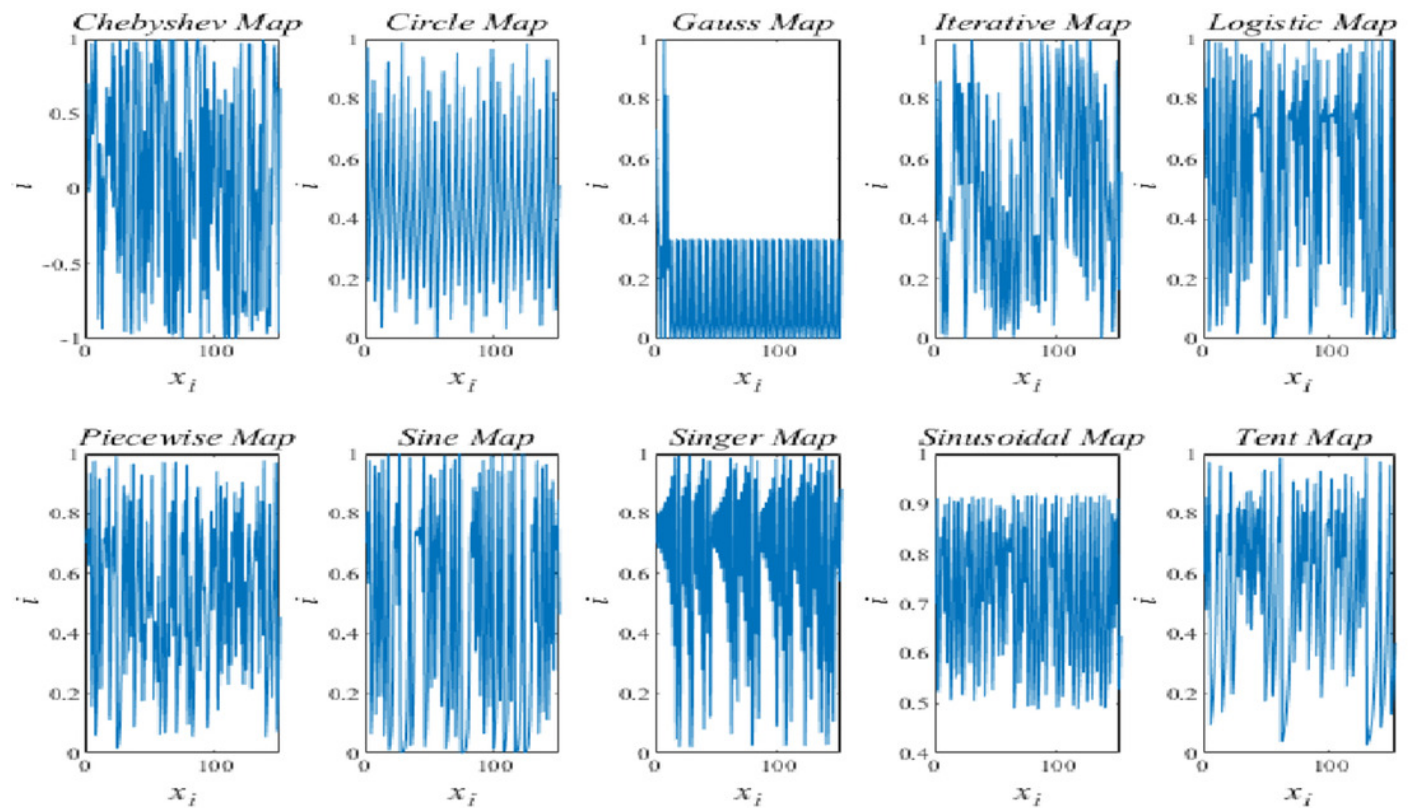
Comparison of the CTSOs approaches based on the maximum of accuracy measure

