A new metaphor-less simple algorithm based on Rao algorithms: A Fully Informed Search Algorithm (FISA)

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Many important engineering optimization problems require a strong and simple optimization algorithm to achieve the best solutions. In 2020, Rao introduced three non-parametric algorithms, known as Rao algorithms, which have garnered significant attention from researchers worldwide due to their simplicity and effectiveness in solving optimization problems. In our simulation studies, we have developed a new version of the Rao algorithm called the Fully Informed Search Algorithm (FISA), which demonstrates acceptable performance in optimizing real-world problems while maintaining the simplicity and non-parametric nature of the original algorithms. We evaluate the effectiveness of the suggested FISA approach by applying it to optimize the shifted benchmark functions, such as those provided in CEC 2005 and CEC 2014, and by using it to design mechanical system components. We compare the results of FISA to those obtained using the original RAO method. The outcomes obtained indicate the efficacy of the proposed new algorithm, FISA, in achieving optimized solutions for the aforementioned problems.

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2 # algorithms: A Fully Informed Search Algorithm (FISA)

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15 ## ABSTRACT

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17 algorithm to achieve the best solutions. In 2020, Rao introduced three non-parametric algorithms,
18 known as Rao algorithms, which have garnered significant attention from researchers worldwide
19 due to their simplicity and effectiveness in solving optimization problems.
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21 Informed Search Algorithm (FISA), which demonstrates acceptable performance in optimizing
22 real-world problems while maintaining the simplicity and non-parametric nature of the original
23 algorithms. We evaluate the effectiveness of the suggested FISA approach by applying it to
24 optimize the shifted benchmark functions, such as those provided in CEC 2005 and CEC 2014,
25 and by using it to design mechanical system components. We compare the results of FISA to those
26 obtained using the original RAO method.
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28 optimized solutions for the aforementioned problems.
29
30 *Keywords*: Optimization, Rao algorithms, Fully Informed Search Algorithm (FISA), constrained
31 engineering optimization.

33 ## INTRODUCTION

34 The objective of maximizing profits or minimizing losses is a crucial concern in several fields,
35 including engineering. In brief, an optimization problem refers to the situation where the aim is to

36 maximize or minimize a function. With the development of technology, optimization problems
37 have become increasingly complex and abundant across a wide range of scientific fields. The
38 complexity and interdependence of modern engineering systems and problems necessitate the
39 selection of the best optimization method to solve them. Metaheuristic algorithms are among the
40 strongest, simplest, and most commonly used optimization methods in recent years (Gogna &
41 Tayal, 2013; Zervoudakis & Tsafarakis, 2020).

42 In general, a mathematical model in the optimization process has three main parts: objective
43 function, design variables, and problem and system constraints. Design or decision variables are
44 the independent variables that must be optimally determined and are denoted by the D-dimensional
45 vector X. According to the problem's nature, they can have a combination of several types of
46 discrete and continuous decision variables. The objective function, also called the cost function, is
47 a function of the decision variables that should be minimized or maximized. The goal of solving
48 optimization problems is to obtain an acceptable solution that minimizes/maximizes objective
49 function while satisfying the problem constraints. The constraints are the same as the physical and
50 design constraints of the problem that must be satisfied in the optimization process so that a
51 practical optimal solution can be obtained (Gogna & Tayal, 2013).

52 Optimization problems can be broadly categorized into two types: unconstrained optimization
53 problems and constrained optimization problems. In the latter case, the design space is limited by
54 one or more constraints, which can take the form of equality or inequality equations. These
55 constraints determine the acceptable region in the design space where the optimal solution must
56 be found.

57 **Optimization algorithms**

58 Optimization algorithms can be broadly classified into two types: exact methods and approximate
59 methods. Exact methods are capable of guaranteeing an optimal solution, but they may require
60 significant computational resources and time. In contrast, approximate methods focus on finding
61 good solutions in a reasonable amount of time. Heuristic algorithms are a popular type of
62 approximate methods that are designed to quickly generate high-quality solutions to a wide range
63 of problems. The effectiveness of heuristic algorithms depends on the nature of the problem being
64 solved. (Gogna & Tayal, 2013).

65 **Metaheuristic methods**

66 Metaheuristics refer to methods that guide the search process and are often inspired by nature.
67 Unlike heuristic algorithms, due to their problem-independent nature, these algorithms can be
68 utilized to optimize a diverse range of problems. These methods are among the most important
69 and promising research in the optimization domain.

70 The general principles of metaheuristic methods are as follows:

71 • Employing a given number of repetitive efforts
72 • Employing one or more agents (particles, neurons, ants, chromosomes, etc.)
73 • Operation (in multi-factor mode with a cooperation-competition mechanism)
74 • Creating methods of self-change and self-transformation

75 Nature has two great tactics:

76   1.   Rewarding strong personal characteristics and punishing weaker ones.

77   2.   Introducing random mutations, which can lead to the birth of new individuals.

78 Recently, many optimization techniques have been proposed that operate on the basis of natural

79 behaviors and social behaviors. In the initialization stage, these algorithms randomly generate

80 solutions and in the later stages, they rely on natural processes to produce better answers.

81 Some of the popular and widely used types of metaheuristics are Particle swarm optimization

82 algorithm (PSO) (Kennedy & Eberhart, 1995), differential evolution (DE) (Storn, Price & others,

83 1995), genetic algorithm (GA) (Whitley, 1994), firefly algorithm (Yang, 2010a), ant colony

84 optimization (Dorigo & Di Caro), bat algorithm (Yang, 2010b), teaching–learning-based

85 optimization (TLBO) (Rao, Savsani & Vakharia, 2011), grey wolf optimizer (GWO) (Mirjalili,

86 Mirjalili & Lewis, 2014), artificial bee colony algorithm (ABC) (Karaboga & Basturk, 2007),

87 imperialist competitive algorithm (ICA) (Atashpaz-Gargari & Lucas, 2007), moth-flame

88 optimization algorithm (MFO) (Mirjalili, 2015), gravitational search algorithm (GSA) (Rashedi,

89 Nezamabadi-pour & Saryazdi, 2009), shuffled frog-leaping algorithm (SFLA) (Eusuff, Lansey &

90 Pasha, 2006), whale optimization algorithm (WOA) (Mirjalili & Lewis, 2016), etc...

91 Now the question is why all these new optimization algorithms, either modified or combined, are

92 needed. The main reason for this is the inability to determine with certainty which optimization or

93 metaheuristic algorithm is appropriate for resolving a problem, and only through the comparison

94 of the outcomes can it be asserted which algorithm provides a superior approach. In addition, based

95 on (Mirjalili, Mirjalili & Lewis, 2014), an algorithm may perform well for some groups of

96 functions but not for some other groups. Therefore, the motivation to modify algorithms or

97 introduce new algorithms has been very high in recent years (Mirjalili & Lewis, 2016).

98 In 2020, Ravipudi Venkata Rao suggested three effective and powerful straightforward algorithms

99 for optimization problems without the use of metaphors (Rao, 2020). These methods use the most

100 and least optimal solutions in each iteration and the casual interrelations between possible

101 solutions. Additionally, these methods need no control parameter other than the population size

102 and the number of iterations.

103 Rao algorithms have been successfully used by researchers in the short time since they have been

104 introduced, some of which include the engineering design optimization (Rao & Pawar, 2020a,b,

105 2022, 2023), weight optimization of reinforced concrete cantilever retaining wall (Elif N. Kalemci,

106 2020), cropping pattern under a constrained environment (Kumar & Yadav, 2019), construction

107 scheduling (Yılmaz & Dede, 2023), photovoltaic cell parameter estimation (Premkumar et al.,

108 2020; Wang et al., 2020), optimization of energy systems (Rao et al., 2022), and optimal power

109 flow (Sahay, Upputuri & Kumar, 2023).

