PeerJ Computer Science

WASPAS method and Aczel-Alsina aggregation operators for managing complex interval-valued intuitionistic fuzzy information and their applications in decision-making

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ABSTRACT

Aczel-Alsina t-norm and t-conorm are a valuable and feasible technique to manage ambiguous and inconsistent information because of their dominant characteristics of broad parameter values. The main theme of this analysis is to explore Aczel-Alsina operational laws in the presence of the complex interval-valued intuitionistic fuzzy (CIVIF) set theory. Furthermore, we derive the theory of aggregation frameworks based on Aczel-Alsina operational laws for managing the theory of CIVIF information. The CIVIF Aczel-Alsina weighted averaging (CIVIFAAWA), CIVIF Aczel-Alsina ordered weighted averaging (CIVIFAAOWA), CIVIF Aczel-Alsina hybrid averaging (CIVIFAAHA), CIVIF Aczel-Alsina weighted geometric (CIVIFAAWG), CIVIF Aczel-Alsina ordered weighted geometric (CIVIFAAOWG) and CIVIF Aczel-Alsina hybrid geometric (CIVIFAAHG) operators are proposed, and their well-known properties and particular cases are also detailly derived. Further, we derive the theory of the WASPAS method for CIVIF information and evaluate their positive and negative aspects. Additionally, we demonstrate the multi-attribute decision-making (MADM) strategy under the invented works. Finally, we express the supremacy and dominancy of the invented methods with the help of sensitive analysis and geometrical shown of the explored works.

Subjects Data Science, Optimization Theory and Computation, Scientific Computing and Simulation
Keywords Complex interval-valued intuitionistic fuzzy sets, Aczel-Alsina aggregation operators, Decision-making strategy, WASPAS method

Submitted 6 January 2023
Accepted 3 April 2023
Published 11 May 2023

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Academic editor
Željko Stević

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DOI 10.7717/peerj-cs.1362

INTRODUCTION

MADM strategies aims to identify the best of a few relative other options or positioning choices as per their importance as far as the assessed objective. The techniques are utilized for choosing the most acceptable other option/arrangement, because there is no such option for which all rules' esteems are awesome. MADM strategy is the sub-part of the decision-making technique that has been used in the region of discrete fields. However, it is massively difficult to apply the MADM technique to the phenomena of fuzzy sets rather than crisp sets. To achieve this idea in the real scenario, Zadeh (1965) explored the fuzzy set (FS), which only depends on the supporting grade (SG) $\overline{\overline{\mathcal{M}}}_{\overline{\overline{\mathfrak{C}_C}}} \in [0,1]$.

In facilitating that sort of situation, FS suffers from an obvious deficiency for not describing the data in the shape of yes or no, not addressing expert opinion, namely, non-SG (NSG). To conquer this imperfection, Atanassov (1986) proposed the methodology of intuitionistic FS (IFS) with an SG and NSG. The well-known prominent of IFS is as followed: $0 \leq \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}} + \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}} \leq 1$. Interval-valued (IV) data is mostly utilized to depict the ambiguity and problematic occurrences, like the difference in temperature, the vacillation of stock cost, and the scope of circulatory strain. Besides, the IV information might be gotten from various areas or sources. For this, the IV intuitionistic FS (IVIFS), was stated by Atanassov & Gargov (1989) with SG $\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}}(\widetilde{x_E})\right]$ and NSG $\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}}(\widetilde{x_E})\right]$ such that $0 \leq \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}}(\widetilde{x_E}) + \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}}(\widetilde{x_E}) \leq 1$.

Based on above discussions, we have obtained the result that the prevailing theories neglect to manage two-domination data in the shape of SG and NSG, and simultaneously neglect to survive with inconsistent and fluctuational at a provided phase of time. However, the data got from "medical research" such that the biometric and facial acknowledgment data set consistently changes with the entry of the time. Along these lines, Ramot et al. (2002) extended the scope of SG from a genuine subset to the unit circle of the complicated plane and henceforth established the principle of complex FS (CFS). The mathematical structure of SG in the circumstances of CFS is of the form $\overline{\overline{\mathcal{M}}}_{\overline{\overline{\mathfrak{C}_C}}}(\widetilde{x_E}) = \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}}(\widetilde{x_E}) e^{i2\pi\left(\overline{\overline{\mathcal{M}}}_{I}(\widetilde{x_E})\right)}$ with $\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}}(\widetilde{x_E}) \in [0,1]$. Since CFS restricts only up to SG and does not take into account NSG, Alkouri & Salleh (2012) produced the principle of complex IFS (CIFS) in the shape of SG $\overline{\overline{\mathcal{M}}}_{\overline{\overline{\mathfrak{C}_C}}}(\widetilde{x_E}) = \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}}(\widetilde{x_E}) e^{i2\pi\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}}(\widetilde{x_E})\right)}$ and NSG $\overline{\overline{\mathcal{N}}}_{\overline{\overline{\mathfrak{C}_C}}}(\widetilde{x_E}) = \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}}(\widetilde{x_E}) e^{i2\pi\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}}(\widetilde{x_E})\right)}$, with $0 \leq \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}}(\widetilde{x_E}) + \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}}(\widetilde{x_E}) \leq 1$ and $0 \leq \overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}}(\widetilde{x_E}) + \overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}}(\widetilde{x_E}) \leq 1$. Recently, Garg & Rani (2019a) studied the form of CIFS in the interval environment and proposed the mathematical structure of CIVIFS. Because of its strong ability in dealing with uncertain information, CIVIFS has been promoted in many ways, but the results in Aczel-Alsina operational laws still need to be enriched.

LITERATURE REVIEW

Under the powerful characteristic of FS, many scholars have conducted a lot of extended research. For illustration, ordered weighted averaging aggregation operators (*Yager, 1988*), immediate probabilities (*Yager, Engemann & Filev, 1995*), modeling decision-making under immediate probabilities (*Engemann, Filev & Yager, 1996*), mixed uncertain satisfaction (*Yager, 2017*), aggregation function (*Durante & Ricci, 2018*), deviation-based aggregation (*Decký, Mesiar & Stupňanová, 2018*), generalized averaging aggregation operators (*Beliakov et al., 2011*; *Liu et al., 2016*; *Yang & Yao, 2021*), and analysis of fuzzy research under bibliometric indicators (*Merigó, Gil-Lafuente & Yager, 2015*).

Due to strong data-inclusive features, IFS and IVIFS have been extended in distinct regions, including bipolar soft sets (*Mahmood, 2020*), analysis of image quality under measures (*Hassaballah & Ghareeb, 2017*), decision-making framework (*Gao et al., 2021*; *Zeng, Hu & Llopis-Albert, 2023*), distance and similarity measures (*Garg & Rani, 2021*; *Peng, Xiaohe & Jianbo, 2021*), hybrid variable approach (*Liu et al., 2021*; *Xue, Deng & Garg, 2021*; *Zhang et al., 2022*), construction of shadowed sets (*Yang & Yao, 2021*), time-series mapping (*Bas, Yolcu & Egrioglu, 2021*), transportation problem (*Bharati, 2021*), and combined compromise solution approach (*Alrasheedi et al., 2021*; *Su et al., 2023*).

Due to its dominant structure, several scholars have shown their interest in CFS and applied it to diverse regions. For convenience, cross-entropy measures (*Liu, Ali & Mahmood, 2020*), complex fuzzy soft sets (CFSS) (*Thirunavukarasu, Suresh & Ashokkumar, 2017*), IV CFSS (*Selvachandran & Singh, 2018*; *Dai, Bi & Hu, 2019*), and complex multi-fuzzy soft sets (*Al-Qudah & Hassan, 2019*). Further, *Garg & Rani (2019a)*; *Garg & Rani (2020a)* invented the CIVIFS and the advanced aggregation operators under CIFS. *Garg & Rani (2020b)* modified the theory of robust and geometric aggregation operators under CIFS. *Garg & Rani (2019b)* proposed the methodology of aggregation operators under generalized CIFS. *Ali et al. (2021)* combined the principle of CIFS and soft set and explored some aggregation operators. Especially, the statistical metrics evaluated by *Menger (1942)*, Einstein aggregation operators for IFS invented by *Wang & Liu (2012)*, Archimedean aggregation operators for IFS stated by *Xia, Xu & Zhu (2012)*, Hamacher aggregation operators for interval-valued IFS presented by *Liu (2013)*, and so on.

MOTIVATION AND MAIN CONTRIBUTION

CIVIFS theory is the modified version of the FS, IFS, IVIFS, CFS, and CIFS because of their valuable and dominant structure. Further, the theory of Aczel-Alsina is also very famous and reliable because it is the generation of the algebraic t-norm and t-conorm. Moreover, discovering the theory of aggregation operators in the presence of Aczel-Alsina information for managing CIVIF values is a very challenging task for new fuzzy scholars, because up to date no one can derive the theory of Aczel-Alsina aggregation operators for CIVIF values. Furthermore, deriving the theory of the WASPAS technique (*Zavadskas et al., 2012*) is also a very awkward and challenging task for fuzzy researchers.

Keeping the benefits of the above prevailing operators, the major contribution of this analysis is illustrated below:

(1) To initiate the Aczel-Alsina operational laws and their related results.
(2) To invent the principle of CIVIFAAWA, CIVIFAAOWA, CIVIFAAHA, CIVIFAAWG, CIVIFAAOWG, and CIVIFAAHG operators, and illustrated their well-known properties and results.
(3) To derive the theory of the WASPAS method for CIVIFSs.
(4) To demonstrate the MADM strategy under the invented works.
(5) To express the supremacy and dominancy of the invented works with the help of sensitive analysis and geometrical shown of the explored works.

Presentation of our analysis is implemented in the shape: Section 2 covers all the prevailing methodologies. In Section 3, we initiate the Aczel-Alsina operational laws and

Table 1 **Representation of the notation used in the proposed work.**

Notation	meanings	Notation	meanings	Notation	Meanings
$\overline{\overline{\mathfrak{C}}}_{C_{ij}}$	Entry of matrix	$\overline{\overline{\mathcal{M}}}_{\overline{\overline{\mathfrak{C}}}_C}(\widetilde{x}_E)$	Complex Interval-valued truth grade	$\overline{\overline{\mathcal{N}}}_{\overline{\overline{\mathfrak{C}}}_C}(\widetilde{x}_E)$	Complex Interval-valued falsity grade
$\boldsymbol{W_j}$	Weight vector	$\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}}(\widetilde{x}_E)$	Lower bound of real part in truth grade	$\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}}(\widetilde{x}_E)$	Lower bound of real part in falsity grade
$\boldsymbol{S_i}$	Score value	$\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}}(\widetilde{x}_E)$	upper bound of real part in truth grade	$\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}}(\widetilde{x}_E)$	upper bound of real part in falsity grade
$^{\circ}$F	Scaler	$\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}}(\widetilde{x}_E)$	Lower bound of imaginary part in truth grade	$\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}}(\widetilde{x}_E)$	Lower bound of imaginary part in falsity grade
$\widetilde{\mathbb{X}}_U$	Universal set	$\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}}(\widetilde{x}_E)$	upper bound of imaginary part in truth grade	$\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}}(\widetilde{x}_E)$	upper bound of imaginary part in falsity grade
$\overline{\overline{\mathfrak{R}}}_{\overline{\overline{\mathfrak{C}}}_C}(\widetilde{x}_E)$	Complex Interval-valued neutral grade	$\overline{\overline{\mathfrak{R}}}^{L}_{\overline{\overline{R}}}(\widetilde{x}_E)$	Lower bound of real part in neutral grade	$\overline{\overline{\mathfrak{R}}}^{L}_{\overline{\overline{I}}}(\widetilde{x}_E)$	Lower bound of imaginary part in neutral grade
$\widetilde{\mathfrak{x}}_E$	Element of universal set	$\overline{\overline{\mathfrak{R}}}^{U}_{\overline{\overline{R}}}(\widetilde{x}_E)$	upper bound of real part in neutral grade	$\overline{\overline{\mathfrak{R}}}^{U}_{\overline{\overline{I}}}(\widetilde{x}_E)$	upper bound of imaginary part in neutral grade

their related results. Section 4 produces the principle of CIVIFAAWA, CIVIFAAOWA, CIVIFAAHA, CIVIFAAWG, CIVIFAAOWG, and CIVIFAAHG operators, and illustrates their well-known properties. In Section 5, we derive the WASPAS method for CIVIFSs. In Section 6, we demonstrate the effectiveness of the MADM strategy under the invented works. The conclusion of this study is illustrated in Section 7.