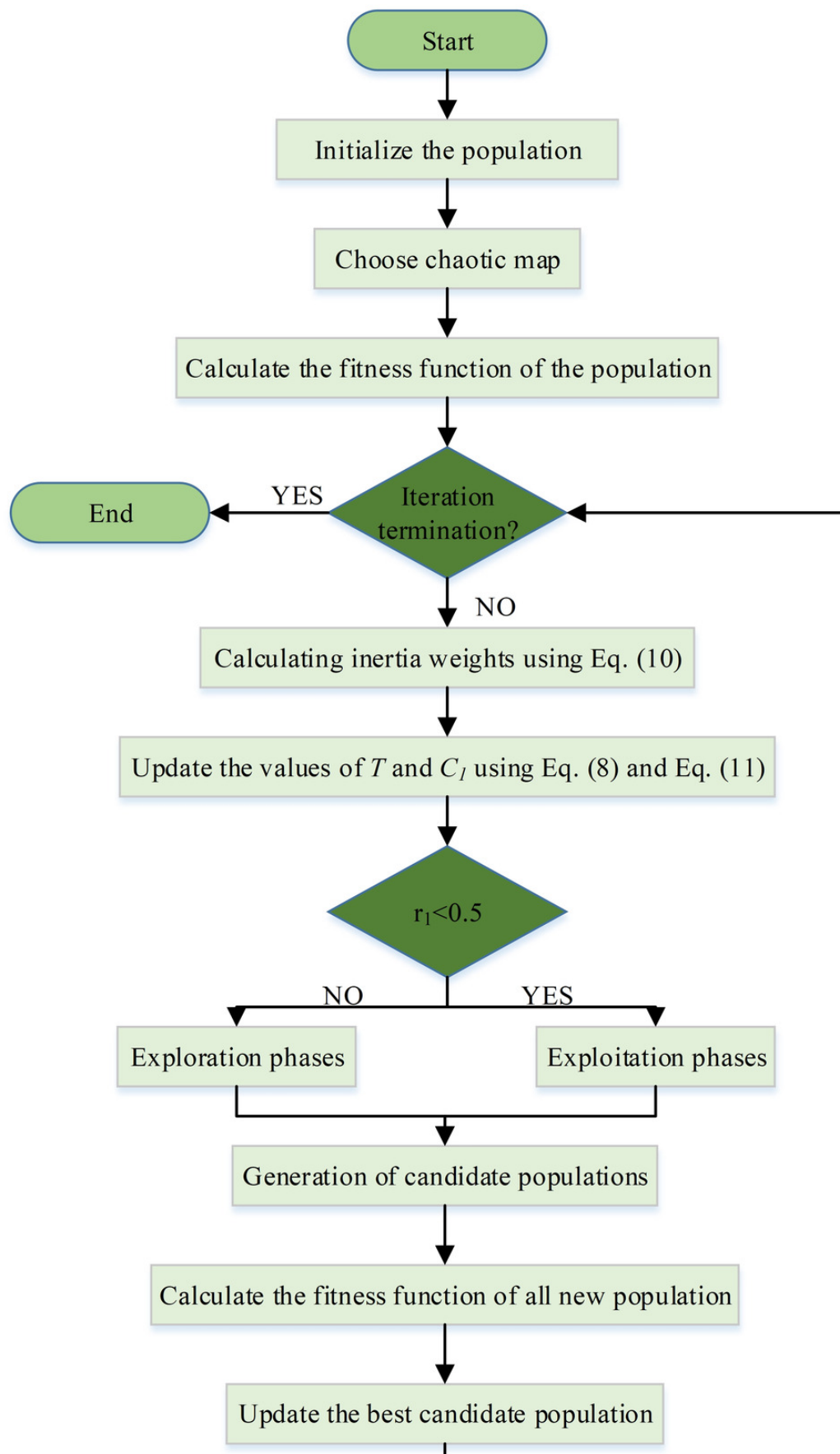
|  | Australian | Ionosphere | Spectheart | Sonar | wine | heart | thyroid | Tic-tac-toe | vehicle | krvskp |
|---|---|---|---|---|---|---|---|---|---|---|
| CTSO-1 | 81.8841 | **95.7143** | **84.9057** | 95.1220 | 97.1429 | 81.4815 | 97.5000 | 80.1047 | 70.4142 | 95.9311 |
| CTSO-2 | 82.6087 | 94.2857 | 83.0189 | 95.1220 | 97.1429 | 81.4815 | 97.9861 | 81.6754 | 73.3728 | 94.9922 |
| CTSO-3 | 81.1594 | 94.2857 | 83.0189 | 95.1220 | 97.1429 | 79.6296 | **98.2639** | **82.1990** | **75.1479** | 93.5837 |
| CTSO-4 | 80.4348 | 94.2857 | **84.9057** | 95.1220 | 94.2857 | **85.1852** | 97.5000 | **82.1990** | 70.4142 | 94.3662 |
| CTSO-5 | 82.6087 | **95.7143** | 81.1321 | 95.1220 | 97.1429 | **85.1852** | 97.9167 | 79.5812 | 72.1893 | 94.9922 |
| CTSO-6 | 82.6087 | 94.2857 | 83.0189 | 95.1220 | 97.1429 | 81.4815 | 98.0556 | 81.1518 | 72.1893 | 95.1487 |
| CTSO-7 | 82.6087 | **95.7143** | **84.9057** | 95.1220 | 97.1429 | **85.1852** | 98.1944 | **82.1990** | 71.0059 | 92.6448 |
| CTSO-8 | 81.1594 | **95.7143** | 83.0189 | **97.5610** | 97.1429 | **85.1852** | 97.9861 | 81.1518 | 70.4142 | **96.0876** |
| CTSO-9 | **84.0580** | **95.7143** | **84.9057** | 95.1220 | **100.0000** | 83.3333 | 98.1944 | **82.1990** | 72.1893 | 94.8357 |
| CTSO-10 | **84.0580** | 92.8571 | 83.0189 | **97.5610** | 97.1429 | **85.1852** | 97.6389 | 81.1518 | 71.0059 | 93.8967 |
| TSO | 72.4638 | 91.4286 | 77.3585 | 90.2439 | 91.4286 | 74.0741 | 97.2222 | 76.4398 | 68.0473 | 90.7668 |
| AO | 78.2609 | 91.4286 | 75.4717 | 90.2439 | 91.4286 | 75.9259 | 96.8056 | 76.9634 | 68.6391 | 89.8279 |
| SMA | 82.6087 | 92.8571 | 81.1321 | 90.2439 | 91.4286 | 75.9259 | 97.2222 | 75.3927 | 67.4556 | 90.9233 |
| WOA | 82.6087 | 94.2857 | 83.0189 | 95.1220 | 94.2857 | 79.6296 | 97.7083 | 79.5812 | 71.5976 | 94.0532 |

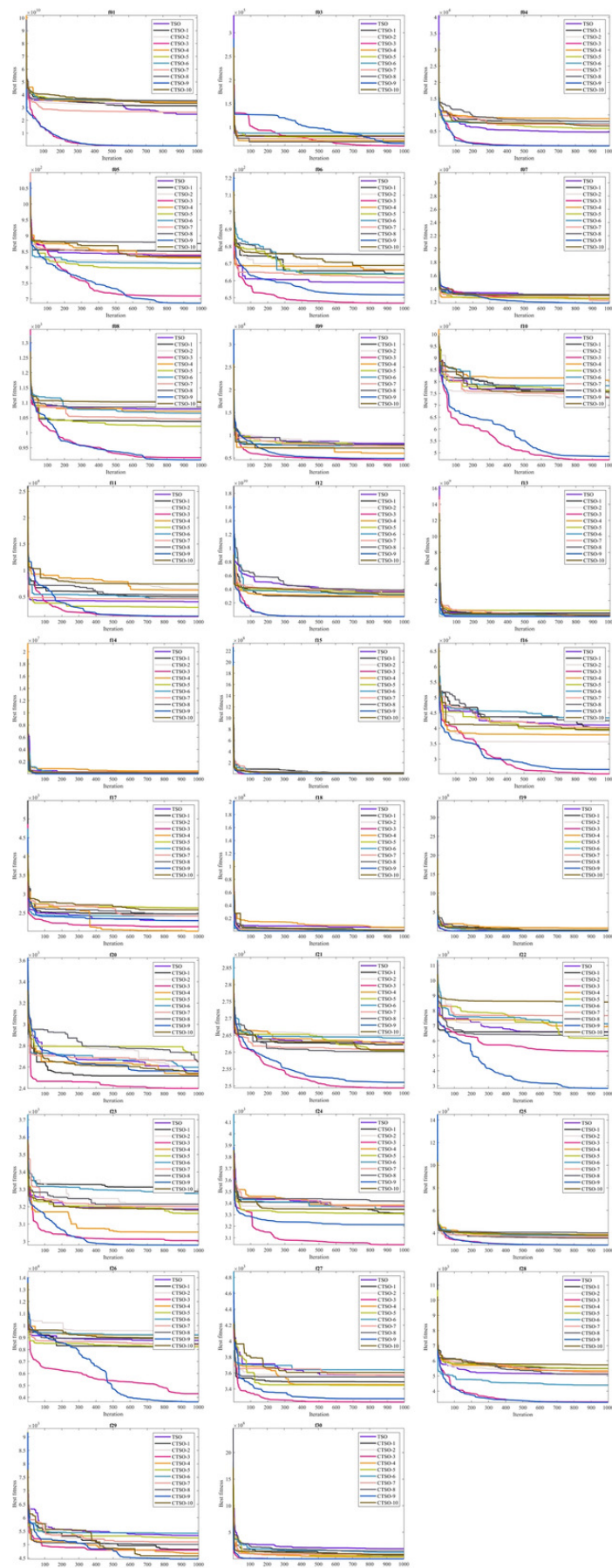
Comparison of for three bar truss problem