110 A significant proportion of optimization problems encountered in practical applications contain

111 shifted functions, for which the performance of Rao algorithms may not be much optimal, as

112 demonstrated in the simulation section. This paper introduces a new algorithm, called Fully

113 Informed Search Algorithm (FISA), which is based on Rao algorithms and address this drawback.

114 FISA not only outperforms the original Rao algorithms in optimizing shifted functions but also

115 preserves their simplicity and requires no control parameters. The suggested algorithm's

116 performance is evaluated by optimizing benchmark problems with shifted functions and real-world
117 problems. The results demonstrate that FISA outperforms not only the original Rao algorithms but
118 also other state-of-the-art methods, indicating its superior performance.
119 The paper continues in four sections: the formulation of the suggested and Rao algorithms is
120 presented in the following section. Reporting the outcomes of simulations and presenting and
121 discussing the findings are done in next section. Lastly, the conclusions are presented.
122

PROPOSED ALGORITHM

124 **Rao algorithms**
125 The basic formulations of Rao algorithms, which are very simple algorithms without control
126 parameters, rely on the difference vectors obtained by subtracting the position (location) of the
127 worst individual from the location of the finest individual in the present iteration. This ensures that
128 the population always moves towards a better solution. These algorithms consist of three distinct
129 movements (position update) vectors for updating the position which are defined as follows (Rao,
130 2020):
131 Rao-1 algorithm:

$$X_{i,j}^{new} = X_{i,j}^{Iter} + r_{1,j}\left(X_{best,j}^{Iter} - X_{worst,j}^{Iter}\right)$$

(1)

132 Rao-2 algorithm:

$$\begin{cases} if\ f\left(X_i^{Iter}\right) < f\left(X_k^{Iter}\right) \\ \quad X_{i,j}^{new} = X_{i,j}^{Iter} + r_{1,j}\left(X_{best,j}^{Iter} - X_{worst,j}^{Iter}\right) + r_{2,j}\left(\left|X_{i,j}^{Iter}\right| - \left|X_{k,j}^{Iter}\right|\right); \\ else \\ \quad X_{i,j}^{new} = X_{i,j}^{Iter} + r_{1,j}\left(X_{best,j}^{Iter} - X_{worst,j}^{Iter}\right) + r_{2,j}\left(\left|X_{k,j}^{Iter}\right| - \left|X_{i,j}^{Iter}\right|\right) \end{cases}$$

(2)

133 Rao-3 algorithm:

$$\begin{cases} if\ f\left(X_i^{Iter}\right) < f\left(X_k^{Iter}\right) \\ \quad X_{i,j}^{new} = X_{i,j}^{Iter} + r_{1,j}\left(X_{best,j}^{Iter} - \left|X_{worst,j}^{Iter}\right|\right) + r_{2,j}\left(\left|X_{i,j}^{Iter}\right| - X_{k,j}^{Iter}\right); \\ else \\ \quad X_{i,j}^{new} = X_{i,j}^{Iter} + r_{1,j}\left(X_{best,j}^{Iter} - \left|X_{worst,j}^{Iter}\right|\right) + r_{2,j}\left(\left|X_{k,j}^{Iter}\right| - X_{i,j}^{Iter}\right) \end{cases}$$

(3)

134

135 In the above equations, X_i^{Iter} represents the ith solution's location in the present iteration $Iter$; j
136 (changing from 1 to D) represents the jth dimension of each solution; X_{best}^{Iter} and X_{worst}^{Iter} represent
137 the position of the highest and lowest performing members of the population during the present
138 iteration, in that order; r_1 and r_2 are two randomly selected values between 0 and 1 with the

139 dimension of D; X_k^{Iter} represents the position of the kth solution, which is indiscriminately chosen;

140 and $f(.)$ represents the numerical output of the function being optimized of the corresponding
141 solution in the present iteration. The location of the ith solution in the next iteration is obtained
142 using equation (4):

$$\begin{cases} X_i^{Iter+1} = X_i^{new} & if\ f\left(X_i^{new}\right) \le f\left(X_i^{Iter}\right) \\ X_i^{Iter+1} = X_i^{Iter} & else \end{cases} \tag{4}$$

143 **The Proposed Fully Informed Search Algorithm (FISA)**
144 The performance of Rao algorithms in optimizing shifted functions, which may be the case for
145 many real-world problems, may be suboptimal. This will be demonstrated in the simulation
146 section. Therefore, we propose a new algorithm, the Fully Informed Search Algorithm (FISA),
147 which is based on Rao algorithms and is designed to address this issue. FISA improves the
148 optimization of shifted functions while retaining the simplicity and absence of control parameters
149 of the original algorithms. Similar to Rao algorithms, FISA moves the population towards better
150 solutions. In summary, FISA can be summarized as follows:

$$X_{i,j}^{new} = X_{i,j}^{Iter} + r_{1,j}\left(MX_{best,j}^{Iter} - X_{i,j}^{Iter}\right) + r_{2,j}\left(X_{i,j}^{Iter} - MX_{worst,j}^{Iter}\right) \tag{5}$$

151
152 In fact, in FISA, each member moves away from the mean position of the individuals within the
153 population that have worse fitness values and approaches the mean position of the individuals that
154 have better fitness values than the associated member. Then, the position of each member is
155 updated using (4). In (5), the values of $MX_{best,j}^{Iter}$ and $MX_{worst,j}^{Iter}$ in each iteration are calculated using
156 (6) and (7), respectively:

$$MX_{best}^{Iter} = \frac{X_{best}^{Iter} + \sum_{l \in Bi} X_l^{Iter}}{length(Bi)+1} \tag{6}$$

$$MX_{worst}^{Iter} = \frac{X_{worst}^{Iter} + \sum_{l \in Wi} X_l^{Iter}}{length(Wi)+1} \tag{7}$$

157 where B_i and W_i are the set of population members that have a better and worse fitness value than
158 the ith member in iteration *Iter*, respectively, and $length(.)$ represents the count of the individuals
159 in the set.
160 The flowchart of FISA is shown in Fig. 1.
161

NUMERICAL RESULTS OF FISA FOR SOLVING BENCHMARK TEST FUNCTIONS

FISA for solving CEC2005 problems

To assess the effectiveness of the suggested algorithm in comparison to the original Rao algorithms, we have chosen 14 real-world shifted functions with 30 dimensions (including Unimodal, Multimodal, and Expanded Multimodal functions), numbered in the order introduced in CEC 2005 (Suganthan et al., 2005), whose data were extracted from (Suganthan). These functions have been successfully utilized in many articles (Ghasemi, Aghaei & Hadipour, 2017; Birogul, 2019; Ghasemi et al., 2019, 2022a,b, 2023; Akbari, Rahimnejad & Gadsden, 2021; Premkumar et al., 2021; Zou et al., 2022). The total count of function evaluations (NFE) during the execution of each algorithm is considered 300,000 based on (Suganthan et al., 2005); accordingly, the number of population members of each algorithm during the optimization is 30; therefore, the convergence curves, in this study, have been extracted for 10,000 iterations. Furthermore, to acquire the optimal solution for each function, each algorithm has been executed independently for 25 runs. A summary of the results comprising the average value, standard deviation, and rank for each algorithm amongst the investigated algorithms is given in Table 1. In this table, N_b and N_w represent the total instances where the corresponding algorithm has the best or the worst result among the studied algorithms, respectively. M_R also represents the mean ranking of each algorithm for 14 test functions.