Before starting the proposed work, all variables and indexes used in this study are defined in Table 1.

PRELIMINARIES

Here, we utilized the weighted sum model (WSM) and the weighted product model (WPM) to review the concept of the WASPAS method (*Zavadskas et al., 2012*). Moreover, the extended WASPAS method was derived from *Zavadskas et al. (2013)*. Some valuable and effective steps of the WASPAS method are listed below:

Step 1: The input data of the technique is represented in the form of a matrix of alternatives and attributes, which is based on the data received from the expert.

Step 2: Normalize the decision matrix in the presence of the information in Eq. (1):

$$\overline{\overline{\mathfrak{C}}}'_{C_{ij}} = \begin{cases} \dfrac{\overline{\overline{\mathfrak{C}}}_{C_{ij}}}{\max_i \overline{\overline{\mathfrak{C}}}_{C_{ij}}} & if\ \ i \in B \\ \dfrac{\min_i \overline{\overline{\mathfrak{C}}}_{C_{ij}}}{\overline{\overline{\mathfrak{C}}}_{C_{ij}}} & if\ \ i \in C \end{cases} \tag{1}$$

where B represented the benefit types of data and C stated the cost type of criteria.

Step 3: Compute WSM and WPM of each alternative:

$$WSM_i = \sum_{j=1}^{m} W_j \overline{\overline{\mathfrak{C}}}'_{C_{ij}} \tag{2}$$

$$WPM_i = \prod_{j=1}^{m} \left(\overline{\overline{\mathfrak{C}}}_{C_{ij}}' \right)^{W_j} \tag{3}$$

Step 4: Calculate the score value by using the theory of WSM and WPM information referring to the following way:

$$S_i = {}^{\circ}F * WSM_i + \left(1 - {}^{\circ}F\right) WPM_i. \tag{4}$$

There exist some special cases: when ${}^{\circ}F = 1$ in Eq. (4), $S_i = WSM_i$; when ${}^{\circ}F = 0$, $S_i = WPM_i$.

Step 5: Deriving the best preference by the score value in Step 4.

Next, the algebraic theories of some prevailing principles like CIVIFSs, the concept of Aczel-Alsina t-norm and t-conorm will be discussed. Of note, the notation $\widetilde{\mathbb{X}}_U$, stated for universal sets.

Definition 1: (*Garg & Rani, 2019a*) The mathematical structure of CIVIFS $\overline{\overline{\mathfrak{C}}}_C$ is shown in the shape of:

$$\overline{\overline{\mathfrak{C}}}_C = \left\{ \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{\mathfrak{C}}}_C}(\widetilde{x}_E), \overline{\overline{\mathcal{N}}}_{\overline{\overline{\mathfrak{C}}}_C}(\widetilde{x}_E) \right) : \widetilde{x}_E \in \widetilde{\mathbb{X}}_U \right\} \tag{5}$$

where the term $\overline{\overline{\mathcal{M}}}_{\overline{\overline{\mathfrak{C}}}_C}(\widetilde{x}_E) = \left[\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}}^L(\widetilde{x}_E), \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}}^U(\widetilde{x}_E) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}}^L(\widetilde{x}_E), \overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}}^U(\widetilde{x}_E)\right]\right)}$ and $\overline{\overline{\mathcal{N}}}_{\overline{\overline{\mathfrak{C}}}_C}(\widetilde{x}_E) = \left[\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}}^L(\widetilde{x}_E), \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}}^U(\widetilde{x}_E) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}}^L(\widetilde{x}_E), \overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}}^U(\widetilde{x}_E)\right]\right)}$ indicate the TD and FD with $0 \le \overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}}^U(\widetilde{x}_E) + \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}}^U(\widetilde{x}_E) \le 1$ and $0 \le \overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}}^U(\widetilde{x}_E) + \overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}}^U(\widetilde{x}_E) \le 1$. Moreover, $\overline{\overline{\mathfrak{R}}}_{\overline{\overline{\mathfrak{C}}}_C}(\widetilde{x}_E) = \left[\overline{\overline{\mathfrak{R}}}_{\overline{\overline{R}}}^L(\widetilde{x}_E), \overline{\overline{\mathfrak{R}}}_{\overline{\overline{R}}}^U(\widetilde{x}_E) \right] e^{i2\pi\left(\left[\overline{\overline{\mathfrak{R}}}_{\overline{\overline{I}}}^L(\widetilde{x}_E), \overline{\overline{\mathfrak{R}}}_{\overline{\overline{I}}}^U(\widetilde{x}_E)\right]\right)} =$ $\left[\left(1 - \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}}^L(\widetilde{x}_E) + \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}}^L(\widetilde{x}_E)\right)\right), \left(1 - \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}}^U(\widetilde{x}_E) + \overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}}^U(\widetilde{x}_E)\right)\right) \right]$ $e^{i2\pi\left[\left(1 - \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}}^L(\widetilde{x}_E) + \overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}}^L(\widetilde{x}_E)\right)\right), \left(1 - \left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}}^U(\widetilde{x}_E) + \overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}}^U(\widetilde{x}_E)\right)\right)\right]}$ states the neutral grade, and $\overline{\overline{\mathfrak{C}}}_{C_j} = \left(\left[\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_j}}}^L(\widetilde{x}_E), \overline{\overline{\mathcal{M}}}_{\overline{\overline{R_j}}}^U(\widetilde{x}_E) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}_{\overline{\overline{I_j}}}^L(\widetilde{x}_E), \overline{\overline{\mathcal{M}}}_{\overline{\overline{I_j}}}^U(\widetilde{x}_E)\right]\right)}, \left[\overline{\overline{\mathcal{N}}}_{\overline{\overline{R_j}}}^L(\widetilde{x}_E), \overline{\overline{\mathcal{N}}}_{\overline{\overline{R_j}}}^U(\widetilde{x}_E) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}_{\overline{\overline{I_j}}}^L(\widetilde{x}_E), \overline{\overline{\mathcal{N}}}_{\overline{\overline{I_j}}}^U(\widetilde{x}_E)\right]\right)} \right)$ denotes the complex interval-valued intuitionistic fuzzy number (CIVIFN).

Definition 2: (*Garg & Rani, 2019a*) Suppose there are two CIVIFNs $\overline{\overline{\mathfrak{C}}}_{C_j} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_j}}}^L(\widetilde{x}_E), \overline{\overline{\mathcal{M}}}_{\overline{\overline{R_j}}}^U(\widetilde{x}_E) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}_{\overline{\overline{I_j}}}^L(\widetilde{x}_E), \overline{\overline{\mathcal{M}}}_{\overline{\overline{I_j}}}^U(\widetilde{x}_E)\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}_{\overline{\overline{R_j}}}^L(\widetilde{x}_E), \overline{\overline{\mathcal{N}}}_{\overline{\overline{R_j}}}^U(\widetilde{x}_E) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}_{\overline{\overline{I_j}}}^L(\widetilde{x}_E), \overline{\overline{\mathcal{N}}}_{\overline{\overline{I_j}}}^U(\widetilde{x}_E)\right]\right)} \end{array} \right)$, $j = 1, 2$, then:

$$\overline{\overline{\mathfrak{C}}}_{C_1} \oplus \overline{\overline{\mathfrak{C}}}_{C_2} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}_{\overline{\overline{R_1}}}^L + \overline{\overline{\mathcal{M}}}_{\overline{\overline{R_2}}}^L - \overline{\overline{\mathcal{M}}}_{\overline{\overline{R_1}}}^L \overline{\overline{\mathcal{M}}}_{\overline{\overline{R_2}}}^L, \overline{\overline{\mathcal{M}}}_{\overline{\overline{R_1}}}^U + \overline{\overline{\mathcal{M}}}_{\overline{\overline{R_2}}}^U - \overline{\overline{\mathcal{M}}}_{\overline{\overline{R_1}}}^U \overline{\overline{\mathcal{M}}}_{\overline{\overline{R_2}}}^U \right] \\ e^{i2\pi\left[\overline{\overline{\mathcal{M}}}_{\overline{\overline{I_1}}}^L + \overline{\overline{\mathcal{M}}}_{\overline{\overline{I_2}}}^L - \overline{\overline{\mathcal{M}}}_{\overline{\overline{I_1}}}^L \overline{\overline{\mathcal{M}}}_{\overline{\overline{I_2}}}^L, \overline{\overline{\mathcal{M}}}_{\overline{\overline{I_1}}}^U + \overline{\overline{\mathcal{M}}}_{\overline{\overline{I_2}}}^U - \overline{\overline{\mathcal{M}}}_{\overline{\overline{I_1}}}^U \overline{\overline{\mathcal{M}}}_{\overline{\overline{I_2}}}^U\right]}, \\ \left[\overline{\overline{\mathcal{N}}}_{\overline{\overline{R_1}}}^L \overline{\overline{\mathcal{N}}}_{\overline{\overline{R_2}}}^L, \overline{\overline{\mathcal{N}}}_{\overline{\overline{R_1}}}^U \overline{\overline{\mathcal{N}}}_{\overline{\overline{R_2}}}^U \right] e^{i2\pi\left[\overline{\overline{\mathcal{N}}}_{\overline{\overline{I_1}}}^L \overline{\overline{\mathcal{N}}}_{\overline{\overline{I_2}}}^L, \overline{\overline{\mathcal{N}}}_{\overline{\overline{I_1}}}^U \overline{\overline{\mathcal{N}}}_{\overline{\overline{I_2}}}^U\right]} \end{array} \right) \tag{6}$$

$$\overline{\overline{\mathfrak{C}}}_{C_1} \otimes \overline{\overline{\mathfrak{C}}}_{C_2} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_1} \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_2}, \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_1} \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_2} \right] e^{i2\pi \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_1} \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_2}, \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_1} \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_2} \right]}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_1} + \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_2} - \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_1} \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_2}, \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_1} + \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_2} - \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_1} \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_2} \right] \\ e^{i2\pi \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_1} + \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_2} - \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_1} \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_2}, \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_1} + \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_2} - \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_1} \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_2} \right]} \end{array} \right) \tag{7}$$