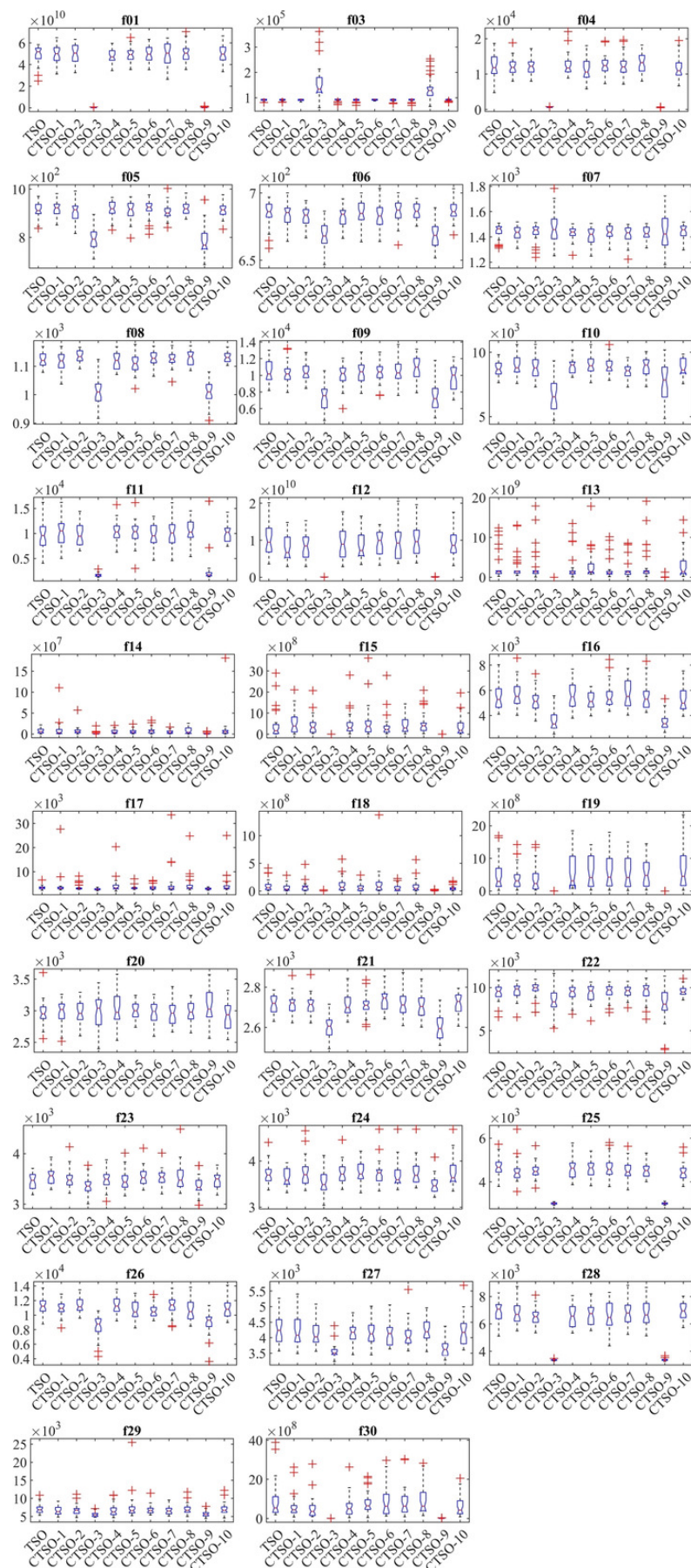
| | Australian | Ionosphere | Spectheart | Sonar | wine | heart | thyroid | Tic-tac-toe | vehicle | krvskp |
|---|---|---|---|---|---|---|---|---|---|---|
| CTSO-1 | 5.7 | 16.6 | 21.1 | 29.6 | 6 | 5.7 | 10.5 | 5.7 | 9.4 | 19.6 |
| CTSO-2 | 5.9 | 15.8 | 22.5 | 29.9 | 6.6 | 5.8 | 10 | 5.3 | 9.5 | 19.4 |
| CTSO-3 | 6.5 | 14.4 | 20.2 | 30.3 | 6.8 | 5 | 9.9 | 5.5 | 8.1 | 18.6 |
| CTSO-4 | 6 | 16.5 | 21.3 | 28.9 | 6.1 | 5.2 | 10.2 | 6 | 9.2 | 19.3 |
| CTSO-5 | 5.6 | 17 | 21.8 | 29.1 | 6.5 | 6.7 | 9.6 | 5.6 | 8.7 | 19.5 |
| CTSO-6 | 6.1 | 15.8 | 21.2 | 29.9 | 5.6 | 5.8 | 9.9 | 5.5 | 8.7 | 20 |
| CTSO-7 | 5.8 | 15.4 | 20.2 | 32.2 | 6.1 | 5.6 | 10 | 6 | 9 | 19.5 |
| CTSO-8 | 6.1 | 15.5 | 21.4 | 28.6 | 6.1 | 6.1 | 10.5 | 6 | 8.6 | 18.5 |
| CTSO-9 | 6 | 16.4 | 21.6 | 29.4 | 6.2 | 5.7 | 10.3 | 5.5 | 9 | 19.2 |
| CTSO-10 | 6.1 | 15.6 | 20.1 | 28.4 | 5.6 | 5.4 | 10.4 | 5.6 | 8.2 | 19.2 |
| TSO | 6.9 | 16.2 | 19.7 | 29.5 | 5.6 | 5.5 | 9.4 | 5.7 | 9.4 | 19.2 |
| AO | 5.3 | 16.4 | 21 | 30.1 | 5.9 | 5.8 | 10.9 | 5.4 | 9.3 | 19.1 |
| SMA | 1.1 | 7.3 | 1 | 29.7 | 5.7 | 3.5 | 7.4 | 6.7 | 9.3 | 20.3 |
| WOA | 2.6 | 8.3 | 11.9 | 22.4 | 5.1 | 5.6 | 7.4 | 6 | 7.5 | 20.9 |