After a concise investigation of Table 1, it can be observed that the FISA algorithm noticeably outperformed the original Rao algorithms, particularly for functions F2, F4, and F6. The proposed algorithm surpassed the Rao-2 and Rao-3 algorithms for all investigated functions, except for the F8 test function where it achieved the same performance as the Rao algorithms. Although the FISA algorithm showed slightly lower performance than the Rao-1 algorithm for test functions F1, F7, and F9, these results were close, and FISA outperformed the Rao-1 algorithm for the remaining 10 test functions. These findings indicate that the proposed algorithm has a strong capability to achieve optimal solutions for practical problems. Additionally, the convergence behaviors of different algorithms for solving the selected functions are presented in Fig. 2, which provides evidence of the superior convergence behavior of the suggested algorithm.

FISA for solving CEC2014 problems

In the second part of demonstrating the efficacy of the suggested method, FISA, in comparison with the RAO algorithms, 30 test functions from CEC 2014 Test Functions, with dimension 30 are selected (Askari & Younas, 2021; Liang, Qu & Suganthan, 2013; Liu & Nishi, 2022; Meng et al., 2022; Band et al., 2022), whose data were extracted from (Suganthan). The optimal value for all functions is 0. The population number was selected as 30 and the stopping criterion was selected as 10,000 iterations for all algorithms; so that the NFE for each algorithm is equal to 300,000. The experiment is conducted by running each algorithm independently 25 times for every function.

After reviewing the results presented in Table 2, taking into account the average value, standard deviation, and also the best optimal value of 25 runs, we can see that the suggested algorithm has

202 a significant advantage over the Rao algorithms. The last row of the table shows the number of
203 test functions in which each algorithm achieved their best solution, denoted as Nb. It is evident
204 that the proposed method outperformed the other algorithms by obtaining the best solution in 23
205 out of 30 test functions. This indicates the strength and efficiency of the suggested technique as a
206 new optimization algorithm. The convergence characteristics of algorithms for some selected test
207 functions are displayed in Fig. 3.
208

209 **NUMERICAL RESULTS OF FISA IN SOLVING ENGINEERING**
210 **PROBLEMS**
211 To demonstrate the effectiveness and optimization efficiency of FISA, three widely recognized
212 engineering problems, namely optimal design of a pressure vessel, tension/compression spring,
213 and welded beam, have been selected. Then we perform the optimization operation for these
214 optimization problems under the same conditions for all algorithms. The number of populations
215 for each algorithm is chosen as 60, and the number of iterations of the algorithms for each run is
216 chosen as 2000. In addition, each optimization operation is performed in 30 separate runs for each
217 problem, using all the parameters as suggested by the respective algorithm designers in their
218 original publications.
219

220 **Pressure vessel optimal design**
221 The goal of the problem is to minimize the overall costs of a pressure vessel, comprising material,
222 forming, and welding expenses. As depicted in Fig. 4, this problem involves four design variables:
223 shell thickness (denoted as x_1 or T_s), head thickness (denoted as x_2 or T_h), inner radius (denoted as
224 x_3 or R), and cylindrical section length (denoted as x_4 or L). While x_3 and x_4 are continuous
225 variables, x_1 and x_2 are discrete variables represented as integer multiples of 0.0625 in.
226 The problem's objective function is nonlinear and it has both a linear and a nonlinear inequality
227 constraint, which are illustrated below (Askarzadeh, 2016):
228 Minimize:

$$f(X) = 0.6224x_1 x_3 x_4 + 1.7781x_2 x_3^2 + 3.1661x_1^2 x_4 + 19.84x_1^2 x_3 \tag{8}$$

229 subject to :

$$g_1(X) = -x_1 + 0.0193x_3 \le 0, \tag{9}$$

$$g_2(X) = -x_2 + 0.00954x_3 \le 0, \tag{10}$$

$$g_3(X) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1,296,000 \le 0, \tag{11}$$

$$g_4(X) = x_4 - 240 \le 0, \tag{12}$$

230 $0 \le x_i \le 100$, i=1, 2

231 $10 \le x_i \le 200$, i=3, 4

232

233 Table 3 compares the outcomes achieved by the proposed algorithm for the problem and other
234 widely used standard algorithms, including quantum-inspired PSO (QPSO) and Gaussian QPSO
235 (G-QPSO) (Coelho, dos Santos Coelho & Coelho, 2010), ABC (Akay & Karaboga, 2012), a GA
236 equipped with a constraint-handling via dominance-based tournament selection (GA4) (Coello
237 Coello et al., 2002), co-evolutionary PSO (CPSO) (He & Wang, 2007), co-evolutionary DE (CDE)
238 (Huang, Wang & He, 2007), unified PSO (UPSO) (Parsopoulos & Vrahatis, 2005), Crow search
239 algorithm (CSA) (Askarzadeh, 2016), hybridizing a genetic algorithm with an artificial immune
240 system (HAIS-GA) (Bernardino et al., 2008), bacterial foraging optimization algorithm (BFOA)
241 (Mezura-Montes & Hernández-Ocana, 2008), evolution strategies (ES) (Mezura-Montes &
242 Coello, 2008), A modification of the T-Cell Algorithm (Aragón, Esquivel & Coello, 2010), a GA
243 enhanced with a self-adaptive penalty method (GA3) (Coello Coello, 2000), queuing search (QS)
244 algorithm (Zhang et al., 2018), and a GA equipped with a constraint-handling via automatic
245 dynamic penalization (ADP) method (BIANCA) (Montemurro, Vincenti & Vannucci, 2013).
246 Furthermore, the best solution found for the problem using the suggested technique was shown in
247 Table 4. Tables 3 and 4 demonstrate that FISA outperforms other algorithms with the smallest
248 value for the best solution, and ranking first for the worst solution and average value. This confirms
249 that FISA performs well and reliably in tackling the pressure vessel optimal design problem.
250

251 **Tension/compression spring optimal design**
252 According to Fig. 5, the goal of the problem includes reducing the tension/compression spring
253 weight while satisfying four inequality limitations (one linear and three nonlinear). The problem
254 comprises three continuous design variables: wire diameter (denoted as d or x_1), mean coil
255 diameter (denoted as D or x_2), and the number of active coils (denoted as P or x_3) (Askarzadeh,
256 2016).
257 Minimize:
258

$$f(X) = (x_3 + 2)x_2 x_1^2 \tag{13}$$

259

260 subject to:

$$g_1(X) = 1 - \frac{x_2^3 x_3}{71,785 x_1^4} \le 0, \tag{14}$$

261

$$g_2(X) = \frac{4x_2^2 - x_1 x_2}{12,566(x_1^3 x_2 - x_1^4)} + \frac{1}{5,108 x_1^2} - 1 \le 0, \tag{15}$$

262

$$g_3(X) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \le 0, \tag{16}$$

263

$$g_4(X) = \frac{x_1 + x_2}{1.5} - 1 \le 0. \tag{17}$$

264

265    $0.05 \le x_1 \le 2$, $0.25 \le x_2 \le 1.3$, $2 \le x_3 \le 15$.