$$\overline{\overline{\psi_S \mathfrak{C}}}_{C_1} = \left(\begin{array}{c} \left[\left(1 - \left(1 - \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_1} \right)^{\overline{\overline{\psi_S}}} \right), \left(1 - \left(1 - \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_1} \right)^{\overline{\overline{\psi_S}}} \right) \right] e^{i2\pi \left[\left(1 - \left(1 - \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_1} \right)^{\overline{\overline{\psi_S}}} \right), \left(1 - \left(1 - \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_1} \right)^{\overline{\overline{\psi_S}}} \right) \right]}, \\ \left[\overline{\overline{\mathcal{N}}}^{L\,\overline{\overline{\psi_S}}}_{\overline{\overline{R}}_1}, \overline{\overline{\mathcal{N}}}^{U\,\overline{\overline{\psi_S}}}_{\overline{\overline{R}}_1} \right] e^{i2\pi \left[\overline{\overline{\mathcal{N}}}^{L\,\overline{\overline{\psi_S}}}_{\overline{\overline{I}}_1}, \overline{\overline{\mathcal{N}}}^{U\,\overline{\overline{\psi_S}}}_{\overline{\overline{I}}_1} \right]} \end{array} \right) \tag{8}$$

$$\overline{\overline{\mathfrak{C}}}_{C_1}^{\overline{\overline{\psi_S}}} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^{L\,\overline{\overline{\psi_S}}}_{\overline{\overline{R}}_1}, \overline{\overline{\mathcal{M}}}^{U\,\overline{\overline{\psi_S}}}_{\overline{\overline{R}}_1} \right] e^{i2\pi \left[\overline{\overline{\mathcal{M}}}^{L\,\overline{\overline{\psi_S}}}_{\overline{\overline{I}}_1}, \overline{\overline{\mathcal{M}}}^{U\,\overline{\overline{\psi_S}}}_{\overline{\overline{I}}_1} \right]}, \\ \left[\left(1 - \left(1 - \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_1} \right)^{\overline{\overline{\psi_S}}} \right), \left(1 - \left(1 - \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_1} \right)^{\overline{\overline{\psi_S}}} \right) \right] e^{i2\pi \left[\left(1 - \left(1 - \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_1} \right)^{\overline{\overline{\psi_S}}} \right), \left(1 - \left(1 - \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_1} \right)^{\overline{\overline{\psi_S}}} \right) \right]} \end{array} \right) \tag{9}$$

Definition 3: (*Garg & Rani, 2019b*) By taking any two CIVIFNs $\overline{\overline{\mathfrak{C}}}_{C_j} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_j}(\tilde{x}_E), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_j}(\tilde{x}_E) \right] e^{i2\pi \left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_j}(\tilde{x}_E), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_j}(\tilde{x}_E) \right] \right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_j}(\tilde{x}_E), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_j}(\tilde{x}_E) \right] e^{i2\pi \left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_j}(\tilde{x}_E), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_j}(\tilde{x}_E) \right] \right)} \end{array} \right)$, then the score value (SV) and accuracy value (AV) are determined by the following formulas:

$$\overline{\overline{\mathcal{S}_{SV}}} \left(\overline{\overline{\mathfrak{C}}}_{C_1} \right) = \frac{1}{4} \left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_1} + \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_1} - \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_1} - \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_1} + \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_1} + \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_1} - \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_1} - \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_1} \right), \tag{10}$$

$$\overline{\overline{\mathcal{H}_{AV}}} \left(\overline{\overline{\mathfrak{C}}}_{C_1} \right) = \frac{1}{4} \left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_1} + \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_1} + \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_1} + \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_1} + \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_1} + \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_1} + \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_1} + \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_1} \right) \tag{11}$$

Definition 4: (*Garg & Rani, 2019a*) By taking any two CIVIFNs $\overline{\overline{\mathfrak{C}}}_{C_j} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_j}(\tilde{x}_E), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_j}(\tilde{x}_E) \right] e^{i2\pi \left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_j}(\tilde{x}_E), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_j}(\tilde{x}_E) \right] \right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_j}(\tilde{x}_E), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_j}(\tilde{x}_E) \right] e^{i2\pi \left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_j}(\tilde{x}_E), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_j}(\tilde{x}_E) \right] \right)} \end{array} \right)$, $j = 1, 2$, then

(1) If $\overline{\overline{\mathcal{S}_{SV}}} \left(\overline{\overline{\mathfrak{C}}}_{C_1} \right) > \overline{\overline{\mathcal{S}_{SV}}} \left(\overline{\overline{\mathfrak{C}}}_{C_2} \right)$, then $\overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_2}$;

(2) If $\overline{\overline{\mathcal{S}_{SV}}} \left(\overline{\overline{\mathfrak{C}}}_{C_1} \right) < \overline{\overline{\mathcal{S}_{SV}}} \left(\overline{\overline{\mathfrak{C}}}_{C_2} \right)$, then $\overline{\overline{\mathfrak{C}}}_{C_1} < \overline{\overline{\mathfrak{C}}}_{C_2}$;

(3) If $\overline{\overline{\mathcal{S}_{SV}}} \left(\overline{\overline{\mathfrak{C}}}_{C_1} \right) = \overline{\overline{\mathcal{S}_{SV}}} \left(\overline{\overline{\mathfrak{C}}}_{C_2} \right)$, then

(i) If $\overline{\overline{\mathcal{H}_{AV}}} \left(\overline{\overline{\mathfrak{C}}}_{C_1} \right) > \overline{\overline{\mathcal{H}_{AV}}} \left(\overline{\overline{\mathfrak{C}}}_{C_2} \right)$, then $\overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_2}$;

(ii) If $\overline{\overline{\mathcal{H}_{AV}}} \left(\overline{\overline{\mathfrak{C}}}_{C_1} \right) < \overline{\overline{\mathcal{H}_{AV}}} \left(\overline{\overline{\mathfrak{C}}}_{C_2} \right)$, then $\overline{\overline{\mathfrak{C}}}_{C_1} < \overline{\overline{\mathfrak{C}}}_{C_2}$;

(iii) If $\overline{\overline{\mathcal{H}_{AV}}}\left(\overline{\overline{\mathfrak{C}_{C_1}}}\right)=\overline{\overline{\mathcal{H}_{AV}}}\left(\overline{\overline{\mathfrak{C}_{C_2}}}\right)$, then $\overline{\overline{\mathfrak{C}_{C_1}}}=\overline{\overline{\mathfrak{C}_{C_2}}}$.

Definition 5: (*Klement & Mesiar, 1997*) Suppose $\overline{\overline{\mathbb{T}_{TN}}}:[0,1]\times[0,1]\to[0,1]$ states a TN, then

(1) $\overline{\overline{\mathbb{T}_{TN}}}\left(\widetilde{x_E},\widetilde{x_E}'\right)=\overline{\overline{\mathbb{T}_{TN}}}\left(\widetilde{x_E}',\widetilde{x_E}\right),\widetilde{x_E},\widetilde{x_E}'\in[0,1]$;

(2) $\overline{\overline{\mathbb{T}_{TN}}}\left(\widetilde{x_E},\widetilde{x_E}'\right)\leq\overline{\overline{\mathbb{T}_{TN}}}\left(\widetilde{x_E},\widetilde{x_E}''\right)$, if $\widetilde{x_E}'\leq\widetilde{x_E}''$;

(3) $\overline{\overline{\mathbb{T}_{TN}}}\left(\widetilde{x_E},\overline{\overline{\mathbb{T}_{TN}}}\left(\widetilde{x_E}',\widetilde{x_E}''\right)\right)=\overline{\overline{\mathbb{T}_{TN}}}\left(\overline{\overline{\mathbb{T}_{TN}}}\left(\widetilde{x_E},\widetilde{x_E}'\right),\widetilde{x_E}''\right)$;

(4) $\overline{\overline{\mathbb{T}_{TN}}}(\widetilde{x_E},1)=\widetilde{x_E}$.

Definition 6: (*Klement & Mesiar, 1997*) Suppose $\overline{\overline{\mathbb{S}_{TN}}}:[0,1]\times[0,1]\to[0,1]$ states a TCN, then

(1) $\overline{\overline{\mathbb{S}_{TN}}}\left(\widetilde{x_E},\widetilde{x_E}'\right)=\overline{\overline{\mathbb{S}_{TN}}}\left(\widetilde{x_E}',\widetilde{x_E}\right),\widetilde{x_E},\widetilde{x_E}'\in[0,1]$;

(2) $\overline{\overline{\mathbb{S}_{TN}}}\left(\widetilde{x_E},\widetilde{x_E}'\right)\leq\overline{\overline{\mathbb{S}_{TN}}}\left(\widetilde{x_E},\widetilde{x_E}''\right)$, if $\widetilde{x_E}'\leq\widetilde{x_E}''$;

(3) $\overline{\overline{\mathbb{S}_{TN}}}\left(\widetilde{x_E},\overline{\overline{\mathbb{S}_{TN}}}\left(\widetilde{x_E}',\widetilde{x_E}''\right)\right)=\overline{\overline{\mathbb{S}_{TN}}}\left(\overline{\overline{\mathbb{S}_{TN}}}\left(\widetilde{x_E},\widetilde{x_E}'\right),\widetilde{x_E}''\right)$;

(4) $\overline{\overline{\mathbb{S}_{TN}}}(\widetilde{x_E},0)=\widetilde{x_E}$.

Definition 7: (*Aczél & Alsina, 1982*) Suppose $\left(\overline{\overline{\mathbb{T}_{TN}}}_A^{\psi}\right)_{\psi\in[0,\infty]}$ states the Aczel-Alsina TN, its expression is listed as follows:

$$\overline{\overline{\mathbb{T}_{TN}}}_A^{\psi}\left(\widetilde{x_E},\widetilde{x_E}'\right)=\begin{cases}\overline{\overline{\mathbb{T}_{TN}}}\left(\widetilde{x_E},\widetilde{x_E}'\right) & if\ \psi=0\\ min\left(\widetilde{x_E},\widetilde{x_E}'\right) & if\ \psi=\infty\\ e^{-\left(\left(-log\widetilde{x_E}\right)^{\psi}+\left(-log\widetilde{x_E}'\right)^{\psi}\right)^{\frac{1}{\psi}}} & otherwise\end{cases}\tag{12}$$

Definition 8: (*Aczél & Alsina, 1982*) Suppose $\left(\overline{\overline{\mathbb{S}_{TN}}}_A^{\psi}\right)_{\psi\in[0,\infty]}$ states the Aczel-Alsina TCN, the detailed expression is shown as follows:

$$\overline{\overline{\mathbb{S}_{TN}}}_A^{\psi}\left(\widetilde{x_E},\widetilde{x_E}'\right)=\begin{cases}\overline{\overline{\mathbb{S}_{TN}}}\left(\widetilde{x_E},\widetilde{x_E}'\right) & if\ \psi=0\\ max\left(\widetilde{x_E},\widetilde{x_E}'\right) & if\ \psi=\infty\\ 1-e^{-\left(\left(-\log(1-\widetilde{x_E})\right)^{\psi}+\left(-\log\left(1-\widetilde{x_E}'\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} & otherwise\end{cases}\tag{13}$$

ACZEL-ALSINA OPERATIONAL LAWS FOR CIVIFSS

This section mainly introduces the Aczel-Alsina operational laws for CIVIFS that keeps the benefits of the IV information and explores these elementary properties.