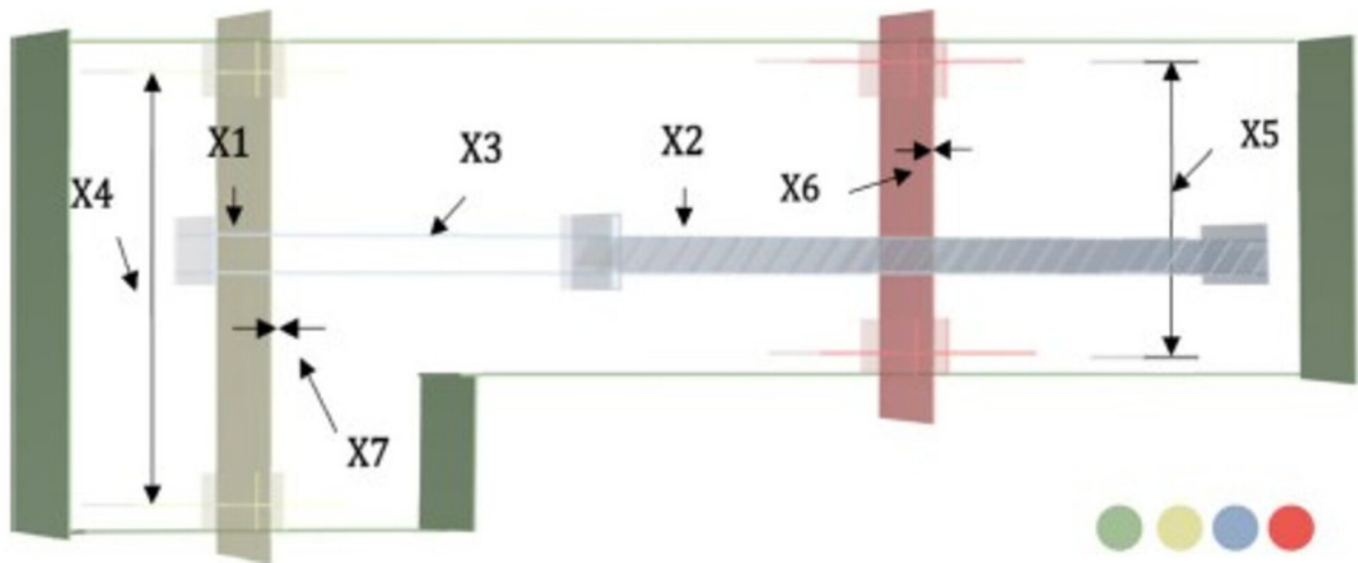
# Figure 1

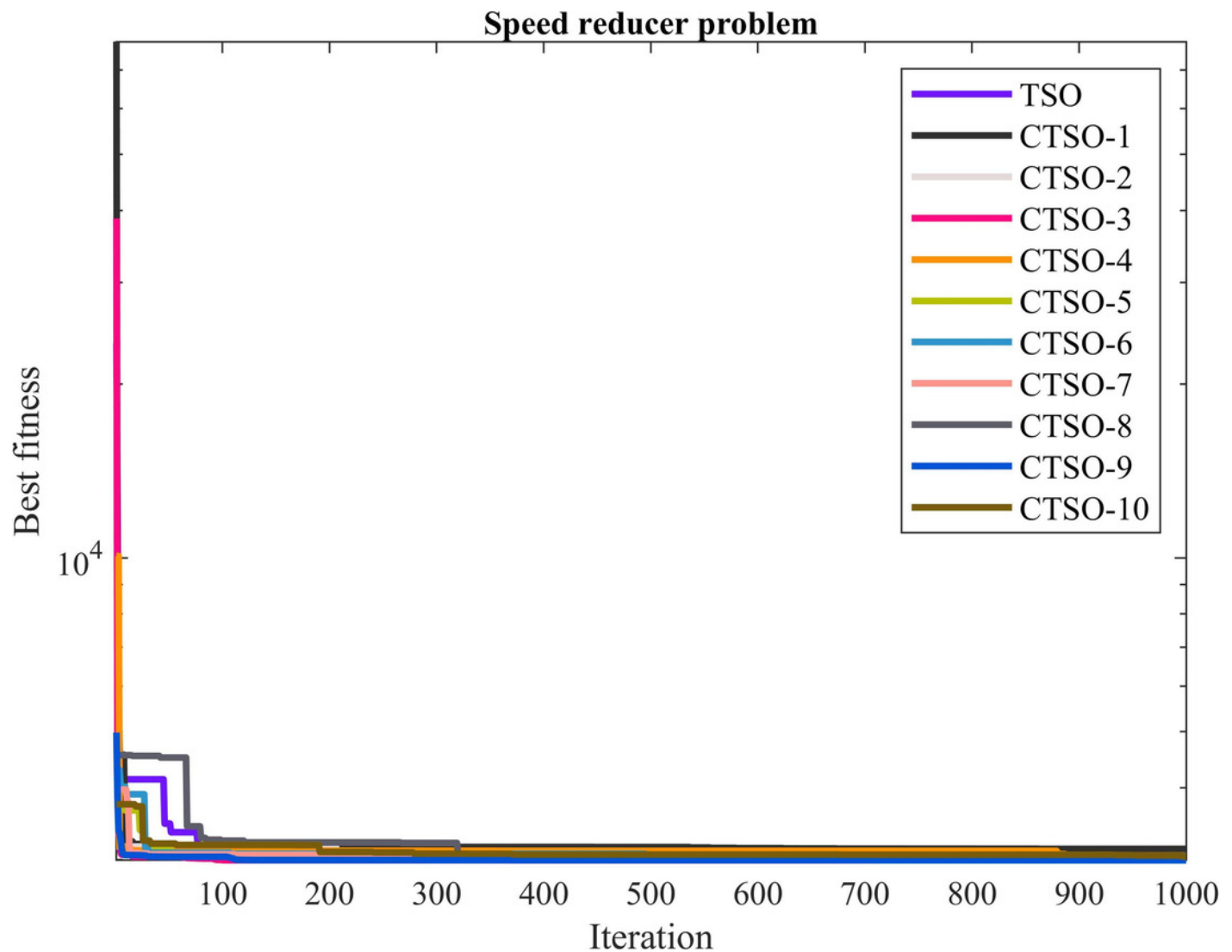
Demonstration of chaotic maps

# Figure 2

