

# Research on Automatic Matching of Online Mathematics Courses and Design of Teaching Activities Based on Multiobjective Optimization Algorithm

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The teaching of the optimization algorithm is a new kind of swarm intelligence optimization technique, which is superior in optimizing many simple functions. Still, it is not evident in processing some complex problems (group and teaching classification). Achieving automatic matching and knowledge transfer in online courses is imperative in mathematics education. This study proposes a design scheme MTCBO-LR (Multiobjective Capability Optimizer-Logistic Regression), based on multitask optimization, which enables precise knowledge transfer and data interaction among many educators. It incorporates the standard TLBO algorithm to optimize, provides a variety of learning tactics for students at different stages of mathematics instruction, and is capable of adaptively adjusting these strategies in response to actual teaching needs. Experimental results on various datasets reveal that the proposed method enhances searchability and group diversity in various optimization extremes and outperforms similar methods in resolving to multitask teaching problems.

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## 16 Abstract

17 The teaching of the optimization algorithm is a new kind of swarm intelligence optimization  
18 technique, which is superior in optimizing many simple functions. Still, it is not evident in  
19 processing some complex problems (group and teaching classification). Achieving automatic  
20 matching and knowledge transfer in online courses is imperative in mathematics education. This  
21 study proposes a design scheme MTCBO-LR (Multiobjective Capability Optimizer-Logistic  
22 Regression), based on multitask optimization, which enables precise knowledge transfer and  
23 data interaction among many educators. It incorporates the standard TLBO algorithm to optimize,  
24 provides a variety of learning tactics for students at different stages of mathematics instruction,  
25 and is capable of adaptively adjusting these strategies in response to actual teaching needs.  
26 Experimental results on various datasets reveal that the proposed method enhances  
27 searchability and group diversity in various optimization extremes and outperforms similar  
28 methods in resolving to multitask teaching problems.

29 **Keywords:** Task optimization; Teaching optimization algorithm; Multiobjective optimization

30

## 31 1 Introduction

32 The current trend in educational tasks is increasingly focused on personalized and specific  
33 development, aiming to meet the unique educational needs of each student and achieve  
34 dynamic resource allocation [1]. Therefore, the school education system should continue to

35 explore and improve teaching methods. Intelligent teaching-assistant systems are a solution that  
36 can automatically allocate appropriate learning resources and tasks based on students' learning  
37 situations and performances, achieving dynamic resource allocation. Meanwhile, teachers  
38 should pay attention to and analyze students' personalized needs, stimulating their learning  
39 interests and potential through diversified teaching methods and strategies. This personalized  
40 and specific teaching approach not only helps improve students' learning effectiveness but also  
41 cultivates their innovation and problem-solving abilities, better adapting to the needs of future  
42 society[2].

43 From design modelling to task optimization, viewing mathematics education as a  
44 multiobjective optimization issue that can be solved by iteratively refining a set of objectives  
45 under constrained decision-making variables is gaining traction as a promising new approach.  
46 Evolutionary multitask, or multi-factor optimization, is a new emerging sub-field in optimization  
47 that combines the ideas of evolutionary computation and multitask learning. Unlike the previous  
48 two, multi-factor optimization integrates multiple interrelated objective functions to solve various  
49 optimization tasks simultaneously and explore potential relationships among tasks to improve  
50 efficiency and effectiveness[3]. Mathematics teaching is a four-satellite activity. Students can  
51 perform multiple tasks. To effectively utilize the commonalities and differences among various  
52 problems, knowledge of one job should be used to assist the solution of another job, which is a  
53 multiobjective problem-solving process.

54 Existing algorithms typically solve singular optimization tasks in isolation and seldom  
55 leverage knowledge gained from one task to help solve another[4]. However, it is plausible that  
56 correlations exist between diverse problems, potentially augmenting the efficiency and  
57 effectiveness of problem-solving. Drawing inspiration from the human ability to manage multiple  
58 tasks, Gupta et al. [5] employed multitask learning in evolutionary computation, introducing a  
59 novel optimization problem category known as multi-factor optimization and a multi-factor  
60 evolutionary algorithm. The objective of this algorithm is to exploit potential synergies among  
61 disparate optimization problems, making efficient use of both shared and distinct characteristics  
62 of the various issues. This paper introduces the MTCBO framework, which is founded on  
63 multitask optimization. The MTCBO algorithm is based on the principles of teaching optimization  
64 algorithms. It capitalizes on the fact that students can transfer knowledge from teachers or  
65 classmates with similar or dissimilar attributes, effectively leveraging the commonalities between  
66 problems. This approach has the potential to greatly enhance the efficiency and performance of  
67 problem-solving.

68 We propose MTCBO-LR, which can realize the function of introducing different learning  
69 strategies at different stages. Three learning strategies are presented to update knowledge at  
70 the teaching level, and two are introduced to update knowledge at the student level. The main  
71 contribution of this paper is

72 (1) Using multiple strategies can maintain the diversity of the population while maintaining a  
73 higher convergence accuracy and speed. The algorithm adaptively selects appropriate learning  
74 strategies, enhances the positive influence of shared information, and improves the overall  
75 optimization ability of the algorithm.

76 (2) The normal distribution perturbation strategy is introduced to help students escape from  
77 local optimality. The selection strategy based on the ranking is adopted to find high-quality  
78 solutions while maintaining the population's diversity, ensuring the algorithm's generalization and  
79 accuracy..

## 80 **2 Related Work**

81 The complexity of mathematics teaching tasks is continuously increasing, leading to  
82 increased complexity in teaching preparation and student acceptance of the teaching process.  
83 To address these challenges, researchers have proposed the concept of multiobjective  
84 optimization. By leveraging multiobjective optimization techniques, it is possible to transfer  
85 knowledge across diverse tasks, thereby improving overall problem-solving capabilities.  
86 Researchers have made significant progress in implementing multiobjective optimization  
87 algorithms and have devised effective methods to leverage information among tasks.

88 For instance, Bali et al. presented a novel evolutionary computing framework that can learn  
89 online and enhance optimization. Zhou [6] proposed the MFEA-AKT algorithm, which utilizes  
90 adaptive knowledge transfer to match appropriate crossover operators adaptively. Gupta  
91 introduced the multi-factor evolutionary algorithm, which combines multitask optimization with  
92 evolutionary algorithms. Liang et al. [7] proposed a novel multi-factor evolutionary algorithm with  
93 a genetic transformation strategy that can improve knowledge transfer efficiency.