Definition 9: For CIVIFNs $\overline{\overline{\mathfrak{C}}}_{C_j}=\left(\begin{array}{c}\left[\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j}^{L}(\widetilde{x_E}),\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j}^{U}(\widetilde{x_E})\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}^{L}(\widetilde{x_E}),\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}^{U}(\widetilde{x_E})\right]\right)},\\ \left[\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j}^{L}(\widetilde{x_E}),\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j}^{U}(\widetilde{x_E})\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j}^{L}(\widetilde{x_E}),\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j}^{U}(\widetilde{x_E})\right]\right)}\end{array}\right)$, $j=1,2$, then

$$\overline{\overline{\mathfrak{C}}}_{C_1}\oplus\overline{\overline{\mathfrak{C}}}_{C_2}=\left(\begin{array}{c}\left[\overline{\overline{\mathbb{S}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{L},\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{L}\right),\overline{\overline{\mathbb{S}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{U},\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{U}\right)\right]e^{i2\pi\left[\overline{\overline{\mathbb{S}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_1}^{L},\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_2}^{L}\right),\overline{\overline{\mathbb{S}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_1}^{U},\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_2}^{U}\right)\right]},\\ \left[\overline{\overline{\mathbb{T}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{L},\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{L}\right),\overline{\overline{\mathbb{T}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{U},\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{U}\right)\right]e^{i2\pi\left[\overline{\overline{\mathbb{T}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{L},\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{L}\right),\overline{\overline{\mathbb{T}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{U},\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{U}\right)\right]}\end{array}\right) \tag{14}$$

$$\overline{\overline{\mathfrak{C}}}_{C_1}\otimes\overline{\overline{\mathfrak{C}}}_{C_2}=\left(\begin{array}{c}\left[\overline{\overline{\mathbb{T}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{L},\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{L}\right),\overline{\overline{\mathbb{T}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{U},\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{U}\right)\right]e^{i2\pi\left[\overline{\overline{\mathbb{T}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_1}^{L},\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_2}^{L}\right),\overline{\overline{\mathbb{T}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_1}^{U},\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_2}^{U}\right)\right]},\\ \left[\overline{\overline{\mathbb{S}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{L},\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{L}\right),\overline{\overline{\mathbb{S}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{U},\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{U}\right)\right]e^{i2\pi\left[\overline{\overline{\mathbb{S}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{L},\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{L}\right),\overline{\overline{\mathbb{S}}}_{TNA}^{\psi}\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{U},\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{U}\right)\right]}\end{array}\right) \tag{15}$$

Definition 10: For any two CIVIFNs $\overline{\overline{\mathfrak{C}}}_{C_j}=\left(\begin{array}{c}\left[\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j}^{L}(\widetilde{x_E}),\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_j}^{U}(\widetilde{x_E})\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}^{L}(\widetilde{x_E}),\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_j}^{U}(\widetilde{x_E})\right]\right)},\\ \left[\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j}^{L}(\widetilde{x_E}),\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_j}^{U}(\widetilde{x_E})\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j}^{L}(\widetilde{x_E}),\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_j}^{U}(\widetilde{x_E})\right]\right)}\end{array}\right)$, $j=1,2$, then the following mathematical formulas hold:

$$\begin{aligned}&\overline{\overline{\mathfrak{C}}}_{C_1}\oplus\overline{\overline{\mathfrak{C}}}_{C_2}\\&=\left(\begin{array}{c}\left[\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]\\ e^{i2\pi\left[\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]},\\ \left[\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]\\ e^{i2\pi\left[\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}\end{array}\right)\end{aligned} \tag{16}$$

$$\begin{aligned}&\overline{\overline{\mathfrak{C}}}_{C_1}\otimes\overline{\overline{\mathfrak{C}}}_{C_2}\\&=\left(\begin{array}{c}\left[\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]\\ e^{i2\pi\left[\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]},\\ \left[\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]\\ e^{i2\pi\left[\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}\end{array}\right)\end{aligned} \tag{17}$$

$$\overline{\overline{\theta}}_S\overline{\overline{\mathfrak{C}}}_{C_1} = \left(\begin{array}{c} \left[\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}, \\ \left[\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]} \end{array} \right) \quad (18)$$

$$\overline{\overline{\mathfrak{C}}}_{C_1}^{\overline{\overline{\theta}}_S} = \left(\begin{array}{c} \left[\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}, \\ \left[\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]} \end{array} \right) \quad (19)$$

Theorem 1: For any two CIVIFNs $\overline{\overline{\mathfrak{C}}}_{C_j} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_j}(\widetilde{x}_E), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_j}(\widetilde{x}_E)\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_j}(\widetilde{x}_E), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_j}(\widetilde{x}_E)\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_j}(\widetilde{x}_E), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_j}(\widetilde{x}_E)\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_j}(\widetilde{x}_E), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_j}(\widetilde{x}_E)\right]\right)} \end{array} \right), j = 1, 2$, then we can derive the subsequent mathematic properties:

(1) $\overline{\overline{\mathfrak{C}}}_{C_1} \oplus \overline{\overline{\mathfrak{C}}}_{C_2} = \overline{\overline{\mathfrak{C}}}_{C_2} \oplus \overline{\overline{\mathfrak{C}}}_{C_1}$;

(2) $\overline{\overline{\mathfrak{C}}}_{C_1} \otimes \overline{\overline{\mathfrak{C}}}_{C_2} = \overline{\overline{\mathfrak{C}}}_{C_2} \otimes \overline{\overline{\mathfrak{C}}}_{C_1}$;

(3) $\overline{\overline{\theta}}_S\left(\overline{\overline{\mathfrak{C}}}_{C_1} \oplus \overline{\overline{\mathfrak{C}}}_{C_2}\right) = \overline{\overline{\theta}}_S\overline{\overline{\mathfrak{C}}}_{C_1} \oplus \overline{\overline{\theta}}_S\overline{\overline{\mathfrak{C}}}_{C_2}$;

(4) $\left(\overline{\theta}_{S_1} + \overline{\theta}_{S_1}\right)\overline{\overline{\mathfrak{C}}}_{C_1} = \overline{\theta}_{S_1}\overline{\overline{\mathfrak{C}}}_{C_1} \oplus \overline{\theta}_{S_2}\overline{\overline{\mathfrak{C}}}_{C_1}$;

(5) $\left(\overline{\overline{\mathfrak{C}}}_{C_1} \otimes \overline{\overline{\mathfrak{C}}}_{C_2}\right)^{\overline{\overline{\theta}}_S} = \overline{\overline{\mathfrak{C}}}_{C_1}^{\overline{\overline{\theta}}_S} \otimes \overline{\overline{\mathfrak{C}}}_{C_2}^{\overline{\overline{\theta}}_S}$;

(6) $\overline{\overline{\mathfrak{C}}}_{C_1}^{\overline{\overline{\theta}}_{S_1}} \otimes \overline{\overline{\mathfrak{C}}}_{C_1}^{\overline{\overline{\theta}}_{S_1}} = \overline{\overline{\mathfrak{C}}}_{C_1}^{\overline{\overline{\theta}}_{S_1} + \overline{\overline{\theta}}_{S_2}}$.

Proof: Next we present the proofs of properties (1), (3) and (5), as we can similarly complete the proofs of properties (2), (4) and (6).

(1) On the basis of Eqs. (16) to (19), then we calculate $\overline{\overline{\mathfrak{C}}}_{C_1} \oplus \overline{\overline{\mathfrak{C}}}_{C_2}$:

$\overline{\overline{\mathfrak{C}}}_{C_1} \oplus \overline{\overline{\mathfrak{C}}}_{C_2}$

$$
=\left(\begin{array}{l}
\left[\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\
e^{i2\pi\left[\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}, \\
\left[\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\
e^{i2\pi\left[\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}
\end{array}\right)
$$

$$
=\left(\begin{array}{l}
\left[\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\
e^{i2\pi\left[\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}, \\
\left[\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\
e^{i2\pi\left[\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{L}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{U}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}
\end{array}\right)
$$

$$
=\overline{\overline{\mathfrak{C}}}_{C_2}\oplus\overline{\overline{\mathfrak{C}}}_{C_1}.
$$

Hence, we investigated $\overline{\overline{\mathfrak{C}}}_{C_1}\oplus\overline{\overline{\mathfrak{C}}}_{C_2}=\overline{\overline{\mathfrak{C}}}_{C_2}\oplus\overline{\overline{\mathfrak{C}}}_{C_1}$, the property (1) is proved.

(2) Suppose $\overline{\overline{\theta}}_S\left(\overline{\overline{\mathfrak{C}}}_{C_1}\oplus\overline{\overline{\mathfrak{C}}}_{C_2}\right)$, then

$$
\overline{\overline{\theta}}_S\left(\overline{\overline{\mathfrak{C}}}_{C_1}\oplus\overline{\overline{\mathfrak{C}}}_{C_2}\right)
$$

$$
=\overline{\overline{\theta}}_S\left(\begin{array}{l}
\left[\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\
e^{i2\pi\left[\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}, \\
\left[\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\
e^{i2\pi\left[\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}
\end{array}\right)
$$

$$=\left(\begin{array}{l}\left[\left[\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]\right.\\ \quad e^{i2\pi\left[\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}\Bigg],\\ \quad\left[\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]\\ \quad e^{i2\pi\left[\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{L}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{U}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}\end{array}\right)$$

$$=\left(\begin{array}{l}\left[\left[\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]\right.\\ \quad e^{i2\pi\left[\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_1}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_1}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}\Bigg],\\ \quad\left[\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_1}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]\\ \quad e^{i2\pi\left[\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_1}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}\end{array}\right)$$

$$\oplus\left(\begin{array}{l}\left[\left[\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]\right.\\ \quad e^{i2\pi\left[\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{M}}}_{\overline{\overline{I}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}\Bigg],\\ \quad\left[\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{R}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]\\ \quad e^{i2\pi\left[\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{N}}}_{\overline{\overline{I}}_2}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}\end{array}\right)$$

$$=\overline{\overline{\theta}}_S\overline{\overline{\mathfrak{C}}}_{C_1}\oplus\overline{\overline{\theta}}_S\overline{\overline{\mathfrak{C}}}_{C_2}.$$

Hence, $\overline{\overline{\theta}}_S\left(\overline{\overline{\mathfrak{C}}}_{C_1}\oplus\overline{\overline{\mathfrak{C}}}_{C_2}\right)=\overline{\overline{\theta}}_S\overline{\overline{\mathfrak{C}}}_{C_1}\oplus\overline{\overline{\theta}}_S\overline{\overline{\mathfrak{C}}}_{C_2}$, the property (3) is proved.

(3) Suppose $\left(\overline{\overline{\mathfrak{C}}}_{C_1} \otimes \overline{\overline{\mathfrak{C}}}_{C_2}\right)^{\overline{\overline{\theta}}_S}$, then

$$
\begin{aligned}
&\left(\overline{\overline{\mathfrak{C}}}_{C_1} \otimes \overline{\overline{\mathfrak{C}}}_{C_2}\right)^{\overline{\overline{\theta}}_S} \\
&= \begin{pmatrix}
\left[\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_1}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_1}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\
e^{i2\pi\left[\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_1}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_1}\right)\right)^{\psi}+\left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}, \\
\left[\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_1}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_1}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\
e^{i2\pi\left[\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_1}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_1}\right)\right)^{\psi}+\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}
\end{pmatrix}^{\overline{\overline{\theta}}_S} \\
&= \begin{pmatrix}
\left[\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\
e^{i2\pi\left[\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}, \\
\left[\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\
e^{i2\pi\left[\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_1}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}
\end{pmatrix} \\
&\otimes \begin{pmatrix}
\left[\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\
e^{i2\pi\left[\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}, \\
\left[\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\
e^{i2\pi\left[\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\overline{\overline{\theta}}_S\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_2}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}
\end{pmatrix} \\
&= \overline{\overline{\mathfrak{C}}}_{C_1}^{\overline{\overline{\theta}}_S} \otimes \overline{\overline{\mathfrak{C}}}_{C_2}^{\overline{\overline{\theta}}_S}
\end{aligned}
$$

Hence, $\left(\overline{\overline{\mathfrak{C}_{C_1}}} \otimes \overline{\overline{\mathfrak{C}_{C_2}}}\right)^{\overline{\overline{\theta_S}}} = \overline{\overline{\mathfrak{C}_{C_1}}}^{\overline{\overline{\theta_S}}} \otimes \overline{\overline{\mathfrak{C}_{C_2}}}^{\overline{\overline{\theta_S}}}$, and the property (5) is proved.