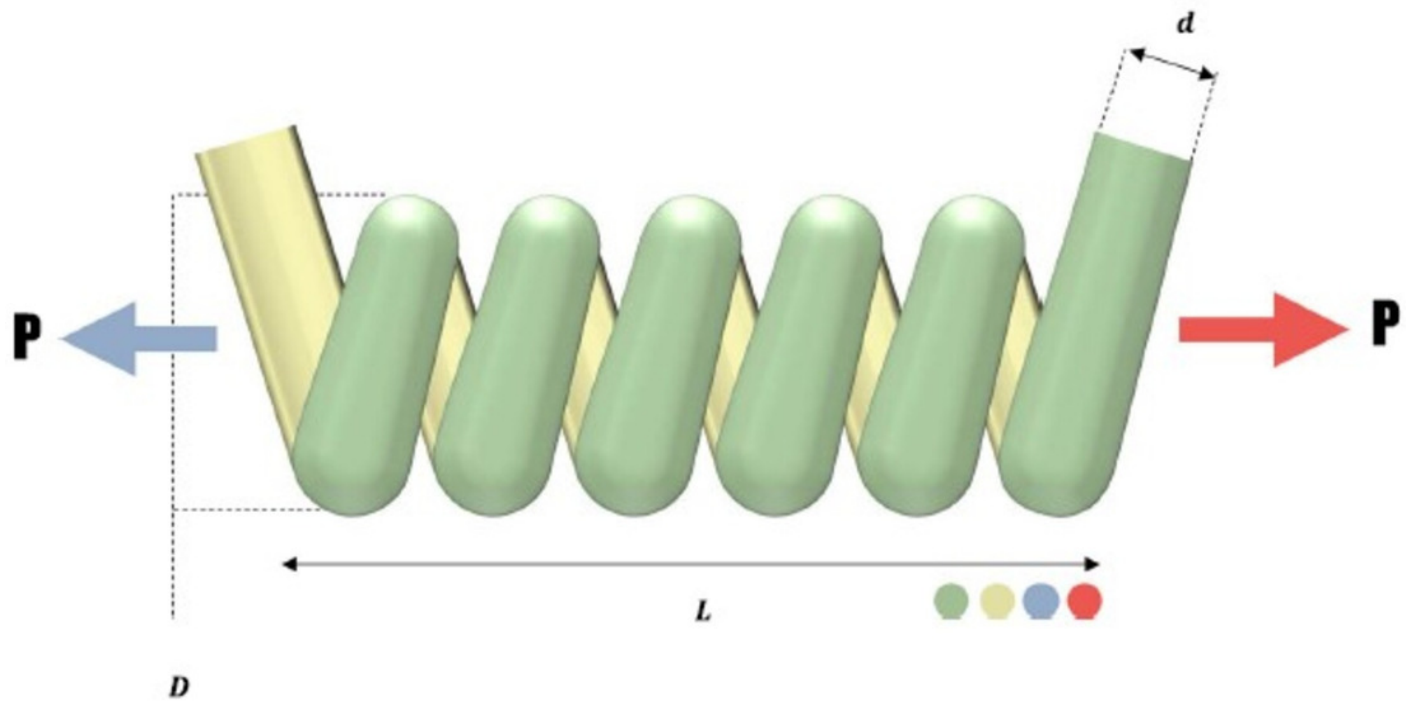
Flowchart of CTSOs

# Figure 3

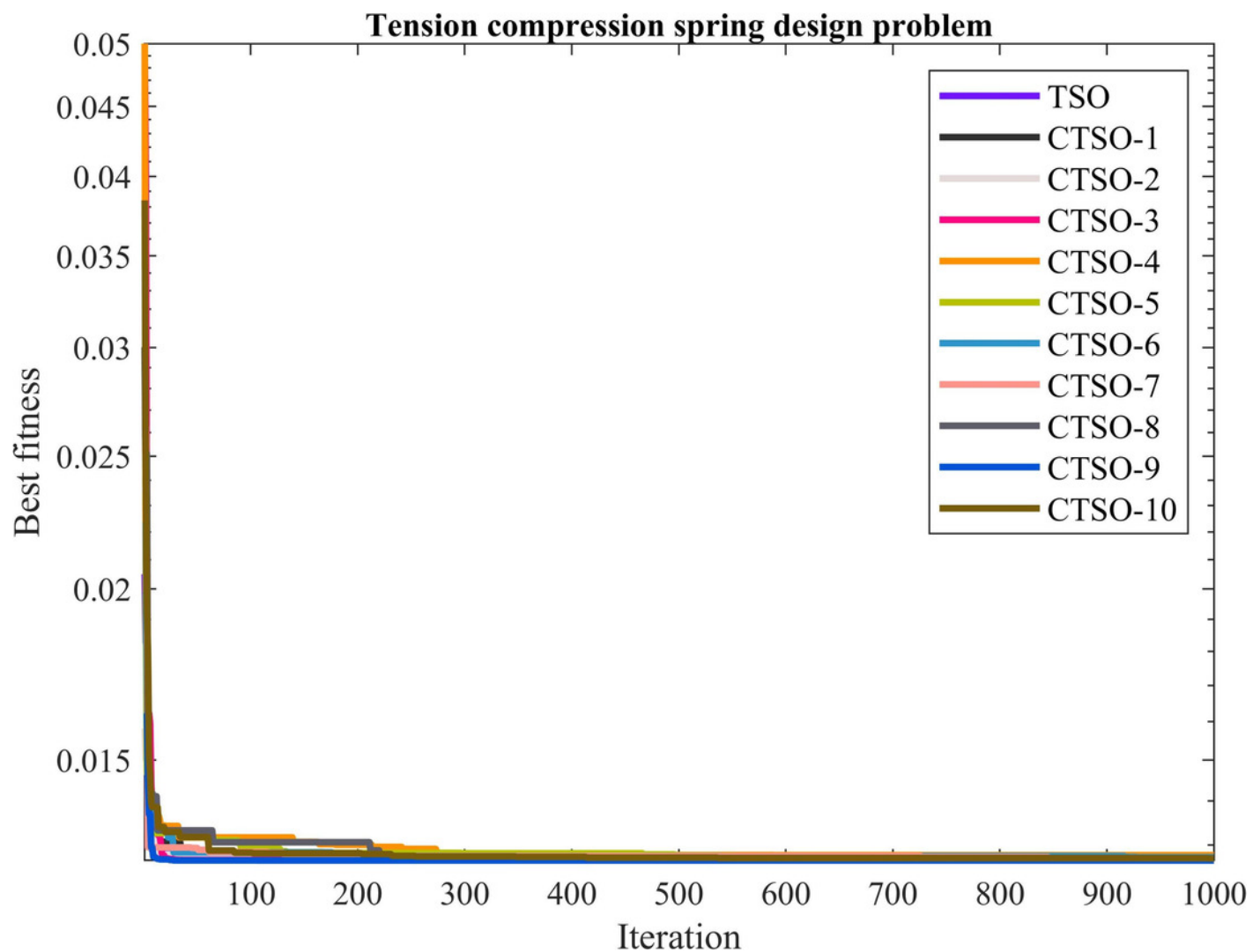
Convergence curve graph of CEC'17 test functions