266

267   Table 5 describes the outcomes of the proposed algorithm for the problem in comparison with
268   other standard and well-known algorithms including BFOA (Mezura-Montes & Hernández-
269   Ocana, 2008), T-Cell (Aragón, Esquivel & Coello, 2010), CDE (Huang, Wang & He, 2007), a
270   cultural algorithm (CA) (Coello Coello & Becerra, 2004), CPSO (He & Wang, 2007), GA4 (Coello
271   Coello et al., 2002), GA3 (Coello Coello, 2000), TEO (Kaveh & Dadras, 2017), G-QPSO (Coelho,
272   dos Santos Coelho & Coelho, 2010), SBO (Ray & Liew, 2003), evolutionary algorithms ((l + k)-
273   ES) (Mezura-Montes & Coello, 2005), grey wolf optimizer (GWO) (Mirjalili, Mirjalili & Lewis,
274   2014), UPSO (Parsopoulos & Vrahatis, 2005), QS (Zhang et al., 2018), SDO (Zhao, Wang &
275   Zhang, 2019), BIANCA (Montemurro, Vincenti & Vannucci, 2013), and water cycle algorithm
276   (WCA) (Eskandar et al., 2012). Furthermore, Table 6 shows the optimal solution obtained for the
277   problem using the suggested algorithm. The tables demonstrate that FISA outperforms all other
278   algorithms in terms of the best value, with the worst solution and average value being the smallest.
279   This suggests that FISA is more effective than other competitive optimizers in solving this
280   problem.
281

282   **Welded beam optimal design**
283   The goal of the problem is the minimization of the cost of a welded beam. The problem has four
284   continuous decision variables, as depicted in Fig. 6, namely x1 (h), x2 (l), x3 (t), and x4 (b), along
285   with two linear and five nonlinear inequality limitations (Askarzadeh, 2016).
286   Minimize:
287

$$f(X) = 1.10471 x_2 x_1^2 + 0.04811 x_3 x_4 (14 + x_2) \tag{18}$$

288
289   Subject to:

$$g_1(X) = \tau(x) - \tau_{max} \le 0, \tag{19}$$

290

$$g_2(X) = \sigma(x) - \sigma_{max} \le 0, \tag{20}$$

291

$$g_3(X) = x_1 - x_4 \le 0, \tag{21}$$

292

$$g_4(X) = 0.10471 x_1^2 + 0.04811 x_3 x_4 (14 + x_2) - 5 \le 0. \tag{22}$$

293

$$g_5(X) = 0.125 - x_1 \leq 0, \tag{23}$$

294

$$g_6(X) = \delta(x) - \delta_{max} \leq 0, \tag{24}$$

295

$$g_7(X) = P - P_c(x) \leq 0, \tag{25}$$

296   Where

$$\tau(x) = \sqrt{\left(\tau'\right)^2 + 2\tau'\tau'' \frac{x_2}{2R} + \left(\tau''\right)^2} \tag{26}$$

$$\tau' = \frac{P}{\sqrt{2}x_1 x_2}, \tau'' = \frac{MR}{J}, \tag{27-28}$$

$$M = P\left(L + \frac{x_2}{2}\right), R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}, \delta(x) = \frac{4PL^3}{Ex_3^3 x_4} \tag{29-31}$$

$$J = 2\left[\sqrt{2}x_1 x_2 \left\{\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right\}\right], \sigma(x) = \frac{6PL}{x_4 x_3^2}, \tag{32-33}$$

$$P_c(x) = \frac{4.013E\sqrt{\frac{x_4^6 x_3^2}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right), \tag{34}$$

297    $P = 6,000$ lb; $L = 14$ in; $E = 30e6$ psi $G = 12e6$ psi, $\tau_{max} = 13,000$ psi, $\sigma_{max} = 30,000$ psi

298    $\delta_{max} = 0.25$ in, $0.1 \leq x_1 \leq 2$, $0.1 \leq x_2 \leq 10$, $0.1 \leq x_3 \leq 10$, $0.1 \leq x_4 \leq 2$.

299

300   The performance of the suggested method for the problem is presented in Table 7, where it is
301   compared to other widely used algorithms, including a cooperative PSO with stochastic
302   movements (EPSO) (Ngo, Sadollah & Kim, 2016), BFOA (Mezura-Montes & Hernández-Ocana,
303   2008), T-Cell (Aragón, Esquivel & Coello, 2010), CDE (Huang, Wang & He, 2007), a hybrid real-
304   parameter GA (HSA-GA) (Hwang & He, 2006), CPSO (He & Wang, 2007),TEO (Kaveh &
305   Dadras, 2017), SBO (Ray & Liew, 2003), Derivative-Free Filter Simulated Annealing Method
306   (FSA) (Hedar & Fukushima, 2006), GA4 (Coello Coello et al., 2002), (l + k)-ES (Mezura-Montes
307   & Coello, 2005), GWO (Mirjalili, Mirjalili & Lewis, 2014), SFO (Shadravan, Naji & Bardsiri,
308   2019), HGSO (Hashim et al., 2019), UPSO (Parsopoulos & Vrahatis, 2005), WCA (Eskandar et
309   al., 2012), BIANCA (Montemurro, Vincenti & Vannucci, 2013). Furthermore, Table 8 displays
310   the optimal solution obtained by the suggested algorithm for the problem. Tables 7 and 8
311   demonstrate that FISA is capable of discovering the most effective optimization solution. The
312   statistical findings also indicate that FISA surpasses other methods and can more effectively handle
313   the constrained engineering problems.

314

MANAGERIAL IMPLICATIONS

Metaheuristic methods provide managers and decision makers with reliable tools for finding appropriate solutions to real-world problems with a limited computational burden and a limited time. Since there are major difficulties in finding the exact solutions of a wide variety of real-world problems, metaheuristic methods are still the focus of many studies for tackling these issues. The paper proposed a simple non-parametric algorithm, named Fully Informed Search Algorithm. The suggested algorithm's effectiveness was verified by testing it on both shifted benchmark functions and mechanical design problems. The non-parametric nature of the proposed method along with its good performance in finding high quality solution of complicated real-world optimization problems make it a good choice for supporting managers in decision making without having to deal with sophisticated parameter tuning.

CONCLUSIONS

In this paper, a new and powerful variant of Rao algorithms, entitled fully informed search algorithm (FISA), is suggested to enhance the performance of Rao algorithms in optimizing real-parameter shifted functions. The efficacy of the suggested algorithm was assessed compared to three original Rao algorithms for test functions presented in CEC 2005 and CEC 2014 and engineering design optimization problems. The obtained results demonstrated that the proposed algorithm has a much better performance compared to the original Rao algorithms.

On the other hand, each algorithm has its own limitations. For this reason, after presenting each algorithm, its improved and modified versions are published one after another in different formats and forms. Like any other algorithm, the proposed algorithm can have limitations, like low convergence speed or getting stuck in local optima. Here we propose several ways to improve and evolve this algorithm. However, the most effective way to evaluate the performance of an algorithm for any given problem is through experimental testing.

In future studies, the suggested FISA can be utilized for solving various complex optimization problems that occur in the real world. Additionally, our future plans involve creating binary and multiobjective variants of FISA. Also, the optimization of support vector machines or kernel extreme learning machines is possible using FISA. By merging FISA with other algorithms, we can establish new hybrid algorithms that take advantage of the strengths and abilities of both algorithms. Recently, many studies in high-dimensional optimization have been conducted, with the majority focusing on the cooperative co-evolution technique. In upcoming studies, FISA could be integrated into various cooperative co-evolution frameworks with different classes to enhance its effectiveness. Additionally, FISA can tackle other practical large-scale optimization problems. Moreover, initializing with opposite learning in FISA would be an appropriate domain to be explored in the future.