ACZEL-ALSINA AGGREGATION OPERATORS FOR CIVIFS

This section proposes a group of aggregation operators by utilizing the Aczel-Alsina operational laws for CIVIFSs such that CIVIFAAWA, CIVIFAAOWA, CIVIFAAHA, CIVIFAAWG, CIVIFAAOWG, and CIVIFAAHG operators, and illustrates their well-known properties.

Definition 11: For CIVIFNs $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{array}\right), j = 1, \ldots, n,$ then the CIVIFAAWA operator is interpreted as:

$$CIVIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{W}_1\mathfrak{C}_{C_1}}} \oplus \overline{\overline{\mathfrak{W}_2\mathfrak{C}_{C_2}}} \oplus \cdots \oplus \overline{\overline{\mathfrak{W}_n\mathfrak{C}_{C_n}}} = \oplus_{j=1}^{n}\left(\overline{\overline{\mathfrak{W}_j\mathfrak{C}_{C_j}}}\right) \tag{20}$$

where $\overline{\overline{\mathfrak{W}}} = \left(\overline{\overline{\mathfrak{W}}}_1, \overline{\overline{\mathfrak{W}}}_2, \ldots, \overline{\overline{\mathfrak{W}}}_n\right)^T$ means the weight of C_j, with a rule $\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j = 1$.

Theorem 2: For CIVIFNs $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{array}\right), j = 1, \ldots, n,$ then by using Eq. (20), we elaborate

$$CIVIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right)$$

$$= \left(\begin{array}{c} \left[\left(1 - e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(1 - e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}, \\ \left[\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]} \end{array}\right) \tag{21}$$

Furthermore, we derive the theory of idempotency, boundedness, and monotonicity for the information in Eq. (21).

Property 1: For CIVIFNs $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_j}(\tilde{x}_E), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_j}(\tilde{x}_E)\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_j}(\tilde{x}_E), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_j}(\tilde{x}_E)\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_j}(\tilde{x}_E), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_j}(\tilde{x}_E)\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_j}(\tilde{x}_E), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_j}(\tilde{x}_E)\right]\right)} \end{array}\right), j = 1, 2, \ldots, n,$ if $\overline{\overline{\mathfrak{C}_{C_j}}} = \overline{\overline{\mathfrak{C}}}$, then the detailed expression is shown as:

$$CIVIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{C}}} \tag{22}$$

Proof: Suppose $\overline{\overline{\mathfrak{C}_{C_j}}} = \overline{\overline{\mathfrak{C}}} = \left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}}, \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}}\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}}, \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}}\right]\right)}, \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}}, \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}}\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}}, \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}}\right]\right)}\right),$ then

$$CIVIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \left(\begin{array}{c} \left[\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}_j}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}_j}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}_j}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}_j}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}, \\ \left[\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j\left(-\log\left(\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}_j}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j\left(-\log\left(\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}_j}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j\left(-\log\left(\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}_j}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j\left(-\log\left(\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}_j}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]} \end{array}\right)$$

$$= \left(\begin{array}{c} \left[\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(1-e^{-\left(\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}, \\ \left[\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\left(-\log\left(\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]} \end{array}\right)$$

$$= \left(\begin{array}{c} \left[\left(1-e^{\log\left(1-\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}}\right)}\right), \left(1-e^{\log\left(1-\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}}\right)}\right)\right] e^{i2\pi\left[\left(1-e^{\log\left(1-\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}}\right)}\right), \left(1-e^{\log\left(1-\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}}\right)}\right)\right]}, \\ \left[\left(e^{\log\left(\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}}\right)}\right), \left(e^{\log\left(\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}}\right)}\right)\right] e^{i2\pi\left[\left(e^{\log\left(\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}}\right)}\right), \left(e^{\log\left(\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}}\right)}\right)\right]} \end{array}\right)$$

$$= \left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R}}}, \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R}}} \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I}}}, \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I}}}\right]\right)}, \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R}}}, \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R}}} \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I}}}, \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I}}}\right]\right)} \right).$$

Property 2: For CIVIFNs $\overline{\overline{\mathfrak{C}}}_{C_j} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E}) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E}) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{array} \right), j = 1, 2, \ldots, n$, if

$$\overline{\overline{\mathfrak{C}}}^{-} = \left(\left[\min_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}, \min_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}} \right] e^{i2\pi\left[\min_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}, \min_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}\right]}, \left[\max_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}, \max_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}} \right] e^{i2\pi\left[\max_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}, \max_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}\right]} \right)$$

and $\overline{\overline{\mathfrak{C}}}^{+} = \left(\left[\max_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}, \max_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}} \right] e^{i2\pi\left[\max_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}, \max_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}\right]}, \left[\min_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}, \min_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}} \right] e^{i2\pi\left[\min_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}, \min_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}\right]} \right)$, then

$$\overline{\overline{\mathfrak{C}}}^{-} \le CIVIFAAWA\left(\overline{\overline{\mathfrak{C}}}_{C_1}, \overline{\overline{\mathfrak{C}}}_{C_2}, \ldots, \overline{\overline{\mathfrak{C}}}_{C_n}\right) \le \overline{\overline{\mathfrak{C}}}^{+}$$

Proof: Suppose $\overline{\overline{\mathfrak{C}}}_{C_j} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E}) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E}) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{array} \right)$, if

$\overline{\overline{\mathfrak{C}}}^{-} = \left(\left[\min_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}, \min_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}} \right] e^{i2\pi\left[\min_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}, \min_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}\right]}, \left[\max_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}, \max_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}} \right] e^{i2\pi\left[\max_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}, \max_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}\right]} \right)$ and

$\overline{\overline{\mathfrak{C}}}^{+} = \left(\left[\max_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}, \max_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}} \right] e^{i2\pi\left[\max_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}, \max_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}\right]}, \left[\min_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}, \min_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}} \right] e^{i2\pi\left[\min_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}, \min_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}\right]} \right)$, by using inequality, we have

$$1 - e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(1 - \overline{\overline{\mathcal{M}}}^{-L}_{\overline{\overline{R_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \le 1 - e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(1 - \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \le 1 - e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(1 - \overline{\overline{\mathcal{M}}}^{+L}_{\overline{\overline{R_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}$$

$$1 - e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(1 - \overline{\overline{\mathcal{M}}}^{-L}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \le 1 - e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(1 - \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \le 1 - e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(1 - \overline{\overline{\mathcal{M}}}^{+L}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}$$

$$e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}^{-L}_{\overline{\overline{R_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \le e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \le e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}^{+L}_{\overline{\overline{R_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}$$

$$e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}^{-L}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \le e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \le e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}^{+L}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}$$

Similarly, for the upper part, we have

$$e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}^{-U}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \le e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}} \le e^{-\left(\sum_{j=1}^{n} \overline{\overline{\mathfrak{W}}}_j \left(-\log\left(\overline{\overline{\mathcal{M}}}^{+U}_{\overline{\overline{I_j}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}$$

Therefore, we obtained

$\overline{\overline{\mathfrak{C}^{-}}} \leq CIVIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}},\overline{\overline{\mathfrak{C}_{C_2}}},\ldots,\overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq \overline{\overline{\mathfrak{C}^{+}}}$.

Property 3: For CIVIFNs $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\begin{array}{c}\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\tilde{x}_E)\right]\right)},\\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\tilde{x}_E)\right]\right)}\end{array}\right), j=1,\ldots,n$, if $\overline{\overline{\mathfrak{C}_{C_j}}} \leq \overline{\overline{\mathfrak{C}_{C_j}}}'$, then

$CIVIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}},\overline{\overline{\mathfrak{C}_{C_2}}},\ldots,\overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq CIVIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}',\overline{\overline{\mathfrak{C}_{C_2}}}',\ldots,\overline{\overline{\mathfrak{C}_{C_n}}}'\right)$

Definition 12: For CIVIFNs $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\begin{array}{c}\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\tilde{x}_E)\right]\right)},\\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\tilde{x}_E)\right]\right)}\end{array}\right), j=1,\ldots,n$, then the CIVIFAAOWA operator is invented by:

$$CIVIFAAOWA\left(\overline{\overline{\mathfrak{C}_{C_1}}},\overline{\overline{\mathfrak{C}_{C_2}}},\ldots,\overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{W}_1\mathfrak{C}_{C_{\varphi(1)}}}} \oplus \overline{\overline{\mathfrak{W}_2\mathfrak{C}_{C_{\varphi(2)}}}} \oplus \cdots \oplus \overline{\overline{\mathfrak{W}_n\mathfrak{C}_{C_{\varphi(n)}}}} = \oplus_{j=1}^{n}\left(\overline{\overline{\mathfrak{W}_j\mathfrak{C}_{C_{\varphi(j)}}}}\right) \tag{25}$$

where $\overline{\overline{\mathfrak{W}}} = \left(\overline{\overline{\mathfrak{W}_1}},\overline{\overline{\mathfrak{W}_2}},\ldots,\overline{\overline{\mathfrak{W}_n}}\right)^T$ indicates the weight of C_j, with $\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}} = 1$, with parameter $\varphi(1),\varphi(2),\ldots,\varphi(n)$ based on $\overline{\overline{\mathfrak{C}_{C_{\varphi(j)}}}} \leq \overline{\overline{\mathfrak{C}_{C_{\varphi(j-1)}}}}$.

Theorem 2: For CIVIFNs $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\begin{array}{c}\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\tilde{x}_E)\right]\right)},\\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\tilde{x}_E)\right]\right)}\end{array}\right), j=1,\ldots,n$, then by using Eq. (25), we elaborate

$$CIVIFAAOWA\left(\overline{\overline{\mathfrak{C}_{C_1}}},\overline{\overline{\mathfrak{C}_{C_2}}},\ldots,\overline{\overline{\mathfrak{C}_{C_n}}}\right)$$

$$= \left(\begin{array}{c}\left[\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_{\varphi(j)}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_{\varphi(j)}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_{\varphi(j)}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_{\varphi(j)}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}, \\ \left[\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_{\varphi(j)}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_{\varphi(j)}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_{\varphi(j)}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_{\varphi(j)}}}}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}\end{array}\right) \tag{26}$$

Idempotency-Property 4: By taking CIVIFNs $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\begin{array}{c}\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\tilde{x}_E)\right]\right)},\\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\tilde{x}_E),\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\tilde{x}_E)\right]\right)}\end{array}\right), j=1,2,\ldots,n$, if $\overline{\overline{\mathfrak{C}_{C_j}}} = \overline{\overline{\mathfrak{C}}}$, then

$$CIVIFAAOWA\left(\overline{\overline{\mathfrak{C}_{C_1}}},\overline{\overline{\mathfrak{C}_{C_2}}},\ldots,\overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{C}}} \tag{27}$$