# Figure 4

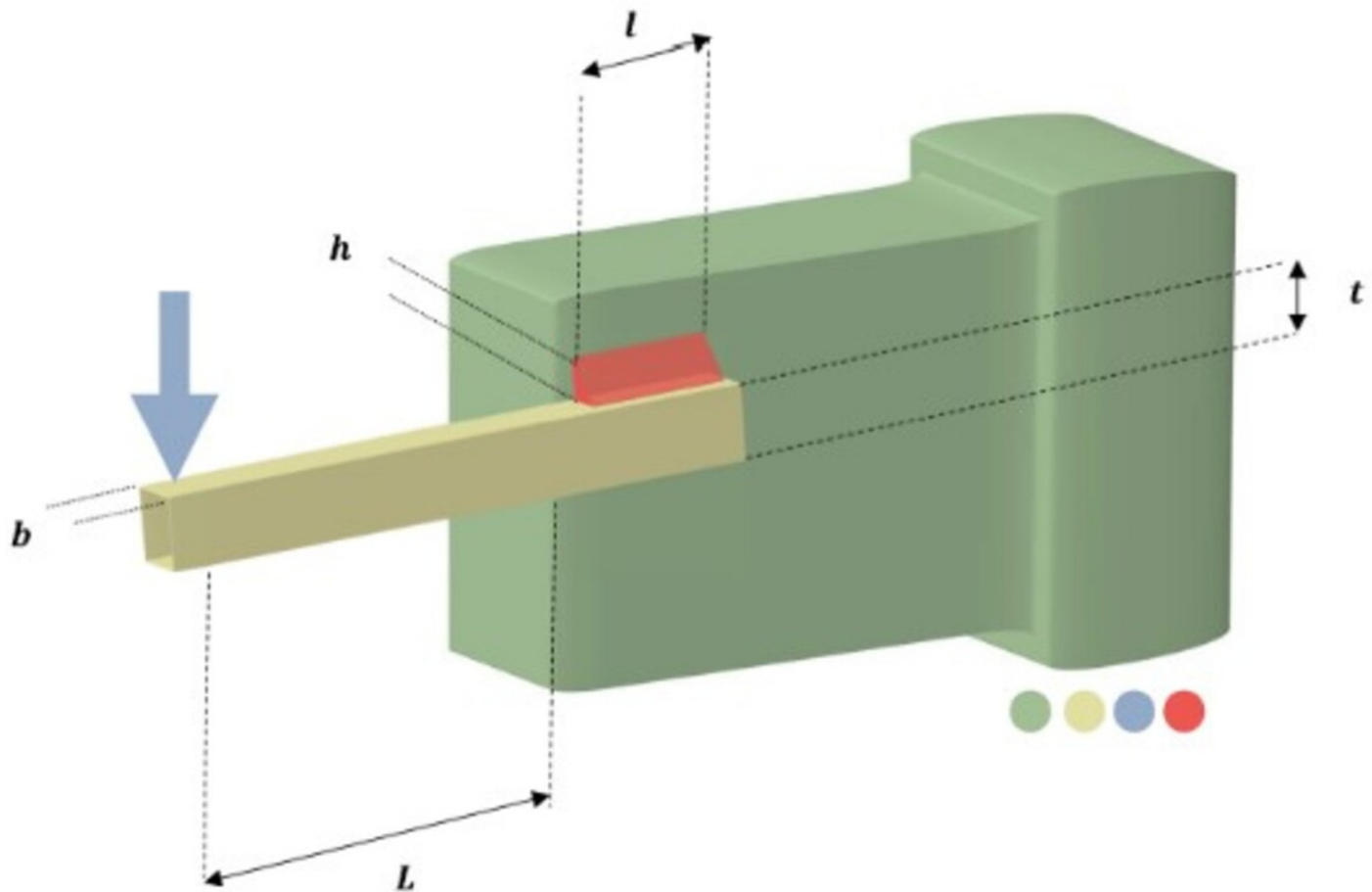
Boxplot of CEC'17 test functions

# Figure 5

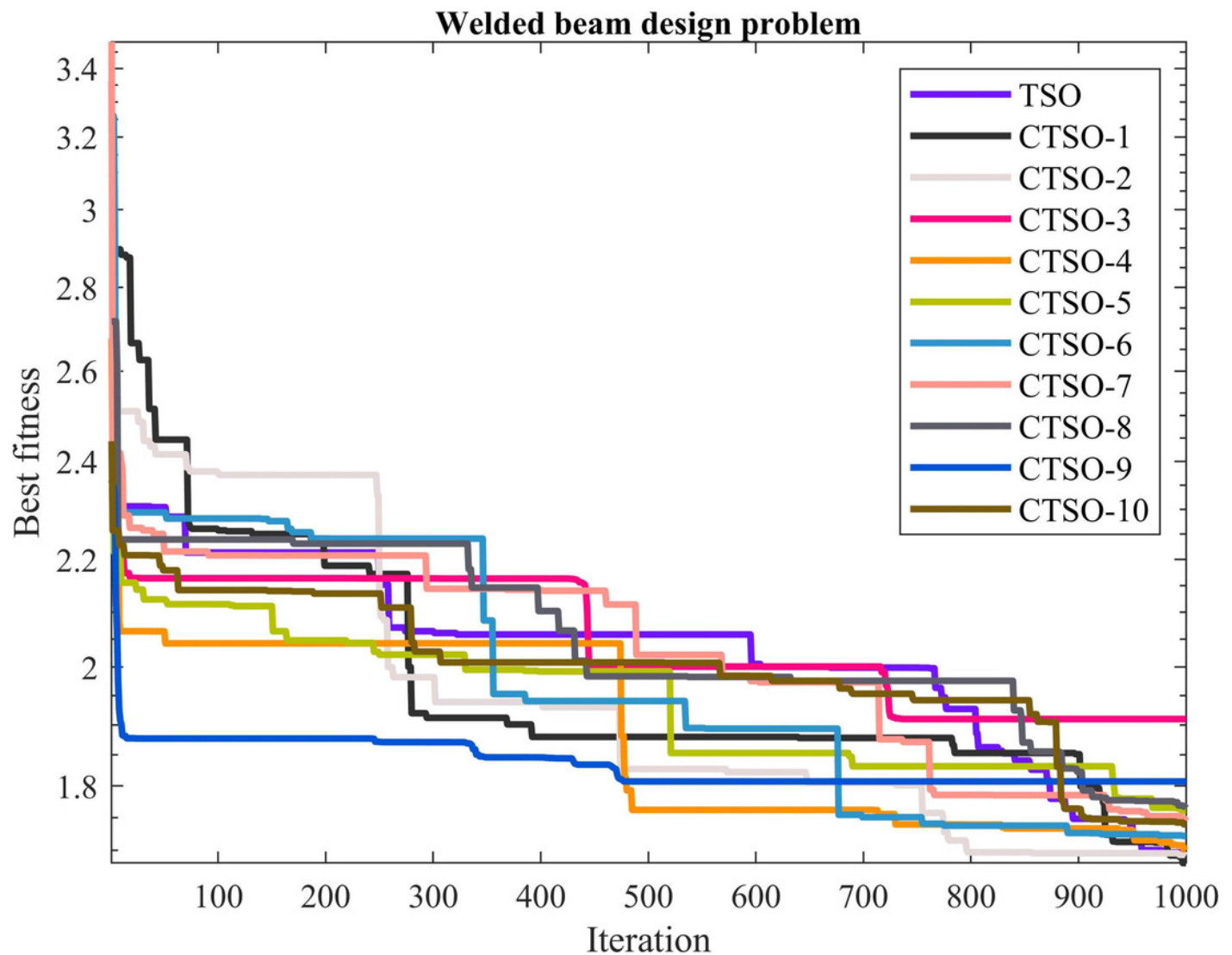