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543

Figure 1

Flowchart of FISA

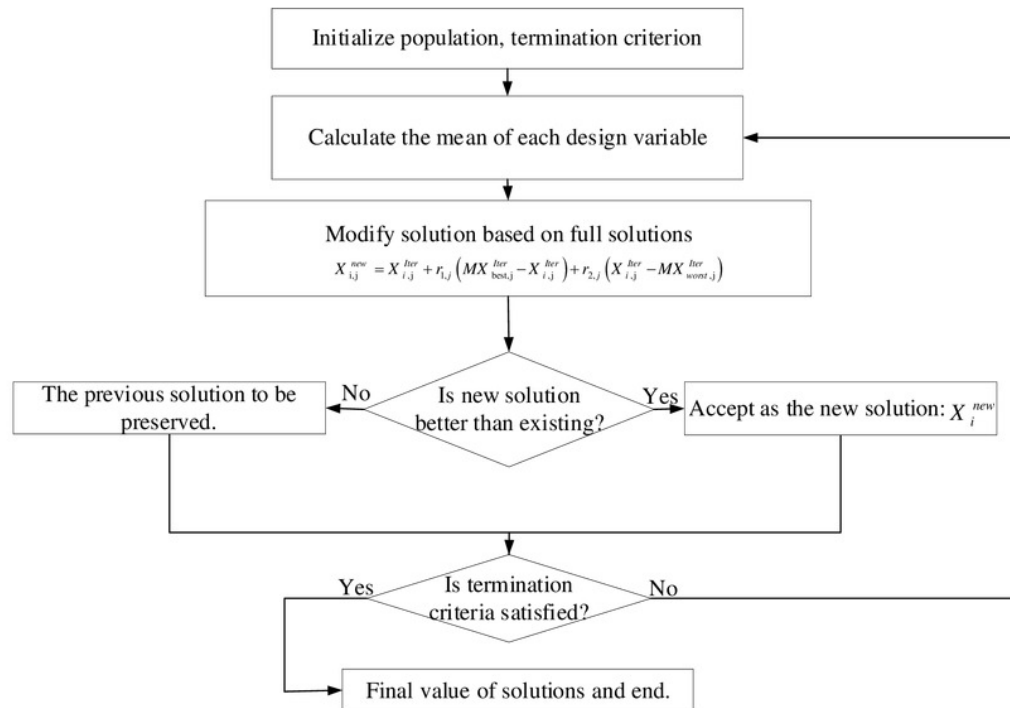



Figure 2

The convergence characteristics of FISA and the Rao algorithms for some selected CEC2005 benchmark functions.

(A) F1 test function. (B) F2 test function. (C) F10 test function. (D) F14 test function.

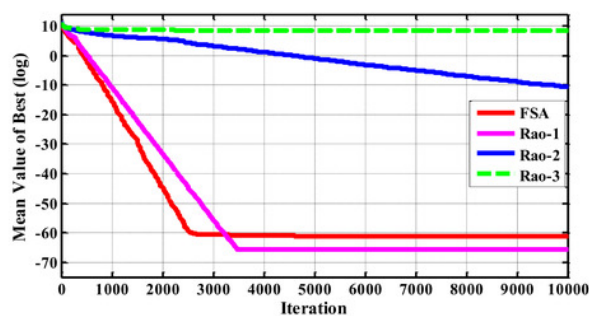
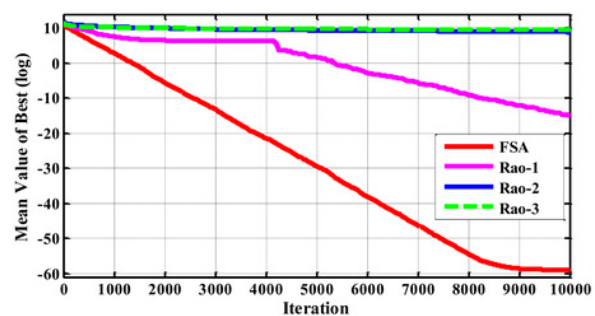
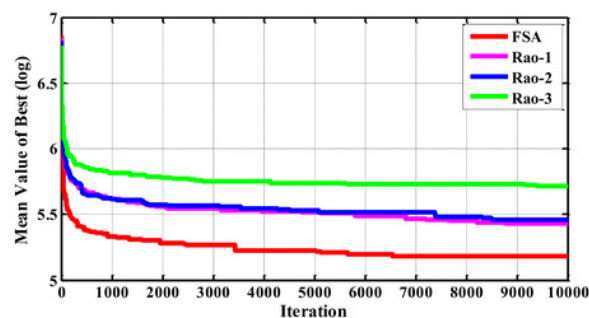
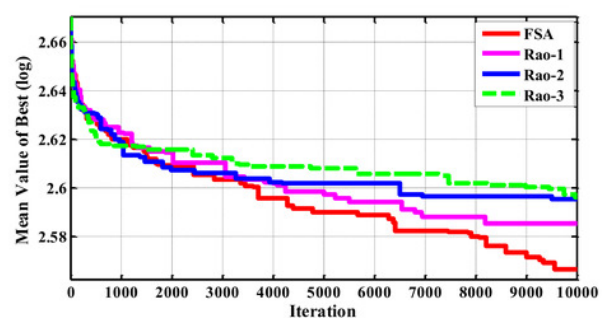



Figure 3

The convergence characteristics of FISA and the Rao algorithms for some selected CEC2014 benchmark functions.

(A) F2 test function. (B) F6 test function. (C) F15 test function. (D) F20 test function. (E) F24 test function. (F) F29 test function.