94 These approaches facilitate exploring and exploiting subspaces for each task and the  
95 combined search space, enabling individuals to discover lesser-known areas and enhance  
96 optimization performance. The advent and evolution of multiobjective optimization algorithms  
97 offer effective techniques and concepts for tackling mathematical teaching tasks.

98 Research into multitask evolutionary algorithms has received considerable attention to  
99 lessen the computational load; Huang et al. proposed the SAEF-AKT framework, which uses an  
100 adaptive knowledge transfer strategy and creates a proxy model by mining past searches for  
101 information. Similarly, Wu [8] introduced the MFEA/D-DRA algorithm that employs decomposition  
102 and dynamic resource allocation strategies to transform the problem into several sub-problems  
103 for adaptive resource allocation. Furthermore, Bali et al. presented the cognitive evolutionary  
104 multitask engine, which analyzes the data produced during multitask optimization and lowers  
105 optimization costs by adjusting the degree of online genetic transfer.

106 Some investigations have proposed novel algorithms and frameworks to expedite the  
107 optimization process. For instance, the collaboration protocol evolutionary framework introduced  
108 by Chen et al. can partition the problem into low-dimensional subproblems and use a locally-  
109 search algorithm grounded on quasi-Newton methods to achieve knowledge exchange and local  
110 search for solving high-dimensional optimization problems. Similarly, Hao et al.[9] developed the  
111 EMHH algorithm, a graph-based evolutionary multitask hyperheuristic algorithm that addresses  
112 multitask problems through cooperative action. Additionally, Xu et al.[10] presented the MTO-  
113 FWA algorithm, which employs transfer sparks to transmit genetic information to enhance  
114 optimization efficiency. Furthermore, Feng et al. integrated multi-factor and differential evolution  
115 algorithms to resolve multiobjective optimization problems. Rao et al.[11] devised a heuristic

116 swarm intelligence optimization algorithm named Teaching-Learning-Based Optimization  
 117 (TLBO), which optimizes by emulating classroom teaching processes. TLBO outperforms  
 118 traditional swarm intelligence optimization algorithms with fewer control parameters, a more  
 119 straightforward overall structure[12-13], faster-running speed, and more straightforward  
 120 implementation. Consequently, it has been widely adopted in various fields, capturing the  
 121 attention of many researchers and continually improving[14-15]. The MTCBO-LR algorithm  
 122 incorporates the normal distribution perturbation strategy. After generating new descendant  
 123 students, the MTCBO-LR algorithm utilizes a rank-based selection strategy to retain students  
 124 with superior quality.

### 125 3 Method

#### 126 3.1MTCBO

127 EMO algorithm plays a significant role in scientific research and engineering applications[16].  
 128 Therefore, solving multiobjective optimization problems is of great importance. Here, we first  
 129 define EMO:

$$\min F(x) = (f_1(x), f_2(x), \dots, f_m(x)) \quad (1)$$

130 The EMO algorithm is a method for solving optimization problems with multiple conflicting  
 131 objective functions by seeking numerous optimal solutions to balance the relationship between  
 132 these objective functions. EMO algorithms are widely used in scientific research and engineering  
 133 applications, such as machine learning, logistics planning, power system optimization, etc.. They  
 134 can help people better understand and solve practical problems.

135 The present study introduces an enhanced single-objective multitask optimization algorithm,  
 136 MTCBO. The primary iteration of this algorithm consists of two scenarios. During the initial  
 137 stages of the algorithm, if the number of solutions in the archive set Arc is less than Na, i.e., Arc  
 138 is small in size, it is challenging to estimate the trend of the Pareto Front (PF) accurately. We  
 139 propose the MTCBO approach for optimizing the MOP method [17] to address this. This  
 140 technique uses crossover and mutation operators on P populations to create new offspring  
 141 populations. The proposed method subsequently updates the archive set Arc and applies fast,  
 142 non-dominated sorting and crowded distance selection to choose the next generation population  
 143 in the population.

144 These two individuals are selected from different sub-populations if this probability condition  
 145 is unmet. Finally, we update the archive set Arc and sub-populations p1, p2, ...,

$$\{x_1, x_2, \dots, x_k\} = \{\arg \min P_1(x_1), \arg \min P_2(x_2), \dots, \arg \min P_K(x_K)\} \quad (2)$$

#### 146 3.2 MTCBO-LR

##### 147 3.2.1 Adaptive Knowledge Transfer

148 To improve the MTCBO algorithm, we propose two methods of adaptive knowledge transfer  
 149 for improvement. Each student chooses learning strategies from the two strategies according to  
 150 their situation, and students update knowledge through appropriate learning strategies in  
 151 different optimization stages. Different from the greedy selection strategy of the original TLBO

152 algorithm, the selection of new offspring is based on individual factor value and factor diversity.

153 Following the above principles, we divided all students into different classes, each class was  
154 assigned different learning strategies based on skill factors, and the performance of the

155 strategies varied. Among them, teacher  $X_{teacher, \tau}^f$  and  $X_{teacher, n}^f$  were the best individuals from

156 class and class factor value, respectively. Teaching assistants  $X_{k, \tau}^m$  and  $X_{k, n}^m$  were randomly

157 selected individuals from class  $P_{\tau}$  and class  $P_n$ , respectively. Mean and were the average scores

158 of class  $P_{\tau}$  and class  $P_n$ , respectively. The individual update formula is as follows:

$$X_{i, new, \tau, new} = \begin{cases} X_{i, \tau} + \text{rand}(1, D) \cdot \left( \left( X_{teacher, \tau}^f + X_{k, \tau}^n \right) / 2 - TF * X_{mean, \tau} \right) & \text{if } \text{rand} \geq rmp \\ X_{i, \tau} + \text{rand}(1, D) \cdot \left( \left( X_{teacher, n}^f + X_{k, n}^m \right) / 2 - TF * X_{mean, n} \right) & \text{otherwise} \end{cases} \quad (3)$$