Speed reducer problem

# Figure 6

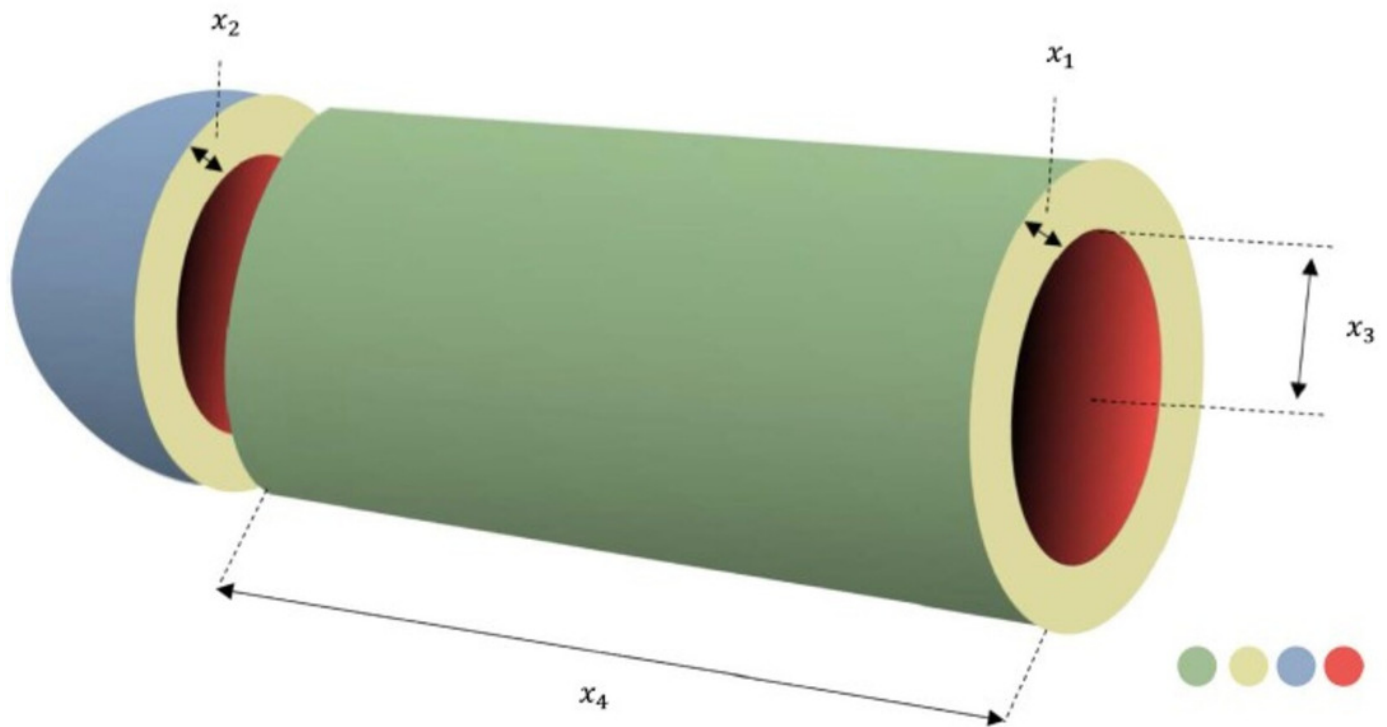
The convergence graph of the methods used on the speed reducer problem