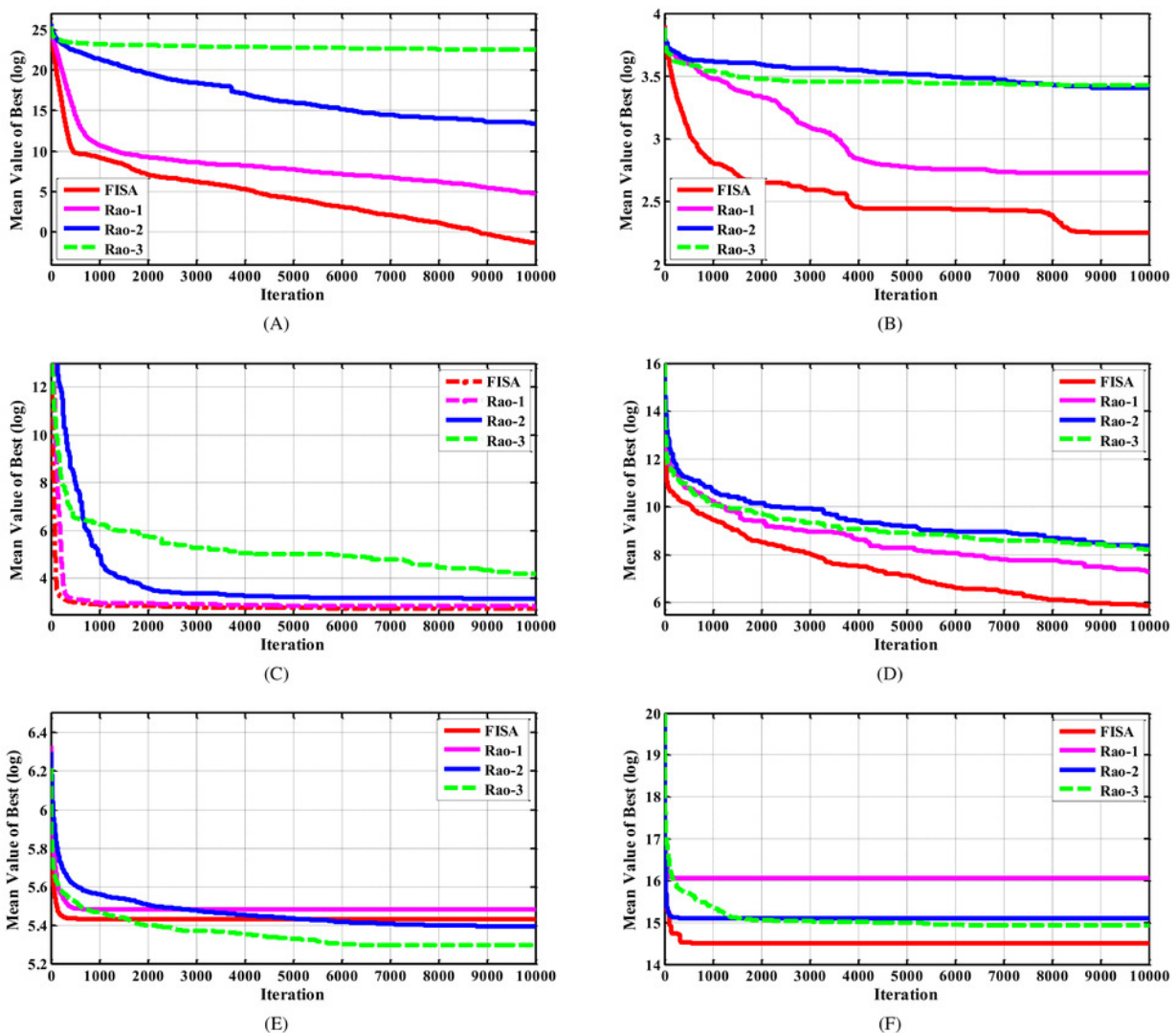



Figure 4

Schematic of the pressure vessel design problem

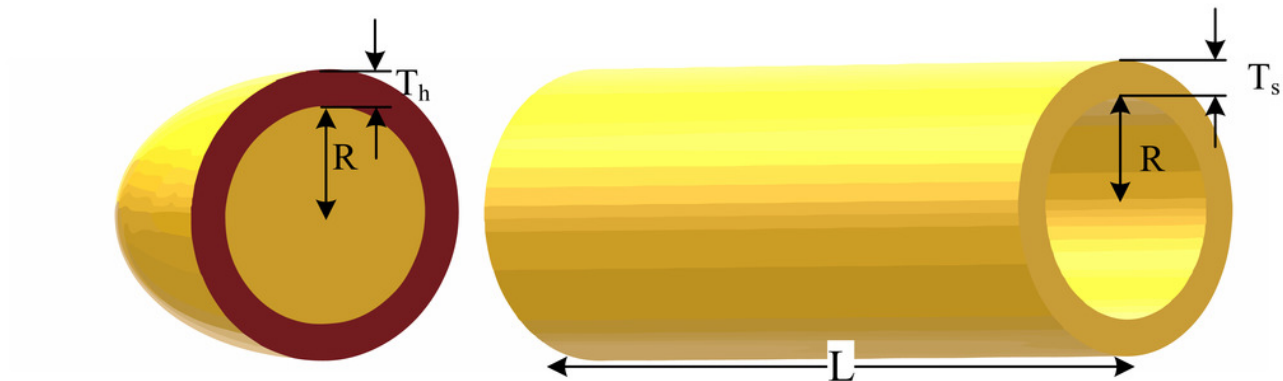



Figure 5

The tension/compression spring optimal design problem

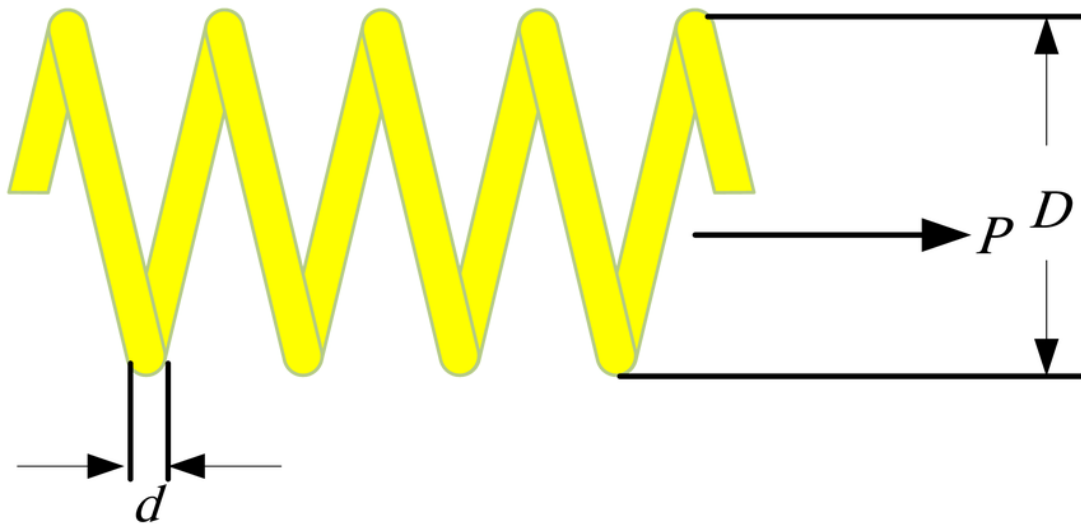
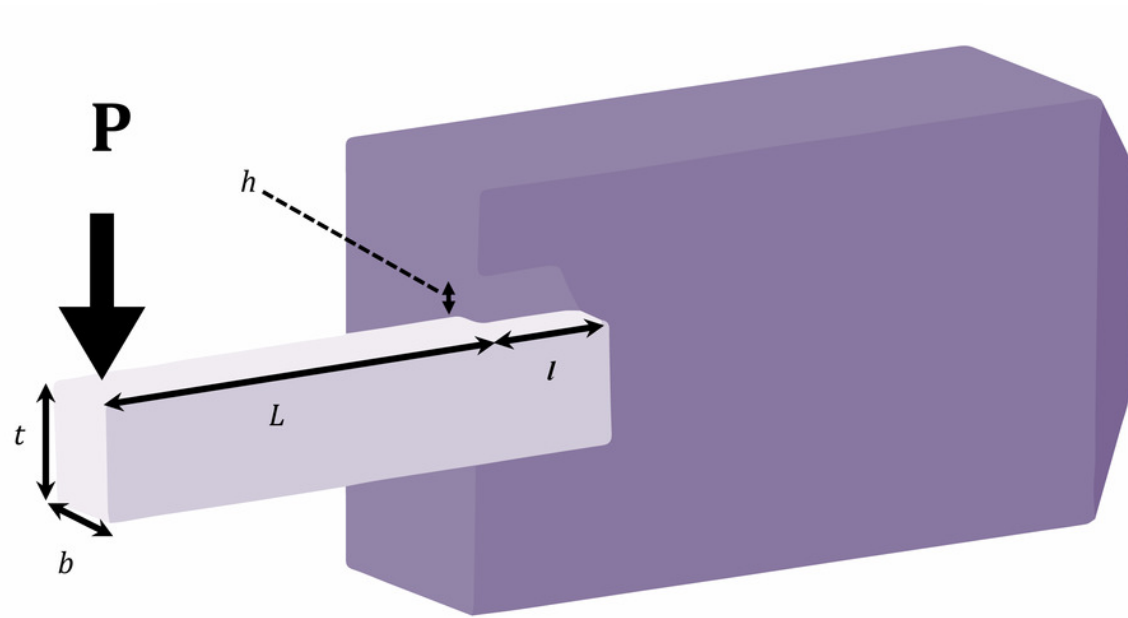



Figure 6

Schematic of welded beam optimal design problem

Schematic of welded beam optimal design problem (Askarzadeh, 2016)

1 **Table 1:**

2 The optimal results obtained by Rao algorithms and FISA on the 30-D real-parameter test

3 functions of CEC 2005.