159 X Strategy 1: This strategy aims to increase diversity among each student. Teachers and  
160 teaching assistants provide instruction based on the difference between their average level and  
161 the class's average score.

$$X_{i, new, \tau, new} = \begin{cases} X_{i, \tau} + \text{rand}(1, D) \cdot \left( \left( X_{teacher, \tau}^f + X_{k, \tau}^n \right) / 2 - TF * X_{mean, \tau} \right) & \text{if } \text{rand} \geq rm \\ X_{i, \tau} + \text{rand}(1, D) \cdot \left( \left( X_{teacher, n}^f + X_{k, n}^m \right) / 2 - TF * X_{mean, n} \right) & \text{otherwise} \end{cases} \quad (4)$$

162 X Strategy 2: A reverse learning mechanism is implemented in this learning strategy.

$$X_i^* = U + L - X_i \quad (5)$$

163 The value of RMP allows for balancing between exploration and exploitation of the search  
164 space. When RMP[18] is close to 1, students will always receive knowledge transfer from  
165 individuals with the same attribute, which can help scan critical areas of the search space but  
166 increase the risk of falling into local optima. Conversely, when rmp is lower than 1,  
167 communication between individuals from different cultures can aid in escaping local optima.  
168 Figure 1 illustrates the algorithm flow of MTCBO-LR, which follows the above principles.

169

170 Figure 1 Algorithm flow of MTCBO-LR

171

### 172 3.3 Adaptive Matching and Vertical Spread Strategies

173 In the later stage of learning, students are likely to encounter learning bottlenecks that make  
174 it difficult to continue improving. Through a certain degree of "perturbation", students can be  
175 helped to break out of the bottleneck, explore a wider search space, and find better solutions.  
176 The criterion that initiates this strategy is as follows: when the optimal value of any task factor

177 remains unchanged for ten successive generations, all students will undergo perturbation, and  
 178 their knowledge will be updated according to their respective learning strategies. The formula for  
 179 updating individuals is as follows:

$$X_{i_{new}, \tau_{new}} = X_{i, \tau} + \text{normrnd}(0, 0.01, 1, D) \quad (6)$$

180 In the later stages of learning, students are likely to encounter learning bottlenecks that are  
 181 difficult to overcome. By introducing a certain degree of "disturbance", students can break  
 182 through bottlenecks, explore a broader search space, and find better solutions[19]. The trigger  
 183 condition for this strategy is: when the optimal factor value of any task does not change for ten  
 184 consecutive according to the learning strategy. The individual update formula is as follows:

185 Where  $\text{ormrnd}(0, 0.01, 1, D)$  is a  $1 \times D$  normal distribution matrix, 0 and 0.01 are the mean  
 186 and standard deviation, respectively.

187 The MTCBO-LR method refers to two MFEA algorithms, the original MFEA and MFEA-AKT.  
 188 Unlike the original MFEA, which uses a single crossover operator, MTCBO-LR uses multiple  
 189 crossover operators with different search performances for knowledge transfer. Specifically,  
 190 during the algorithm execution process, MTCBO-LR randomly selects two parent individuals to  
 191 generate offspring individuals. If these two parent individuals have the same skill factor, the SBX  
 192 crossover operator is used for crossover operation. Otherwise, whether to perform crossover or  
 193 mutation operation on these two parent individuals is determined according to the preset random  
 194 mating probability. Suppose parents have different skill factors and need to perform crossover  
 195 operations. In that case, MTCBO-LR will randomly assign  $cf$  for activating the crossover factor  $W$   
 196 or  $p$  and use the corresponding crossover operator to perform crossover operations on  $W$  and  $p$ .  
 197 In addition, the generated individuals, i.e., migration offspring, will use the crossover factor  $Cfa$   
 198 as their attribute. If two offspring  $C1$  and  $C2$ , are generated by a crossover operation or a  
 199 mutation operation without knowledge transfer, then  $C1$  and  $C2$  will inherit their corresponding  
 200 parent crossover factor, respectively.

201

$$\begin{aligned} c_1 &= (p_1^1, p_1^2, \dots, p_2^i, p_2^{i+1}, \dots, p_2^j, p_1^{j+1}, \dots, p_1^n) \\ c_2 &= (p_2^1, p_2^2, \dots, p_1^i, p_1^{i+1}, \dots, p_1^j, p_2^{j+1}, \dots, p_2^n) \end{aligned} \quad (7)$$

### 202 3.4 Selection strategy based on KNN

203 The KNN model classifier was trained using a historical transfer data set containing positive  
 204 and negative transfer individuals[20], denoted as HTS. Let  $M$  represent the set of all trained KNN  
 205 model classifiers. The following steps were taken to train the KNN model classifier: (1) A  
 206 similarity matrix was constructed based on the distance between individuals in HTS. The initial  
 207 label of all training data was set to "unlabeled." The maximum local neighbour  $N$  of each  
 208 individual in HTS, which covers the maximum number of neighbours of the same category, was  
 209 calculated. (2) The maximum local neighbour  $N_i$  of all individuals  $S_i$  with the "unlabeled" label  
 210 was put into set  $Q$ . (3) The maximum value  $N$  in set  $Q$  was found, and a model  $M = \langle Cls(s;),$   
 211  $Sim(s;), Num(s;), Rep(s;)\rangle$ , where  $Num(s;) = N$ , was constructed. This indicates that  $s;$  covers  
 212 the maximum number of neighbours of the same category. Model  $M$  was then put into the model

213 set M.

214 Using the KNN classifier based on the above steps can avoid the problem of randomly  
 215 selecting individuals from the current population, which often leads to repeat visits to hopeless  
 216 regions in the search space and low efficiency in traditional algorithms. Mathematical proofs  
 217 suggest that introducing a conflicting solution and its opposite has greater potential than  
 218 introducing two unrelated random solutions to approach the optimal global solution without prior  
 219 information[21]. Therefore, we defined a conflicting point in this paper. Additionally, a selection  
 220 strategy was designed that considers both factor value and factor diversity to update individuals.