Monotonicity-Property 5: By taking CIVIFNs $\overline{\overline{\mathfrak{C}_{C_j}}}$
$= \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^L_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^U_{\overline{\overline{R_j}}}(\widetilde{x_E}) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^L_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^U_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^L_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^U_{\overline{\overline{R_j}}}(\widetilde{x_E}) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^L_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^U_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{array} \right), j = 1, 2, \ldots, n,$
if $\overline{\overline{\mathfrak{C}}}^- = \left(\left[\min_j \overline{\overline{\mathcal{M}}}^L_{\overline{\overline{R_j}}}, \min_j \overline{\overline{\mathcal{M}}}^U_{\overline{\overline{R_j}}} \right] e^{i2\pi\left[\min_j \overline{\overline{\mathcal{M}}}^L_{\overline{\overline{I_j}}}, \min_j \overline{\overline{\mathcal{M}}}^U_{\overline{\overline{I_j}}}\right]},\right.$
$\left. \left[\max_j \overline{\overline{\mathcal{N}}}^L_{\overline{\overline{R_j}}}, \max_j \overline{\overline{\mathcal{N}}}^U_{\overline{\overline{R_j}}} \right] e^{i2\pi\left[\max_j \overline{\overline{\mathcal{N}}}^L_{\overline{\overline{I_j}}}, \max_j \overline{\overline{\mathcal{N}}}^U_{\overline{\overline{I_j}}}\right]} \right)$ and
$\overline{\overline{\mathfrak{C}}}^+ = \left(\left[\max_j \overline{\overline{\mathcal{M}}}^L_{\overline{\overline{R_j}}}, \max_j \overline{\overline{\mathcal{M}}}^U_{\overline{\overline{R_j}}} \right] e^{i2\pi\left[\max_j \overline{\overline{\mathcal{M}}}^L_{\overline{\overline{I_j}}}, \max_j \overline{\overline{\mathcal{M}}}^U_{\overline{\overline{I_j}}}\right]},\right.$
$\left. \left[\min_j \overline{\overline{\mathcal{N}}}^L_{\overline{\overline{R_j}}}, \min_j \overline{\overline{\mathcal{N}}}^U_{\overline{\overline{R_j}}} \right] e^{i2\pi\left[\min_j \overline{\overline{\mathcal{N}}}^L_{\overline{\overline{I_j}}}, \min_j \overline{\overline{\mathcal{N}}}^U_{\overline{\overline{I_j}}}\right]} \right)$, then

$$\overline{\overline{\mathfrak{C}}}^- \leq CIVIFAAOWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq \overline{\overline{\mathfrak{C}}}^+ \tag{28}$$

Boundedness-Property 6: By taking CIVIFNs
$\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^L_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^U_{\overline{\overline{R_j}}}(\widetilde{x_E}) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^L_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^U_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^L_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^U_{\overline{\overline{R_j}}}(\widetilde{x_E}) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^L_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^U_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{array} \right), j = 1, 2, \ldots, n,$ if $\overline{\overline{\mathfrak{C}_{C_j}}} \leq \overline{\overline{\mathfrak{C}_{C_j}}}'$, then

$$CIVIFAAOWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq CIVIFAAOWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}', \overline{\overline{\mathfrak{C}_{C_2}}}', \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}'\right) \tag{29}$$

Definition 13: For CIVIFNs $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^L_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^U_{\overline{\overline{R_j}}}(\widetilde{x_E}) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^L_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^U_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^L_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^U_{\overline{\overline{R_j}}}(\widetilde{x_E}) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^L_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^U_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{array} \right), j = 1, \ldots, n,$
then the CIVIFAAHA operator is invented by:

$$CIVIFAAHA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right)$$
$$= \overline{\overline{\mathfrak{W}_1}}\,\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(1)}}}} \oplus \overline{\overline{\mathfrak{W}_2}}\,\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(2)}}}} \oplus \cdots \oplus \overline{\overline{\mathfrak{W}_n}}\,\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(n)}}}} = \oplus_{j=1}^{n}\left(\overline{\overline{\mathfrak{W}_j}}\,\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j)}}}}\right) \tag{30}$$

Theorem 3: For CIVIFNs $\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^L_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^U_{\overline{\overline{R_j}}}(\widetilde{x_E}) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^L_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^U_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^L_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^U_{\overline{\overline{R_j}}}(\widetilde{x_E}) \right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^L_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^U_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{array} \right), j = 1, \ldots, n,$ then
by using Eq. (30), we elaborate

$CIVIFAAHA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right)$

$$
=\left(\begin{array}{c}
\left[\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_{\varphi(j)}}^{\cdot}}}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R}}_{\varphi(j)}}^{\cdot}}}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\
e^{i2\pi\left[\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I}}_{\varphi(j)}}^{\cdot}}}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I}}_{\varphi(j)}}^{\cdot}}}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}, \\
\left[\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R}}_{\varphi(j)}}^{\cdot}}}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R}}_{\varphi(j)}}^{\cdot}}}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\
e^{i2\pi\left[\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_{\varphi(j)}}^{\cdot}}}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I}}_{\varphi(j)}}^{\cdot}}}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}
\end{array}\right) \tag{31}
$$

Idempotency-Property 7: By taking CIVIFNs

$$
\overline{\overline{\mathfrak{C}_{C_j}}}=\left(\begin{array}{c}\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}),\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}),\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}),\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}),\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}\end{array}\right), j=1,2,\ldots,n, \text{ if } \overline{\overline{\mathfrak{C}_{C_j}}}=\overline{\overline{\mathfrak{C}}}, \text{ then}
$$

$$
CIVIFAAHA\left(\overline{\overline{\mathfrak{C}_{C_1}}},\overline{\overline{\mathfrak{C}_{C_2}}},\ldots,\overline{\overline{\mathfrak{C}_{C_n}}}\right)=\overline{\overline{\mathfrak{C}}} \tag{32}
$$

Monotonicity-Property 8: By taking CIVIFNs

$$
\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\begin{array}{c}\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}),\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}),\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}),\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}),\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}\end{array}\right),
$$

$$
\text{if } \overline{\overline{\mathfrak{C}}}^{-} = \left(\left[\min_j\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}},\min_j\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}\right]e^{i2\pi\left[\min_j\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}},\min_j\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}\right]},\right.
$$

$$
\left.\left[\max_j\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}},\max_j\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}\right]e^{i2\pi\left[\max_j\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}},\max_j\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}\right]}\right) \text{ and}
$$

$$
\overline{\overline{\mathfrak{C}}}^{+} = \left(\left[\max_j\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}},\max_j\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}\right]e^{i2\pi\left[\max_j\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}},\max_j\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}\right]},\right.
$$

$$
\left.\left[\min_j\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}},\min_j\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}\right]e^{i2\pi\left[\min_j\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}},\min_j\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}\right]}\right), \text{ then}
$$

$$
\overline{\overline{\mathfrak{C}}}^{-} \leq CIVIFAAHA\left(\overline{\overline{\mathfrak{C}_{C_1}}},\overline{\overline{\mathfrak{C}_{C_2}}},\ldots,\overline{\overline{\mathfrak{C}_{C_n}}}\right)\leq\overline{\overline{\mathfrak{C}}}^{+} \tag{33}
$$

Boundedness-Property 9: By taking CIVIFNs

$$
\overline{\overline{\mathfrak{C}_{C_j}}}=\left(\begin{array}{c}\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}),\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}),\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}),\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}),\overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}\end{array}\right), j=1,2,\ldots,n, \text{ if } \overline{\overline{\mathfrak{C}_{C_j}}}\leq\overline{\overline{\mathfrak{C}_{C_j}}}^{\prime}, \text{ then}
$$

$$
CIVIFAAHA\left(\overline{\overline{\mathfrak{C}_{C_1}}},\overline{\overline{\mathfrak{C}_{C_2}}},\ldots,\overline{\overline{\mathfrak{C}_{C_n}}}\right)\leq CIVIFAAHA\left(\overline{\overline{\mathfrak{C}_{C_1}}}^{\prime},\overline{\overline{\mathfrak{C}_{C_2}}}^{\prime},\ldots,\overline{\overline{\mathfrak{C}_{C_n}}}^{\prime}\right) \tag{34}
$$

Definition 14: By taking CIVIFNs $\overline{\overline{\mathfrak{C}}}_{C_j}=\left(\begin{array}{c}\left[\overline{\mathcal{M}}^{L}_{\overline{\overline{R}}_j}(\tilde{x}_E),\overline{\mathcal{M}}^{U}_{\overline{\overline{R}}_j}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\mathcal{M}}^{L}_{\overline{\overline{I}}_j}(\tilde{x}_E),\overline{\mathcal{M}}^{U}_{\overline{\overline{I}}_j}(\tilde{x}_E)\right]\right)},\\ \left[\overline{\mathcal{N}}^{L}_{\overline{\overline{R}}_j}(\tilde{x}_E),\overline{\mathcal{N}}^{U}_{\overline{\overline{R}}_j}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\mathcal{N}}^{L}_{\overline{\overline{I}}_j}(\tilde{x}_E),\overline{\mathcal{N}}^{U}_{\overline{\overline{I}}_j}(\tilde{x}_E)\right]\right)}\end{array}\right)$, then the CIVIFAAWG operator is interpreted as:

$$CIVIFAAWG\left(\overline{\overline{\mathfrak{C}}}_{C_1},\overline{\overline{\mathfrak{C}}}_{C_2},\ldots,\overline{\overline{\mathfrak{C}}}_{C_n}\right)=\overline{\overline{\mathfrak{C}}}_{C_1}^{\overline{\overline{\mathfrak{W}}}_1}\otimes\overline{\overline{\mathfrak{C}}}_{C_2}^{\overline{\overline{\mathfrak{W}}}_2}\otimes\cdots\otimes\overline{\overline{\mathfrak{C}}}_{C_n}^{\overline{\overline{\mathfrak{W}}}_n}=\otimes_{j=1}^{n}\left(\overline{\overline{\mathfrak{C}}}_{C_j}^{\overline{\overline{\mathfrak{W}}}_j}\right) \tag{35}$$

Theorem 4: By taking CIVIFNs $\overline{\overline{\mathfrak{C}}}_{C_j}=\left(\begin{array}{c}\left[\overline{\mathcal{M}}^{L}_{\overline{\overline{R}}_j}(\tilde{x}_E),\overline{\mathcal{M}}^{U}_{\overline{\overline{R}}_j}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\mathcal{M}}^{L}_{\overline{\overline{I}}_j}(\tilde{x}_E),\overline{\mathcal{M}}^{U}_{\overline{\overline{I}}_j}(\tilde{x}_E)\right]\right)},\\ \left[\overline{\mathcal{N}}^{L}_{\overline{\overline{R}}_j}(\tilde{x}_E),\overline{\mathcal{N}}^{U}_{\overline{\overline{R}}_j}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\mathcal{N}}^{L}_{\overline{\overline{I}}_j}(\tilde{x}_E),\overline{\mathcal{N}}^{U}_{\overline{\overline{I}}_j}(\tilde{x}_E)\right]\right)}\end{array}\right)$, then by using Eq. (35), we elaborate

$$CIVIFAAWG\left(\overline{\overline{\mathfrak{C}}}_{C_1},\overline{\overline{\mathfrak{C}}}_{C_2},\ldots,\overline{\overline{\mathfrak{C}}}_{C_n}\right)$$

$$=\left(\begin{array}{c}\left[\left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\mathcal{M}}^{L}_{\overline{\overline{R}}_j}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\mathcal{M}}^{U}_{\overline{\overline{R}}_j}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right)\right]\\ e^{i2\pi\left[\left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\mathcal{M}}^{L}_{\overline{\overline{I}}_j}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right),\left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\mathcal{M}}^{U}_{\overline{\overline{I}}_j}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right)\right]},\\ \left[\left(1-e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\mathcal{N}}^{L}_{\overline{\overline{R}}_j}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\mathcal{N}}^{U}_{\overline{\overline{R}}_j}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right)\right]\\ e^{i2\pi\left[\left(1-e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\mathcal{N}}^{L}_{\overline{\overline{I}}_j}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right),\left(1-e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\mathcal{N}}^{U}_{\overline{\overline{I}}_j}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right)\right]}\end{array}\right) \tag{36}$$