F	Rao-1 Mean Std Dev Rank	Rao-2 Mean Std Dev Rank	Rao-3 Mean Std Dev Rank	FISA Mean Std Dev Rank
F1	2.88E-29 7.64E-29 1	2.39E-05 3.43E-05 3	4.11E+03 5.53E+02 4	3.37E-27 5.54E-27 2
F2	3.32E-07 8.76E-07 2	7.53E+03 2.52E+03 3	1.51E+04 2.91E+03 4	2.04E-26 3.89E-27 1
F3	1.61E+07 8.89E+06 2	7.02E+07 2.19E+07 3	8.41E+07 2.36E+07 4	5.83E+06 3.84E+06 1
F4	2.18E+02 5.09E+02 4	2.09E+04 8.84E+03 2	2.11E+04 8.43E+03 3	4.01E-05 9.75E-05 1
F5	3.63E+03 1.70E+03 3	3.35E+03 2.44E+03 2	5.56E+03 2.07E+03 4	2.34E+03 9.42E+03 1
F6	2.19E+01 3.54E+01 2	5.55E+06 1.47E+07 3	1.09E+08 2.51E+07 4	4.01E+00 4.96E+07 1
F7	1.98E-02 1.36E-02 1	2.64E-01 2.50E-01 3	3.03E+02 5.19E+01 4	2.88E-02 1.32E-02 2
F8	2.09E+01 5.39E-02 1	2.09E+01 3.42E-02 1	2.09E+01 6.51E-02 1	2.09E+01 5.69E-02 1
F9	1.63E+02 5.71E+01 1	2.02E+02 1.44E+01 3	2.33E+02 1.21E+01 4	1.91E+02 2.00E+01 2
F10	2.28E+02 2.15E+01 2	2.34E+02 1.48E+01 3	3.03E+02 2.60E+01 4	1.78E+02 1.99E+01 1
F11	4.02E+01 1.25E+00 4	3.93E+01 5.42E-01 3	3.89E+01 1.11E+00 2	3.77E+01 1.65E+00 1
F12	6.11E+04 5.85E+04 2	1.93E+05 1.60E+05 4	1.62E+05 6.39E+04 3	5.28E+04 5.05E+04 1
F13	1.70E+01 8.39E-01	1.78E+01 1.61E+00	3.38E+01 3.59E+00	1.50E+01 1.23E+00

	2	3	4	1
	1.33E+01	1.34E+01	1.34E+01	1.27E+01
F14	2.29E-01	2.08E-01	1.16E-01	2.02E-01
	2	3	3	1
Nb/Nw/MR	4/2/2.071	1/2/2.786	1/10/3.429	11/0/1.214

Mean statistical results of the optimization algorithms

1 Table 2. Mean statistical results of the optimization algorithms.

Number	Mean				Std.				Best			
Functions	Rao-1	Rao-2	Rao-3	FISA	Rao-1	Rao-2	Rao-3	FISA	Rao-1	Rao-2	Rao-3	FISA
F1	9582162	50437582	81428816	2800719	3079516	20123351	39651820	2150031	6946796	14997575	29955287	628863
F2	120.89	659713	6137293771	0.2668	112.9	848640	2928707621	0.2912	0.4824	29241	3010346193	0.008785
F3	4485	40811	38797	31.1	2228.7	10093	9319	31.85	1526	24146	23581	0.4269
F4	61.73	133.42	342.18	24.415	51.53	39.751	69.579	31.2	1.21	64.89	270.3	1.379e-05
F5	20.90	20.87	20.93	20.94	0.06756	0.0954	0.0494	0.06964	20.79	20.65	20.85	20.77
F6	15.33	30.14	30.72	9.535	7.284	5.586	4.372	2.443	6.486	15.29	22.86	6.25
F7	0.005914	0.0994	15.59	0.00689	0.005473	0.1606	5.7095	0.00919	3.41e-13	2.27e-13	10.26	3.41e-13
F8	171.18	194.74	194.73	183.55	45.848	20.150	27.835	10.84	48	169.00	131.77	172.33
F9	223.75	250.58	239.61	183.97	19.06	15.61	20.347	8.097	203.1	230.54	217.6	172.2
F10	5984.46	4211.659	4524	6108	407	1432	1348	369	5393	1312	1425.08	5490
F11	6931.32	6635.03	6865	6566	475.32	354.75	408.21	294.6	5868	5981	6113.02	6321
F12	2.345	2.60	2.53	2.40	0.3794	0.2785	0.3263	0.2383	1.7	2.221	1.746	2.017
F13	0.5111	0.552	1.5011	0.34246	0.1238	0.0895	0.41817	0.0583	0.2819	0.3974	1.083	0.2657
F14	0.5391	0.7987	7.732	0.28264	0.2937	0.342	4.556	0.0343	0.29	0.28242	2.6359	0.243
F15	17.02	23.12	66.97	15.42	2.404	10.571	77.772	1.47	14.30	17.71	27.308	12.92
F16	12.81	12.79	12.83	11.90	0.266	0.1874	0.1794	0.2726	12.38	12.398	12.42	11.47
F17	965058	3981833	2540846	389902	436615	2490205	1244123	326223	129986	1177515	1107044	22824
F18	199137	30841200	36432033	11283	449272	80536940	34940261	9520	25671	11381	7416244	2405
F19	10.584	26.22	25.60	28.62	1.678	33.385	1.80	32.333	8.794	7.106	22.86	5.597
F20	1463	4209.6	3737.2	348.8	807.5	2655	2537	118.5	807.2	1001.8	823.585	191.2
F21	287610	1244033	710914	153382	187045	1293203	354945	121947	132730	235591	299730	27539
F22	586.5	458.5	665.70	378.992	166.88	126.3	143.81	133	352.7	216.55	425	173
F23	315.244	315.257	334.029	315.244	8.78e-13	0.0158	7.95	5.95e-13	315.244	315.245	325.65	315.244
F24	240.398	220.55	200.124	228.68	9.096	13.208	0.0523	7.22	222.9	201.711	200.05	222.4
F25	205.676	212.8	217.134	204.147	3.137	4.553	4.393	0.9164	203.06	205.69	211.18	202.84
F26	124.44	164.57	111.935	110.34	75.72	110.71	33.7	31.53	100.27	100.501	100.61	100.27
F27	763.611	972.56	996.85	623.0	189.926	168.99	77.46	99.14	427.3	713.631	881.77	424.97
F28	1236.6	987.07	1282.0	1070.2	252.42	71.279	216.6	144.7	946.5	901.36	1039.66	900.4
F29	9508567	3573247	3049683	1965559	7769278	4629638	4599150	4156387	4780.58	1711	82249	1494
F30	22053	4434	9726	4809	22730	1248	7326.9	2402	2337.4	2256	4387	1934
Nb	5	4	1	21	2	3	4	21	5	3	1	23

Best statistical results of various algorithms for pressure vessel optimal design problem