221 In this paper, we design a selection strategy considering factor value and diversity to renew  
 222 individuals. It is implemented in the MTCBO-LR algorithm. For the populations corresponding to  
 223 task T, we arranged them from most minor to most significant regarding factor value and  
 224 assigned Fitness Rank (FR) to each individual. The calculation formula of FR is as follows:

225

$$FR_j^i = i, \quad i = 1, 2, \dots, NP \quad (8)$$

226 The smaller the factor value of an individual, the lower its corresponding fitness level. The  
 227 population corresponding to task Tyj is sorted in ascending order of factor diversity, and the  
 228 resulting order is the diversity rank (DR) of the individual, which is defined by the formula:

$$DR_j^i = i, \quad i = 1, 2, \dots, NP \quad (9)$$

229 The smaller the factor diversity of an individual, the smaller the corresponding diversity level.  
 230 In task Tyj, the rankRy of individual Xyi is defined as:

$$R_j^i = \omega \cdot DR_j^i + (1 - \omega) \cdot FR_j^i \quad (10)$$

231 where  $\omega = \frac{G}{Maxgen}$  is the current iteration number and Maxgen is the maximum iteration  
 232 number.

## 233 4 The experiment

### 234 4.1 Experiment Settings

235 This experiment mainly verifies the optimization effect of the proposed method on small-  
 236 scale problems, which is suitable for forming small-class online collaborative learning groups. A  
 237 simulated data set containing seven features of 30 learners (numbered S1-S30) was selected.

238 To assess optimization test problems, this study selected seven sets of classic single-  
 239 objective to multitask optimization problems and ten sets of complex single-objective multitask  
 240 optimization problems for analysis. Each benchmark test problem consisted of two component  
 241 tasks containing a single-objective optimization task. The seven classic Multitask Optimization  
 242 Technique Reports had varying degrees of overlap and similarity between the component tasks.  
 243 The overlap degree refers to the similarity or difference of the optimal global solution of the two  
 244 tasks in the same search space, with three types of overlap degrees. CI meant that the

245 component tasks in the benchmark test problem had precisely the same global optimal solution.  
246 PI meant that the optimal global solution of the component tasks in the benchmark test problem  
247 was partially the same. NI meant that the component tasks in the benchmark test problem had  
248 completely different global optimal solutions. The similarity degree refers to the similarity of the  
249 shape and size of the two tasks in the same search space. It should be noted that since the  
250 MTCBO-LR algorithm processed two tasks once in each iteration, while the SOTLBO algorithm  
251 processed only one task, the total number of iterations of the SOTLBO algorithm was 250 when  
252 reaching the termination condition, while the MTCBO-LR algorithm was 500.

#### 253 4.2 Single-objective Task Benchmarking Problem Performance

254 Assuming the mean and standard deviation were 0 and 0.01, respectively, the other  
255 parameters were the same as the original publication. Table 1 presents the average factor  
256 values and standard deviations of five algorithms independently run 20 times, with the best  
257 experimental results highlighted in bold. The final comparison was based on the average results  
258 of 20 independent runs.

259

260 As shown in Table 1, the MTCBO-LR algorithm performs better in 7 tasks. Specifically, the  
261 MTCBO-LR algorithm has a minor score in all group problems in the seven tasks and has a clear  
262 advantage, especially compared to the traditional SOTLBO algorithm. This is because the  
263 MTCBO-LR algorithm employs multiple strategies to maintain population diversity, thus  
264 maintaining high convergence accuracy. The superiority of the MTCBO-LR algorithm  
265 demonstrates that novel and adaptive learning strategies can reduce the negative transfer of  
266 knowledge and that rank-based selection strategies can maintain population diversity while  
267 finding high-quality solutions, enhancing the algorithm's development and exploration  
268 capabilities. The experimental results are confirmed in Figures 2, 3, and 4, indicating that the  
269 MTCBO-LR algorithm has high effectiveness in solving MTO problems.

#### 270 4.3 Multiobjective Task Benchmark Performance

271 To assess the efficacy of the MTCBO-LR algorithm on multiobjective tasks, we conducted a  
272 comparative analysis across five distinct task combinations. To ensure parity across all trials, we  
273 maintained a population size of 100 and a maximum evaluation threshold of 200,000 as the  
274 termination criterion[22]. Each algorithm underwent 20 independent runs for the sake of  
275 comparison. This study employed the iteration count of the MTCBO-LR algorithm as the  
276 benchmark, set as the abscissa. It used the corresponding true factor value of all algorithms as  
277 the ordinate. Our findings indicate that the MTCBO-LR algorithm outperforms other EMT  
278 algorithms regarding convergence speed across most test sets. The algorithm generates high-  
279 quality solutions within fewer than 150 iterations for the CI+HS and NI+HS test problems. This  
280 superior performance is attributed to the algorithm's multifaceted approach and enhanced  
281 accuracy and speed of convergence. In contrast, other algorithms could not attain comparable  
282 results at termination. Table 2 provides an overview of the parameter settings for the comparison  
283 algorithms.

284 Table 3 details the parameter configurations for both the MFEA-GHS and MFEA-AKT  
285 algorithms. To analyze the convergence performance of the MTCBO-LR algorithm, this study

286 presents graphical visualizations of the convergence of the SOTLBO, MFEA, MFDE, MFPSO,  
287 and MTCBO-LR algorithms across five distinct sets of single-objective multitask test problems. It  
288 is worth noting that each algorithm was run for differing iterations. Using a population size of 100  
289 and a maximum evaluation threshold of 100,000, each algorithm underwent 20 independent runs,  
290 and the convergence curve graphs for four strategies are provided.

#### 291 4.4 Global Strategy Effectiveness Evaluation

292 The MTCBO-LR algorithm incorporates position. These individuals can generate new  
293 populations from their optimal historical positions. The optimal global position (gbest) selection  
294 depends on the distribution of non-dominant solutions. The unsuccessful game strategies learn  
295 from the successful ones and refer to the specific game process network. This paper adopts a  
296 corresponding strategy evaluation, and convergence curves are constructed for MTCBO-LR and  
297 seven different algorithms targeting two different tasks, as shown in Figures 5 and 6. The graphs  
298 illustrate that the convergence curve IGD value of the MTCBO-LR algorithm is higher, and the  
299 convergence speed is faster, indicating better performance.