Definition 15: For CIVIFNs $\overline{\overline{\mathfrak{C}}}_{C_j}=\left(\begin{array}{c}\left[\overline{\mathcal{M}}^{L}_{\overline{\overline{R}}_j}(\tilde{x}_E),\overline{\mathcal{M}}^{U}_{\overline{\overline{R}}_j}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\mathcal{M}}^{L}_{\overline{\overline{I}}_j}(\tilde{x}_E),\overline{\mathcal{M}}^{U}_{\overline{\overline{I}}_j}(\tilde{x}_E)\right]\right)},\\ \left[\overline{\mathcal{N}}^{L}_{\overline{\overline{R}}_j}(\tilde{x}_E),\overline{\mathcal{N}}^{U}_{\overline{\overline{R}}_j}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\mathcal{N}}^{L}_{\overline{\overline{I}}_j}(\tilde{x}_E),\overline{\mathcal{N}}^{U}_{\overline{\overline{I}}_j}(\tilde{x}_E)\right]\right)}\end{array}\right),j=1,\ldots,n,$ then the CIVIFAAOWG operator is interpreted as:

$$CIVIFAAOWG\left(\overline{\overline{\mathfrak{C}}}_{C_1},\overline{\overline{\mathfrak{C}}}_{C_2},\ldots,\overline{\overline{\mathfrak{C}}}_{C_n}\right)$$

$$=\overline{\overline{\mathfrak{C}}}_{C_{\varphi(1)}}^{\overline{\overline{\mathfrak{W}}}_1}\otimes\overline{\overline{\mathfrak{C}}}_{C_{\varphi(2)}}^{\overline{\overline{\mathfrak{W}}}_2}\otimes\cdots\otimes\overline{\overline{\mathfrak{C}}}_{C_{\varphi(n)}}^{\overline{\overline{\mathfrak{W}}}_n}=\otimes_{j=1}^{n}\left(\overline{\overline{\mathfrak{C}}}_{C_{\varphi(j)}}^{\overline{\overline{\mathfrak{W}}}_j}\right) \tag{37}$$

where $\overline{\overline{\mathfrak{W}}}=\left(\overline{\overline{\mathfrak{W}}}_1,\overline{\overline{\mathfrak{W}}}_2,\ldots,\overline{\overline{\mathfrak{W}}}_n\right)^T$ indicates the weight of C_j, with a rule $\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}}}_j=1$, with parameter $\varphi(1),\varphi(2),\ldots,\varphi(n)$ based on $\overline{\overline{\mathfrak{C}}}_{C_{\varphi(j)}}\leq\overline{\overline{\mathfrak{C}}}_{C_{\varphi(j-1)}}$.

Theorem 5: For CIVIFNs $\overline{\overline{\mathfrak{C}}}_{C_j}=\left(\begin{array}{c}\left[\overline{\mathcal{M}}^{L}_{\overline{\overline{R}}_j}(\tilde{x}_E),\overline{\mathcal{M}}^{U}_{\overline{\overline{R}}_j}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\mathcal{M}}^{L}_{\overline{\overline{I}}_j}(\tilde{x}_E),\overline{\mathcal{M}}^{U}_{\overline{\overline{I}}_j}(\tilde{x}_E)\right]\right)},\\ \left[\overline{\mathcal{N}}^{L}_{\overline{\overline{R}}_j}(\tilde{x}_E),\overline{\mathcal{N}}^{U}_{\overline{\overline{R}}_j}(\tilde{x}_E)\right]e^{i2\pi\left(\left[\overline{\mathcal{N}}^{L}_{\overline{\overline{I}}_j}(\tilde{x}_E),\overline{\mathcal{N}}^{U}_{\overline{\overline{I}}_j}(\tilde{x}_E)\right]\right)}\end{array}\right),j=1,\ldots,n,$

then we elaborate

$$CIVIFAAOWG\left(\overline{\overline{\mathfrak{C}}}_{C_1},\overline{\overline{\mathfrak{C}}}_{C_2},\ldots,\overline{\overline{\mathfrak{C}}}_{C_n}\right)$$

$$= \begin{pmatrix} \left[\left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{R_{\varphi(j)}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{R_{\varphi(j)}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right)\right] e^{i2\pi\left[\left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{I_{\varphi(j)}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{I_{\varphi(j)}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right)\right]}, \\ \left[\left(1-e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{L}_{R_{\varphi(j)}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right), \left(1-e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{U}_{R_{\varphi(j)}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right)\right] e^{i2\pi\left[\left(1-e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{L}_{I_{\varphi(j)}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right), \left(1-e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\overline{\mathcal{N}}}^{U}_{I_{\varphi(j)}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}}}_j}\right)^{\frac{1}{\psi}}}\right)\right]} \end{pmatrix} \tag{38}$$

Idempotency-Property 10: By taking CIVIFNs $\overline{\overline{\mathfrak{C}}}_{C_j} = \begin{pmatrix} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{pmatrix}, j = 1, 2, \ldots, n$, if $\overline{\overline{\mathfrak{C}}}_{C_j} = \overline{\overline{\mathfrak{C}}}$, then

$$CIVIFAAOWG\left(\overline{\overline{\mathfrak{C}}}_{C_1}, \overline{\overline{\mathfrak{C}}}_{C_2}, \ldots, \overline{\overline{\mathfrak{C}}}_{C_n}\right) = \overline{\overline{\mathfrak{C}}} \tag{39}$$

Monotonicity-Property 11: By taking CIVIFNs $\overline{\overline{\mathfrak{C}}}_{C_j} = \begin{pmatrix} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{pmatrix}, j = 1, 2, \ldots, n$, if $\overline{\overline{\mathfrak{C}}}^{-} = \left(\left[\min_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}, \min_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}\right] e^{i2\pi\left[\min_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}, \min_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}\right]}, \left[\max_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}, \max_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}\right] e^{i2\pi\left[\max_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}, \max_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}\right]}\right)$ and $\overline{\overline{\mathfrak{C}}}^{+} = \left(\left[\max_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}, \max_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}\right] e^{i2\pi\left[\max_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}, \max_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}\right]}, \left[\min_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}, \min_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}\right] e^{i2\pi\left[\min_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}, \min_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}\right]}\right)$, then

$$\overline{\overline{\mathfrak{C}}}^{-} \leq CIVIFAAOWG\left(\overline{\overline{\mathfrak{C}}}_{C_1}, \overline{\overline{\mathfrak{C}}}_{C_2}, \ldots, \overline{\overline{\mathfrak{C}}}_{C_n}\right) \leq \overline{\overline{\mathfrak{C}}}^{+} \tag{40}$$

Boundedness-Property 12: By taking CIVIFNs $\overline{\overline{\mathfrak{C}}}_{C_j} = \begin{pmatrix} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{pmatrix}, j = 1, 2, \ldots, n$, if $\overline{\overline{\mathfrak{C}}}_{C_j} \leq \overline{\overline{\mathfrak{C}}}'_{C_j}$, then

$$CIVIFAAOWG\left(\overline{\overline{\mathfrak{C}}}_{C_1}, \overline{\overline{\mathfrak{C}}}_{C_2}, \ldots, \overline{\overline{\mathfrak{C}}}_{C_n}\right) \leq CIVIFAAOWG\left(\overline{\overline{\mathfrak{C}}}'_{C_1}, \overline{\overline{\mathfrak{C}}}'_{C_2}, \ldots, \overline{\overline{\mathfrak{C}}}'_{C_n}\right) \tag{41}$$

Definition 16: For CIVIFNs $\overline{\overline{\mathfrak{C}_{C_j}}} = \begin{pmatrix} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{pmatrix}, j = 1, \ldots, n,$

then the CIVIFAAHG operator is shown as:

$$CIVIFAAHG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(1)}}}}^{\overline{\overline{\mathfrak{W}_1}}} \otimes \overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(2)}}}}^{\overline{\overline{\mathfrak{W}_2}}} \otimes \cdots \otimes \overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(n)}}}}^{\overline{\overline{\mathfrak{W}_n}}} = \otimes_{j=1}^{n}\left(\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j)}}}}^{\overline{\overline{\mathfrak{W}_j}}}\right) \tag{42}$$

where $\overline{\overline{\mathfrak{W}}} = \left(\overline{\overline{\mathfrak{W}_1}}, \overline{\overline{\mathfrak{W}_2}}, \ldots, \overline{\overline{\mathfrak{W}_n}}\right)^T$ indicates the weight of C_j, with a rule $\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}} = 1$, with parameter $\varphi(1), \varphi(2), \ldots, \varphi(n)$ based on $\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j)}}}} \leq \overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j-1)}}}}$. Additionally, $\overline{\overline{\dot{\mathfrak{C}}_{C_{\varphi(j)}}}} = n\overline{\overline{\mathfrak{W}''_j}}\,\overline{\overline{\mathfrak{C}_{C_{\varphi(j)}}}}$ with $\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}''_j}} = 1$.

Theorem 6: For CIVIFNs $\overline{\overline{\mathfrak{C}_{C_j}}} = \begin{pmatrix} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{pmatrix}, j = 1, \ldots, n,$

then by using Eq. (42)), we elaborate

$$CIVIFAAHG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \begin{pmatrix} \left[\left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\overline{\dot{\mathcal{M}}}}^{L}_{\overline{\overline{R_{\varphi(j)}}}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}_j}}}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\overline{\dot{\mathcal{M}}}}^{U}_{\overline{\overline{R_{\varphi(j)}}}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}_j}}}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\overline{\dot{\mathcal{M}}}}^{L}_{\overline{\overline{I_{\varphi(j)}}}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}_j}}}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\sum_{j=1}^{n}\left(-\log\left(\overline{\overline{\dot{\mathcal{M}}}}^{U}_{\overline{\overline{I_{\varphi(j)}}}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}_j}}}\right)^{\frac{1}{\psi}}}\right)\right]}, \\ \left[\left(1 - e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\overline{\dot{\mathcal{N}}}}^{L}_{\overline{\overline{R_{\varphi(j)}}}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}_j}}}\right)^{\frac{1}{\psi}}}\right), \left(1 - e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\overline{\dot{\mathcal{N}}}}^{U}_{\overline{\overline{R_{\varphi(j)}}}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}_j}}}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(1-e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\overline{\dot{\mathcal{N}}}}^{L}_{\overline{\overline{I_{\varphi(j)}}}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}_j}}}\right)^{\frac{1}{\psi}}}\right), \left(1-e^{-\left(\sum_{j=1}^{n}\left(-\log\left(1-\overline{\overline{\dot{\mathcal{N}}}}^{U}_{\overline{\overline{I_{\varphi(j)}}}}\right)\right)^{\psi\overline{\overline{\mathfrak{W}_j}}}\right)^{\frac{1}{\psi}}}\right)\right]} \end{pmatrix} \tag{43}$$

Idempotency-Property 13: By taking CIVIFNs

$$\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{array} \right), j = 1, 2, \ldots, n, \text{ if } \overline{\overline{\mathfrak{C}_{C_j}}} = \overline{\overline{\mathfrak{C}}}, \text{ then}$$

$$CIVIFAAHG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) = \overline{\overline{\mathfrak{C}}} \tag{44}$$

Monotonicity-Property 14: By taking CIVIFNs

$$\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{array} \right), j = 1, 2, \ldots, n,$$

$$\text{if } \overline{\overline{\mathfrak{C}}}^{-} = \left(\left[\min_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}, \min_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}\right] e^{i2\pi\left[\min_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}, \min_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}\right]}, \left[\max_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}, \max_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}\right] e^{i2\pi\left[\max_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}, \max_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}\right]}\right) \text{ and}$$

$$\overline{\overline{\mathfrak{C}}}^{+} = \left(\left[\max_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}, \max_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}\right] e^{i2\pi\left[\max_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}, \max_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}\right]}, \left[\min_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}, \min_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}\right] e^{i2\pi\left[\min_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}, \min_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}\right]}\right), \text{ then}$$

$$\overline{\overline{\mathfrak{C}}}^{-} \leq CIVIFAAHG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq \overline{\overline{\mathfrak{C}}}^{+} \tag{45}$$

Boundedness-Property 15: By taking CIVIFNs

$$\overline{\overline{\mathfrak{C}_{C_j}}} = \left(\begin{array}{c} \left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)}, \\ \left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}}(\widetilde{x_E}), \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}}(\widetilde{x_E})\right]\right)} \end{array} \right), j = 1, 2, \ldots, n, \text{ if } \overline{\overline{\mathfrak{C}_{C_j}}} \leq \overline{\overline{\mathfrak{C}_{C_j}}}', \text{ then}$$

$$CIVIFAAHG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right) \leq CIVIFAAHG\left(\overline{\overline{\mathfrak{C}_{C_1}}}', \overline{\overline{\mathfrak{C}_{C_2}}}', \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}'\right) \tag{46}$$

WASPAS METHOD FOR CIVIFSS

The main theme of this section is to illustrate the WASPAS method for CIVIFSs and verify the validity of the proposed method with the help of some numerical examples.