1   Table 3. Best statistical results of various algorithms for pressure vessel optimal design problem

Methods	Best	Mean	Worst	Std.
QPSO (Coelho, dos Santos Coelho & Coelho, 2010)	6059.7209	6440.3786	8017.2816	479.2671
ABC (Akay & Karaboga, 2012)	6059.714339	6245.308144	N.A.	2.05e+02
GA4 (Coello Coello et al., 2002)	6059.9463	6177.2533	6469.3220	130.9297
CPSO (He & Wang, 2007)	6061.0777	6147.1332	6363.8041	86.4545
CDE (Huang, Wang & He, 2007)	6059.7340	6085.2303	6371.0455	43.013
G-QPSO (Coelho, dos Santos Coelho & Coelho, 2010)	6059.7208	6440.3786	7544.4925	448.4711
UPSO (Parsopoulos & Vrahatis, 2005)	6154.70	8016.37	9387.77	745.869
ES (Mezura-Montes & Coello, 2008)	6059.746	6850.00	7332.87	426
T-Cell (Aragón, Esquivel & Coello, 2010)	6390.554	6737.065	7694.066	357
GA3 (Coello Coello, 2000)	6288.7445	6293.8432	6308.4970	7.4133
HAIS-GA (Bernardino et al., 2008)	6832.584	7187.314	8012.615	276
CSA (Askarzadeh, 2016)	6059.71436343	6342.49910551	7332.84162110	384.94541634
BFOA (Mezura-Montes & Hernández-Ocana, 2008)	6060.460	6074.625	N.A.	156
BIANCA (Montemurro, Vincenti & Vannucci, 2013)	6059.9384	6182.0022	6447.3251	122.3256
QS (Zhang et al., 2018)	6059.714	6060.947	6090.526	N.A.
Rao-1 (Rao & Pawar, 2020b)	6059.714334	6069.230694	6093.903548	10.451664
Rao-2 (Rao & Pawar, 2020b)	6059.714334	6062.055668	6090.526202	7.171409
Rao-3 (Rao & Pawar, 2020b)	6059.714334	6061.883052	6090.526202	7.810982
FISA	6059.714334	6061.320721	6066.824063	4.74

The best solutions found for the pressure vessel optimal design problem by FISA

1   Table 4. The best solutions found for the pressure vessel optimal design problem by FISA.

Design variables	FISA
x_1	0.8125
x_2	0.4375
x_3	42.098446
x_4	176.63660
$g_1(X)$	-1.1300e-10
$g_2(X)$	-0.035881
$g_3(X)$	-2.788752e-05
$g_4(X)$	-63.36340
Best	6059.714334

Best statistical results of various algorithms for tension/compression spring optimal design problem

1 Table 5. Best statistical results of various algorithms for tension/compression spring optimal
2 design problem

Methods	Best	Mean	Worst	Std.
CA (Coello Coello & Becerra, 2004)	0.012721	0.013568	0.0151156	8.4e-04
BFOA (Mezura-Montes & Hernández-Ocana, 2008)	0.012671	0.012759	N.A.	1.36e-04
T-Cell (Aragón, Esquivel & Coello, 2010)	0.012665	0.012732	0.013309	9.4e-05
CDE (Huang, Wang & He, 2007)	0.012670	0.012703	0.012790	2.07e-05
CPSO (He & Wang, 2007)	0.0126747	0.012730	0.012924	5.19e-05
TEO (Kaveh & Dadras, 2017)	0.012665	0.012685	0.012715	4.4079e-06
G-QPSO (Coelho, dos Santos Coelho & Coelho, 2010)	0.012665	0.013524	0.017759	1.268e-03
SBO (Ray & Liew, 2003)	0.012669249	0.012922669	0.016717272	5.92e-04
GA4 (Coello Coello et al., 2002)	0.012681	0.012742	0.012973	9.5e-05
GA3 (Coello Coello, 2000)	0.0127048	0.012769	0.012822	3.93e-05
(1 + k)-ES (Mezura-Montes & Coello, 2005)	0.012689	0.013165	N.A.	3.9e-04
UPSO (Parsopoulos & Vrahatis, 2005)	0.01312	0.02294	N.A.	7.2e-03
GWO (Mirjalili, Mirjalili & Lewis, 2014)	0.0126660	N.A.	N.A.	N.A.
WCA (Eskandar et al., 2012)	0.012665	0.012746	0.012952	8.06e-05
BIANCA (Montemurro, Vincenti & Vannucci, 2013)	0.012671	0.012681	0.012913	5.1232e-05
SDO (Zhao, Wang & Zhang, 2019)	0.0126663	0.0126724	0.0126828	6.1899e-06
QS (Zhang et al., 2018)	0.012665	0.012666	0.012669	N.A.
Rao-1 (Rao & Pawar, 2020b)	0.012666	0.012712	0.012846	3.6195e-05
Rao-2 (Rao & Pawar, 2020b)	0.012669	0.013232	0.030455	2.5886e-03
Rao-3 (Rao & Pawar, 2020b)	0.012672	0.013086	0.017773	1.2062e-03
FISA	0.012665	0.012666	0.012675	7.05e-07

The best solutions found for the tension/compression spring optimal design problem by
FISA

1   Table 6. The best solutions found for the tension/compression spring optimal design problem by
2   FISA

Design variables	FISA
x_1	0.0517770562
x_2	0.3588357559
x_3	11.1661043232
$g_1(X)$	-1.310475e-05
$g_2(X)$	-5.853421e-06
$g_3(X)$	-4.057851
$g_4(X)$	-0.727695
Best	0.012665

Best statistical results of various algorithms for welded beam optimal design problem

2 Table 7. Best statistical results of various algorithms for welded beam optimal design problem.

Methods	Best	Mean	Worst	Std.
EPSO (Ngo, Sadollah & Kim, 2016)	1.7248530	1.7282190	1.7472200	5.62e-03
BFOA (Mezura-Montes & Hernández-Ocana, 2008)	2.3868	2.4040	N.A.	1.6e-02
T-Cell (Aragón, Esquivel & Coello, 2010)	2.3811	2.4398	2.7104	9.314e-02
CDE (Huang, Wang & He, 2007)	1.73346	1.768158	1.824105	2.2194e-02
CPSO (He & Wang, 2007)	1.728024	1.748831	1.782143	1.2926e-02
HSA-GA (Hwang & He, 2006)	2.2500	2.26	2.28	7.8e-03
FSA (Hedar & Fukushima, 2006)	2.3811	2.4041	2.4889	N.A.
TEO (Kaveh & Dadras, 2017)	1.725284	1.768040	1.931161	5.81661e-02
SBO (Ray & Liew, 2003)	2.3854347	3.0025883	6.3996785	9.59e-01
GA4 (Coello Coello et al., 2002)	1.728226	1.792654	1.993408	7.47e-02
(l + k)-ES (Mezura-Montes & Coello, 2005)	1.724852	1.777692	N.A.	8.8e-02
UPSO (Parsopoulos & Vrahatis, 2005)	1.92199	2.83721	N.A.	6.83e-01
GWO (Mirjalili, Mirjalili & Lewis, 2014)	1.72624	N.A.	N.A.	N.A.
SFO (Shadravan, Naji & Bardsiri, 2019)	1.73231	N.A.	N.A.	N.A.
HGSO (Hashim et al., 2019)	1.7260	1.7265	1.7325	7.66e-03
WCA (Eskandar et al., 2012)	1.724856	1.726427	1.744697	4.29e-03
BIANCA (Montemurro, Vincenti & Vannucci, 2013)	1.725436	1.752201	1.793233	2.3001e-02
Rao-1 (Rao & Pawar, 2020b)	1.724852	1.724852	1.724852	2.62
Rao-2 (Rao & Pawar, 2020b)	1.724852	1.724852	1.724852	9.83e-04
Rao-3 (Rao & Pawar, 2020b)	1.724852	1.724852	1.724852	2.06e-03
FISA	1.724852	1.724852	1.724852	5.93e-05

The best solutions found for the welded beam optimal design problem by FISA

1   Table 8. The best solutions found for the welded beam optimal design problem by FISA.

Design variables	FISA
x_1	0.20572963980
x_2	3.4704886655
x_3	9.0366239101
x_4	0.2057296398
$g_1(X)$	-2.2653330e-07
$g_2(X)$	-3.1932722e-07
$g_3(X)$	0.0
$g_4(X)$	-3.43298379
$g_5(X)$	-0.08072964
$g_6(X)$	-0.23554032
$g_7(X)$	-1.10549263e-06
Best	1.7248523