#### 300 4.5 Discussion

301 In the MTTLBO algorithm, students learn from teachers and classmates in different tasks,  
302 realizing cross-task knowledge transfer. Introducing a reverse learning mechanism in the  
303 "teaching" stage can prevent convergence too fast and fall into local optimal. The experimental  
304 results show that compared with the existing evolutionary multitasking algorithms, the MTTLBO  
305 algorithm achieves satisfactory results in single-objective multitasking optimization. In the  
306 MTTLBO-MR algorithm, each student learns from other individuals in different tasks, realizing  
307 cross-task knowledge transfer and multi-learning strategy ensures the diversity of search. In the  
308 late iteration period, the normal distribution perturbation strategy is introduced to prevent falling  
309 into local optimal effectively. Specifically, the training results of the model can better enable the  
310 members of the online collaborative learning group to help each other, ensure heterogeneity  
311 within the group, and realize the mixed grouping of learners with different cognitive levels and  
312 learning styles to realize the complementary features among the members of the learning group.  
313 The formed online learning group contains learners with different characteristics. When  
314 completing collaborative learning tasks, various thinking modes of learners of different types  
315 interact and collide within the group to realize brainstorming, improve the efficiency of  
316 collaborative learning, and promote the cultivation of various abilities of learners.

317

## 318 5 Conclusion

319 In this paper, we propose the MTCBO multiobjective optimization strategy and the phased  
320 optimization method of MTCBO-LR, which can give different teaching strategies for students with  
321 varying learning situations to achieve better optimization results. At the same time, we also  
322 propose three different learning strategies for global effectiveness evaluation to guide the  
323 algorithm's exploration and utilization in the search process. These learning strategies can  
324 expand the exploration range, increase the exploration depth, and improve the search accuracy,  
325 thus effectively improving the optimization effect.

326 Our experiments showed that the MTCBO-LR algorithm showed excellent optimization  
327 results in different test problems. This demonstrates the effectiveness of our proposed  
328 multiobjective optimization strategy and phased optimization method, which can significantly  
329 improve the learning effect. The multiobjective optimization strategy of MTCBO proposed in this  
330 paper and the phased optimization method of MTCBO-LR, as well as the introduction of learning  
331 strategies, provide a new idea and method for solving multiobjective optimization problems.  
332 Future research can further explore how to apply these strategies in practical problems and how  
333 to combine them with other optimization algorithms to improve the optimization effect further.

334 Currently, the most lacking mathematics education is reasonable and reliable teaching  
335 strategies and personalized plans. The MTCBO-LR algorithm designed in this paper can  
336 effectively realize the complement process of the corresponding strategy, ensure the successful  
337 implementation and development of teaching, and achieve good teaching results.