# Figure 7

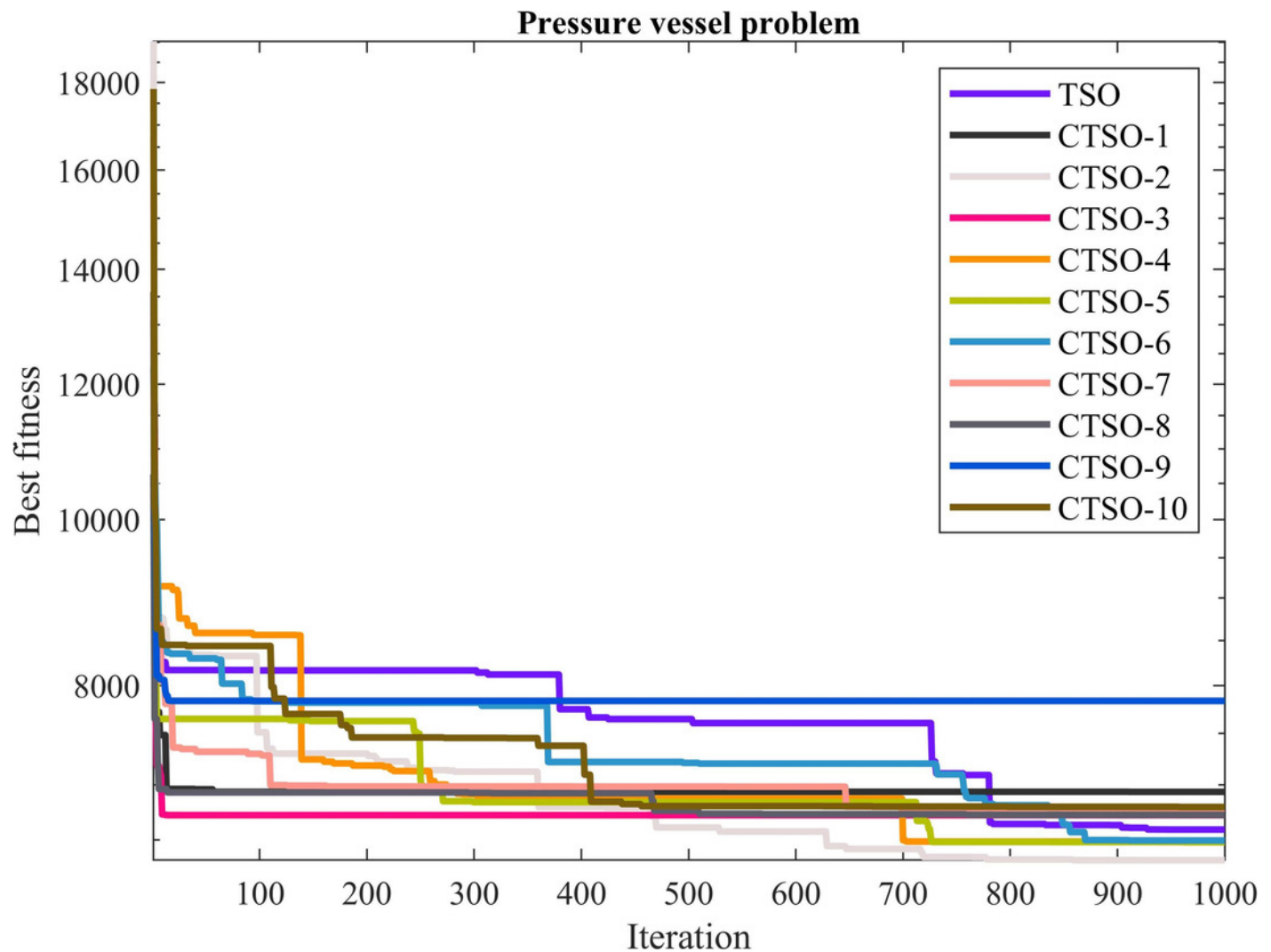
Tension compression spring design problem

# Figure 8

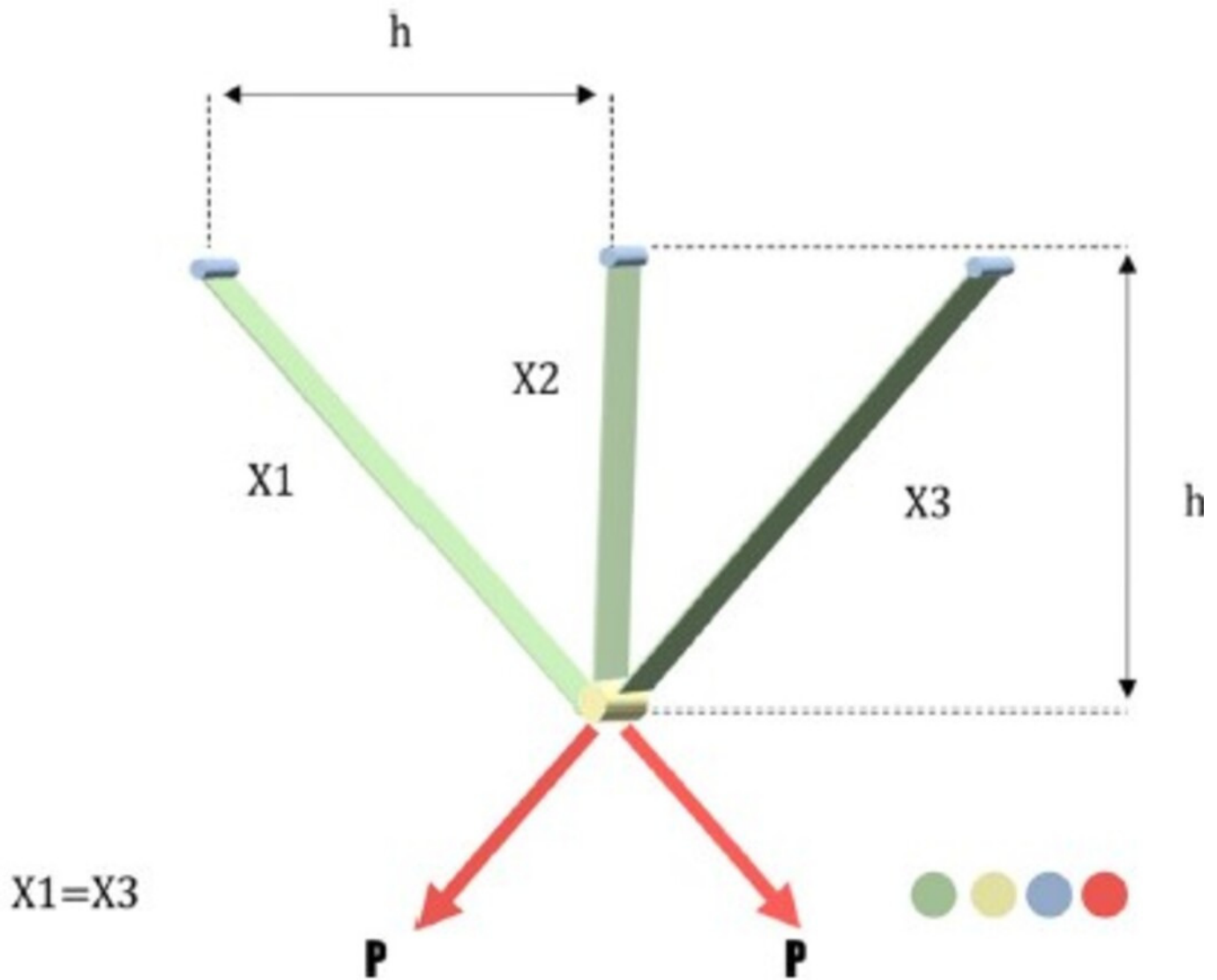
The convergence graph of the methods used on the tension compression spring design problem

# Figure 9

Welded beam design problem

# Figure 10

The convergence graph of the methods used on the welded beam design problem

# Figure 11

Pressure vessel design problem

# Figure 12

The convergence graph of the methods used on the pressure vessel problem

# Figure 13

Three bar truss design problem

# Figure 14

The convergence graph of the methods used on the three-bar truss design problem