338

339

## 340 Reference

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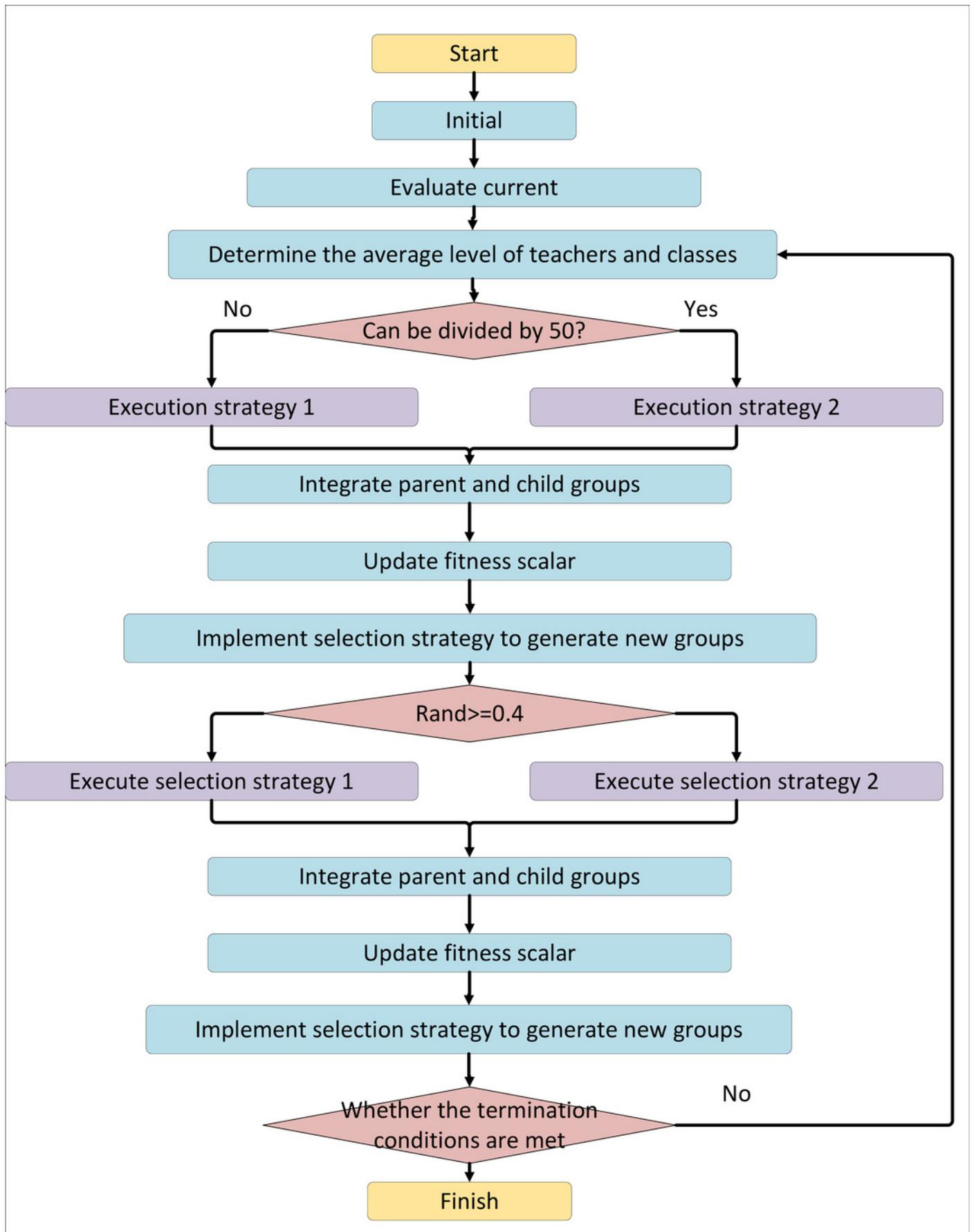
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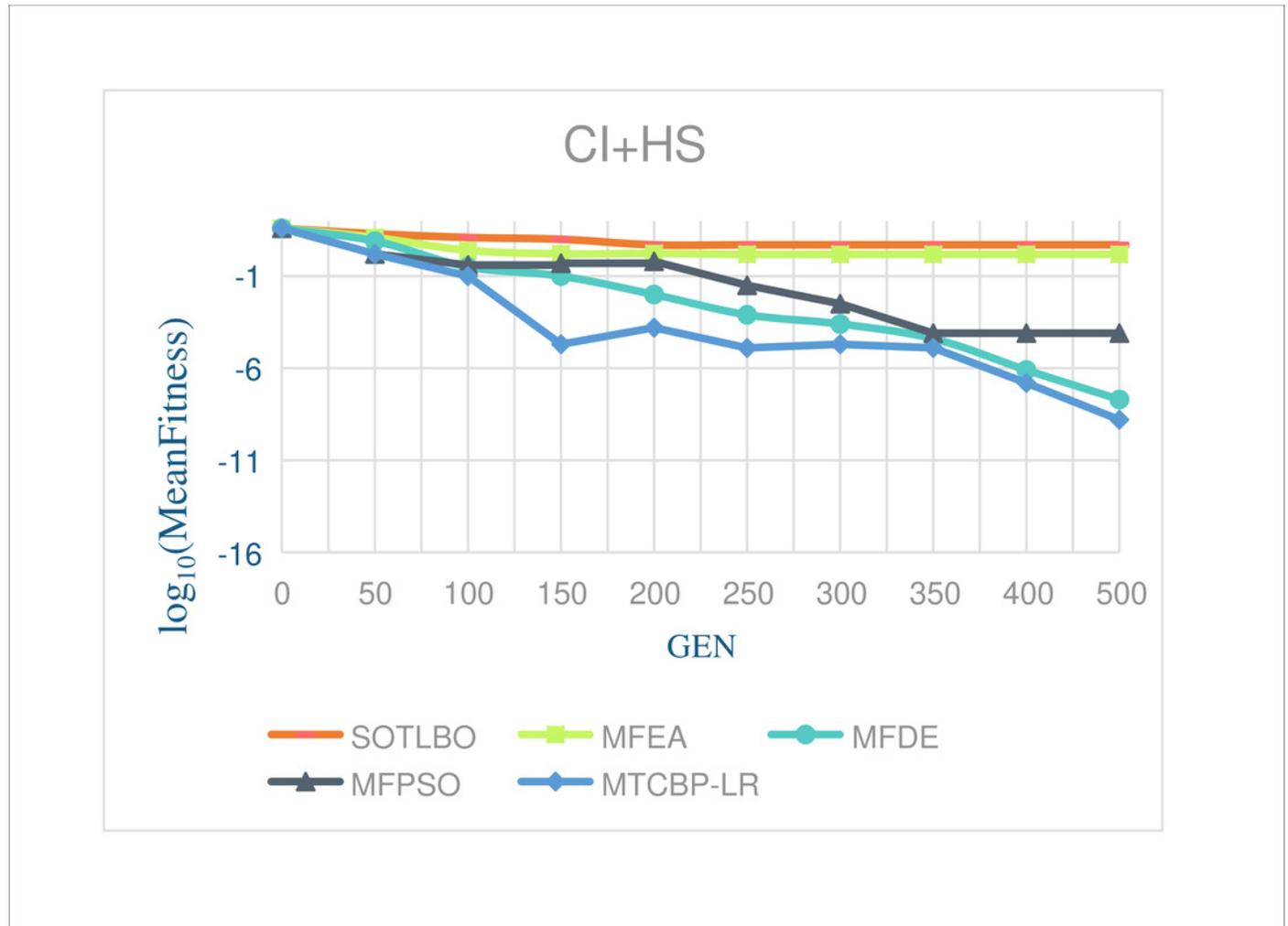
# Figure 1

Figure 1 The algorithm flow of MTCBO-LR



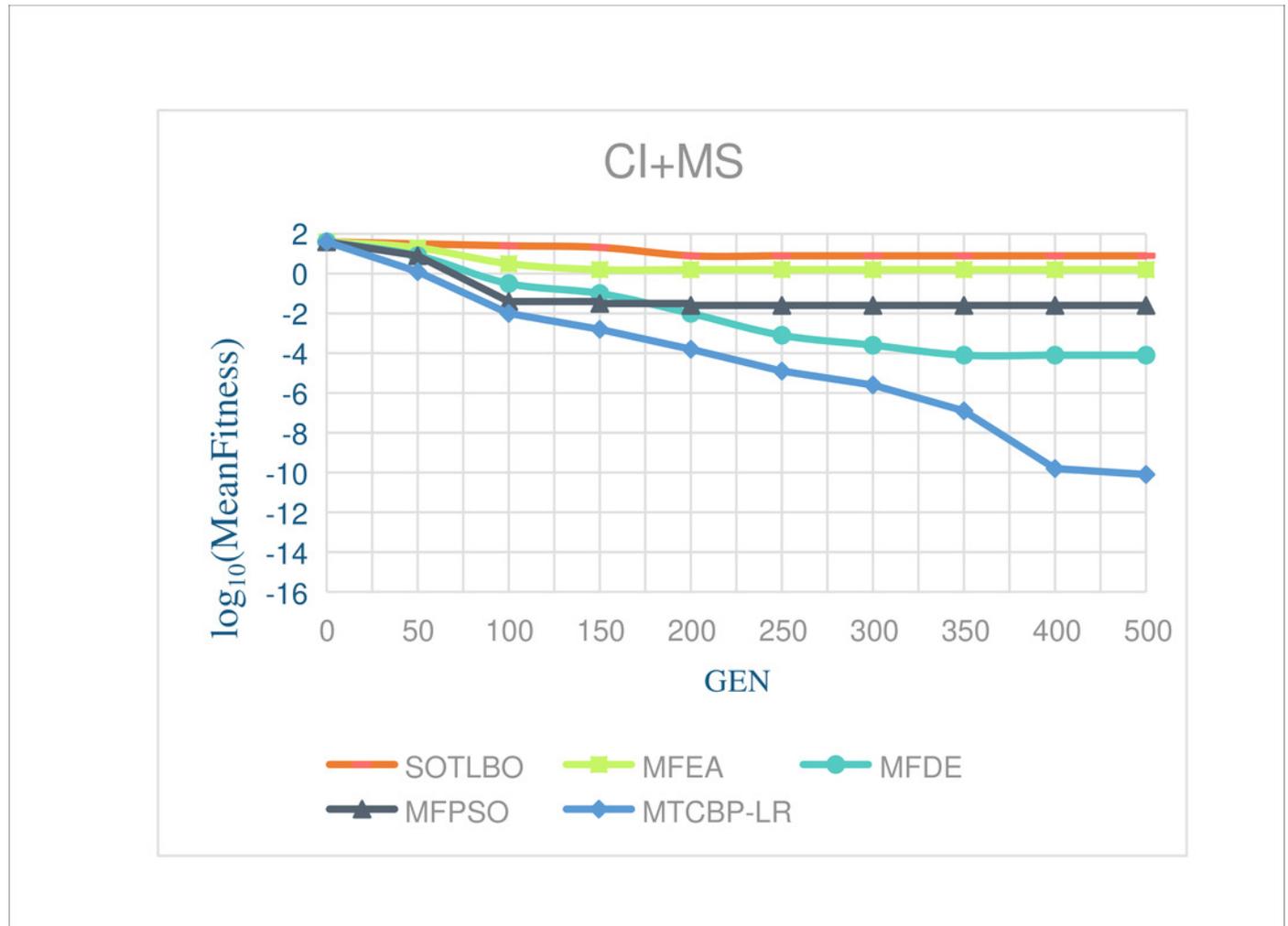
## Figure 2

Figure2 The effectiveness of five algorithms on CI+HS tasks



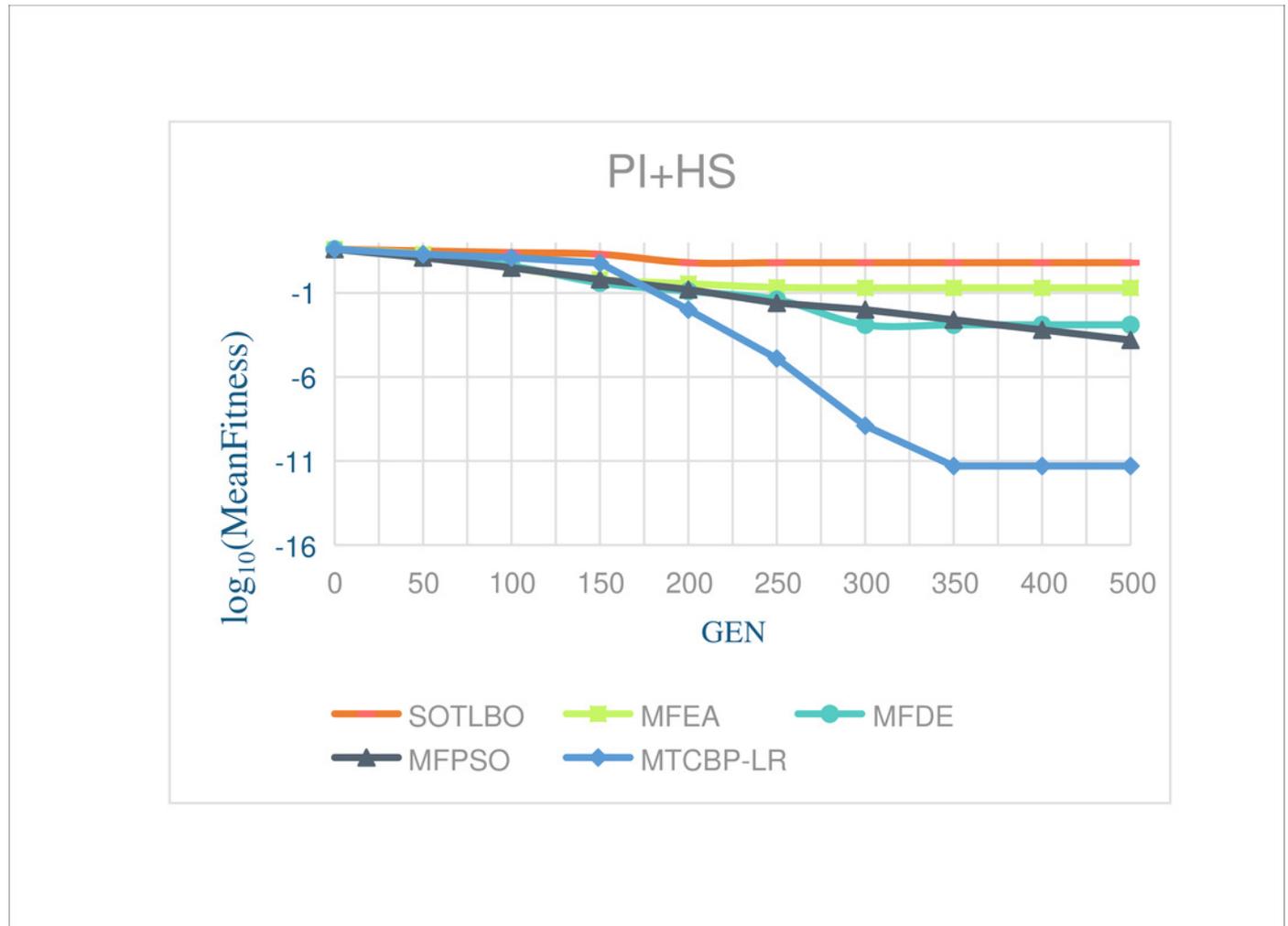
## Figure 3

Figure3 The effectiveness of five algorithms on CI+MS tasks



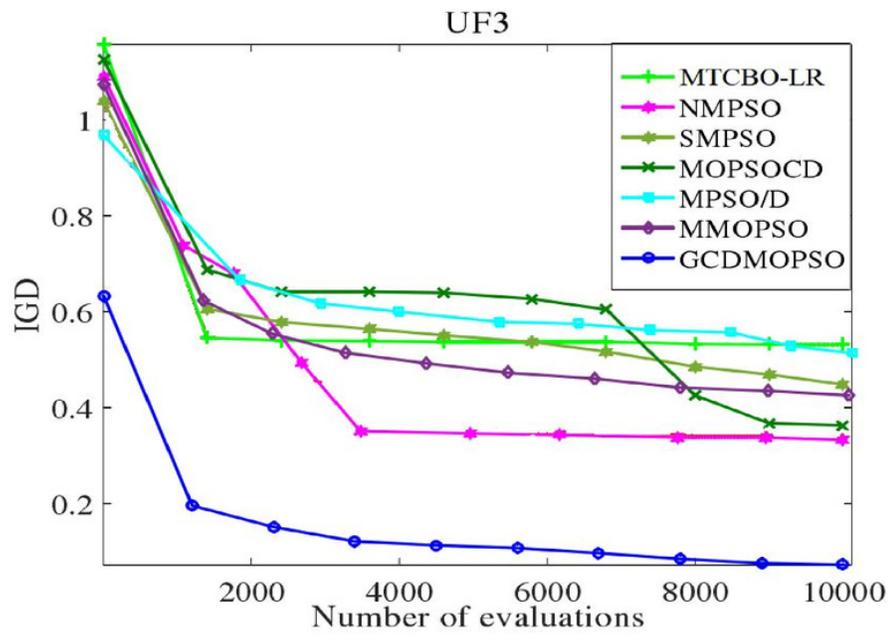
## Figure 4

Figure4 The effectiveness of five algorithms on PI+HS tasks



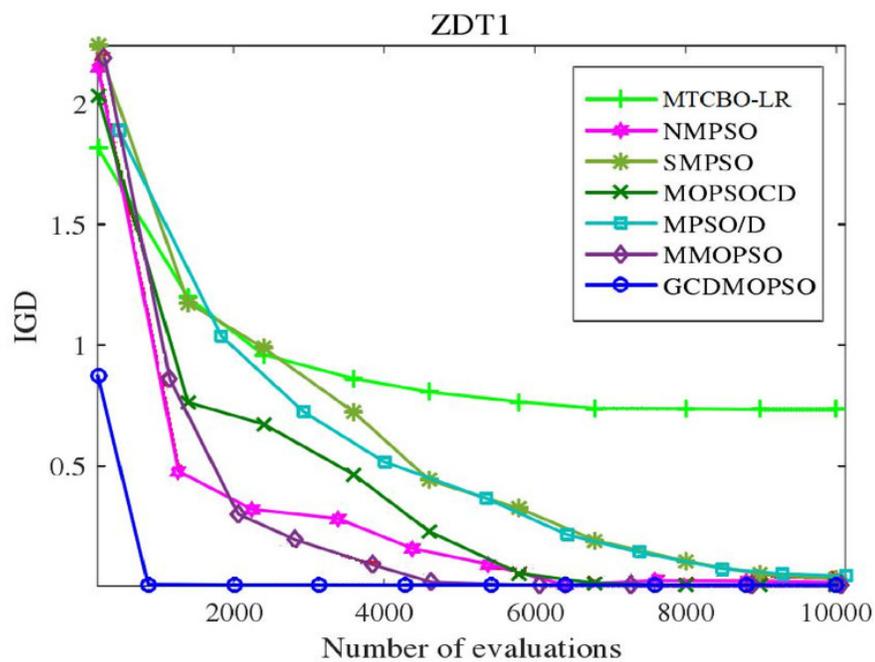
## Figure 5

Figure 5 Convergence curves of different algorithms on UF3



## Figure 6

Figure 6 Convergence curves of different algorithms on ZDT1



**Table 1** (on next page)

Table 1 Score performance of 7 groups of single-target multi-task benchmarks

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Table 1 Score performance of 7 groups of single-target multi-task benchmarks

MCI+HS	MFEA-GHS	EMT	TMOMFEA	MTCBO-LR
MCI+MS	-0.063	- 1.512	-0.523	-0.621
MCI+LS	2.965	1.412	-1.523	-1.421
MPL+HS	1.508	0.052	1.762	-2.402
MPI+MS	-0.107	- 0.145	-1.422	-0.091
MPI+LS	2.991	3.379	2.764	-0.987
MNO+HS	0.215	- 0.445	0.124	-0.447

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**Table 2** (on next page)

Table 2 Parameter settings of algorithms MFEA-GHS and MFEA-AKT

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Table 2 Parameter settings of algorithms MFEA-GHS and MFEA-AKT

MFEA-GHS	MFEA-AKT
zoom ratio: $sr \in [0.5, 1.5]$	polynomial index of variation: 5
number of top individuals: 2	Analog Binary Cross Index: 2
polynomial index of variation: 5	arithmetic cross index: 0.25
Analog Binary Cross Index: 2	geometric cross index: 0.25

2

**Table 3** (on next page)

Table 3 Performance of 7 groups of single-objective multi-task benchmarking problems

1 Table 3 Performance of 7 groups of single-objective multi-task benchmarking problems

CI+HS	WORK	MFEA-GHS	MFEA	MFEA-AKT	MTCBO-LR
CI+MS	Griewank	3.85E-01	3.59E-01	1.55E-05	0.00E+00
	Rastrign	1.41E-01	2.17E+02	3.99E-02	0.00E+00
CI+LS	Ackley	4.12E+00	4.63E+00	6.11E-05	2.21E-01
	Rastrign	3.14E+02	2.79E+01	2.79E-02	0.01E-01
PL+HS	Ackley	2.63E+01	9.09E-01	1.23E-05	1.07E+01
	Schewfel	4.73E-02	2.45E+02	3.57E-02	1.22E+02
PI+MS	Mode	6.17E-01	6.19E-01	2.33E-05	1.16E+01
	Modify	9.02E-02	7.17E-02	5.28E-02	2.87E+03
PI+LS	Modify	5.83E+01	1.59E-01	3.53E-05	1.19E+01
	Schewfel	1.73E+02	5.57E+02	6.10E-02	1.64E+03
NO+HS	Mode	8.91E-01	7.59E-01	2.25E-05	1.87E+02
	Ackley	2.86E+02	8.17E+02	4.16E-02	2.61E-01

2