Some valuable and effective steps of the WASPAS method are listed below:

Step 1: The input data of the technique is represented in the form of a matrix of alternatives and attributes, which is based on the data received from the expert.

Step 2: Further, we normalize the information in decision matrix by using the below theory:

$$\overline{\overline{\mathfrak{C}_{C_{oj}}}} = \left(\begin{array}{c} \left[\max_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_{oj}}}}(\widetilde{x_E}), \max_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_{oj}}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\max_j \overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_{oj}}}}(\widetilde{x_E}), \max_j \overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_{oj}}}}(\widetilde{x_E})\right]\right)}, \\ \left[\min_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_{0j}}}}(\widetilde{x_E}), \min_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_{0j}}}}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\min_j \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_{0j}}}}(\widetilde{x_E}), \min_j \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_{0j}}}}(\widetilde{x_E})\right]\right)} \end{array} \right) \tag{47}$$

where the data in Eq. (48) is used for benefit types of data, such as

$$\overline{\overline{\mathfrak{C}_{C_{ij}}}}^{/} = \begin{cases} 0 & otherwise \\ \dfrac{\overline{\overline{\mathcal{M}_{\overline{\overline{R_{ij}}}}}}^{L}}{1+\overline{\overline{\mathcal{M}_{\overline{\overline{R_{oj}}}}}}^{L}} & \left(\begin{matrix}\overline{\overline{\mathcal{M}_{\overline{\overline{R_{ij}}}}}}^{L} \leq \overline{\overline{\mathcal{M}_{\overline{\overline{R_{oj}}}}}}^{L}, \overline{\overline{\mathcal{M}_{\overline{\overline{R_{ij}}}}}}^{U} \leq \overline{\overline{\mathcal{M}_{\overline{\overline{R_{oj}}}}}}^{U}, \\ \overline{\overline{\mathcal{M}_{\overline{\overline{I_{ij}}}}}}^{L} \leq \overline{\overline{\mathcal{M}_{\overline{\overline{I_{oj}}}}}}^{L}, \overline{\overline{\mathcal{M}_{\overline{\overline{I_{ij}}}}}}^{U} \leq \overline{\overline{\mathcal{M}_{\overline{\overline{I_{oj}}}}}}^{U}\end{matrix}\right) \\ \dfrac{\overline{\overline{\mathcal{N}_{\overline{\overline{R_{ij}}}}}}^{L}}{1+\overline{\overline{\mathcal{N}_{\overline{\overline{R_{oj}}}}}}^{L}} & for\ non-membership\ grade \end{cases} \tag{48}$$

Where the data in Eq. (49) is used for cost types of data, such as

$$\overline{\overline{\mathfrak{C}_{C_{ij}}}}^{/} = \begin{cases} 0 & otherwise \\ \dfrac{\overline{\overline{\mathcal{M}_{\overline{\overline{R_{ij}}}}}}^{L}}{1+\overline{\overline{\mathcal{M}\mathfrak{C}_{C_{oj}}}}^{L}} & \left(\begin{matrix}\overline{\overline{\mathcal{M}_{\overline{\overline{R_{ij}}}}}}^{L} \leq \overline{\overline{\mathcal{M}_{\overline{\overline{R_{oj}}}}}}^{L}, \overline{\overline{\mathcal{M}_{\overline{\overline{R_{ij}}}}}}^{U} \leq \overline{\overline{\mathcal{M}_{\overline{\overline{R_{oj}}}}}}^{U}, \\ \overline{\overline{\mathcal{M}_{\overline{\overline{I_{ij}}}}}}^{L} \leq \overline{\overline{\mathcal{M}_{\overline{\overline{I_{oj}}}}}}^{L}, \overline{\overline{\mathcal{M}_{\overline{\overline{I_{ij}}}}}}^{U} \leq \overline{\overline{\mathcal{M}_{\overline{\overline{I_{oj}}}}}}^{U}\end{matrix}\right) \\ \dfrac{\overline{\overline{\mathcal{N}_{\overline{\overline{R_{ij}}}}}}^{L}}{1+\overline{\overline{\mathcal{N}_{\overline{\overline{\mathfrak{C}_{C_{oj}}}}}}}^{L}} & for\ non-membership\ grade \end{cases} \tag{49}$$

where $\mathfrak{C}_{C_{oj}} = \left(\begin{matrix}\left[\min_j \overline{\overline{\mathcal{M}_{\overline{\overline{R_{oj}}}}}}^{L}(\widetilde{x_E}), \min_j \overline{\overline{\mathcal{M}_{\overline{\overline{R_{oj}}}}}}^{U}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\min_j \overline{\overline{\mathcal{M}_{\overline{\overline{I_{oj}}}}}}^{L}(\widetilde{x_E}), \min_j \overline{\overline{\mathcal{M}_{\overline{\overline{I_{oj}}}}}}^{U}(\widetilde{x_E})\right]\right)}, \\ \left[\max_j \overline{\overline{\mathcal{N}_{\overline{\overline{R_{0j}}}}}}^{L}(\widetilde{x_E}), \max_j \overline{\overline{\mathcal{N}_{\overline{\overline{R_{0j}}}}}}^{U}(\widetilde{x_E})\right] e^{i2\pi\left(\left[\max_j \overline{\overline{\mathcal{N}_{\overline{\overline{I_{0j}}}}}}^{L}(\widetilde{x_E}), \max_j \overline{\overline{\mathcal{N}_{\overline{\overline{I_{0j}}}}}}^{U}(\widetilde{x_E})\right]\right)}\end{matrix}\right).$

Step 3: Utilizing *CIVIFAAWA* and *CIVIFAAWG* operators to obtain the WSM and WPM of each alternative:

$$WSM_i = CIVIFAAWA\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right)$$

$$= \left(\begin{matrix} \left[\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{R_j}}}}}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(1-e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(1-\overline{\overline{\mathcal{M}_{\overline{\overline{I_j}}}}}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]}, \\ \left[\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{R_j}}}}}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right] \\ e^{i2\pi\left[\left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}}^{L}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right), \left(e^{-\left(\sum_{j=1}^{n}\overline{\overline{\mathfrak{W}_j}}\left(-\log\left(\overline{\overline{\mathcal{N}_{\overline{\overline{I_j}}}}}^{U}\right)\right)^{\psi}\right)^{\frac{1}{\psi}}}\right)\right]} \end{matrix}\right) \tag{50}$$

$$WPM_i = CIVIFAAWG\left(\overline{\overline{\mathfrak{C}_{C_1}}}, \overline{\overline{\mathfrak{C}_{C_2}}}, \ldots, \overline{\overline{\mathfrak{C}_{C_n}}}\right)$$

$$= \left(\begin{array}{c} \left[\left(e^{-\left(\sum_{j=1}^{n} \left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{R_j}}} \right) \right)^{\psi \overline{\overline{\mathfrak{W}}}_j} \right)^{\frac{1}{\psi}}} \right), \left(e^{-\left(\sum_{j=1}^{n} \left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{R_j}}} \right) \right)^{\psi \overline{\overline{\mathfrak{W}}}_j} \right)^{\frac{1}{\psi}}} \right) \right] \\ e^{i2\pi \left[\left(e^{-\left(\sum_{j=1}^{n} \left(-\log\left(\overline{\overline{\mathcal{M}}}^{L}_{\overline{\overline{I_j}}} \right) \right)^{\psi \overline{\overline{\mathfrak{W}}}_j} \right)^{\frac{1}{\psi}}} \right), \left(e^{-\left(\sum_{j=1}^{n} \left(-\log\left(\overline{\overline{\mathcal{M}}}^{U}_{\overline{\overline{I_j}}} \right) \right)^{\psi \overline{\overline{\mathfrak{W}}}_j} \right)^{\frac{1}{\psi}}} \right) \right]}, \\ \left[\left(1 - e^{-\left(\sum_{j=1}^{n} \left(-\log\left(1 - \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{R_j}}} \right) \right)^{\psi \overline{\overline{\mathfrak{W}}}_j} \right)^{\frac{1}{\psi}}} \right), \left(1 - e^{-\left(\sum_{j=1}^{n} \left(-\log\left(1 - \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{R_j}}} \right) \right)^{\psi \overline{\overline{\mathfrak{W}}}_j} \right)^{\frac{1}{\psi}}} \right) \right] \\ e^{i2\pi \left[\left(1 - e^{-\left(\sum_{j=1}^{n} \left(-\log\left(1 - \overline{\overline{\mathcal{N}}}^{L}_{\overline{\overline{I_j}}} \right) \right)^{\psi \overline{\overline{\mathfrak{W}}}_j} \right)^{\frac{1}{\psi}}} \right), \left(1 - e^{-\left(\sum_{j=1}^{n} \left(-\log\left(1 - \overline{\overline{\mathcal{N}}}^{U}_{\overline{\overline{I_j}}} \right) \right)^{\psi \overline{\overline{\mathfrak{W}}}_j} \right)^{\frac{1}{\psi}}} \right) \right]} \end{array} \right) \tag{51}$$

Step 4: Compute the score value according to WSM and WPM, the detailed formula is listed as follows.

$$S_i = {}^{\circ}F * \overline{\overline{\mathcal{S}_{SV}}}\left(WSM_i\right) + \left(1 - {}^{\circ}F\right) * \overline{\overline{\mathcal{S}_{SV}}}\left(WPM_i\right) \tag{52}$$

Step 5: Rank the alternatives and derive the best one referring to the score value S_i in Step 4.

Further, we justify the above-mentioned method by some practical examples.

Example 1: To verify the WASPAS technique under the consideration of some CIVIF information, we applied it for practical CIVIF decision matrix to obtain the best alternative. Four alternatives: S_1, S_2, S_3, S_4; and four criteria C_1, C_2, C_3, C_4, and C_{IJ} indicates the assessment information of $S_I\,(I = 1, 2, 3, 4)$ under the criterion $C_J\,(J = 1, 2, 3, 4)$. Some valuable and effective steps of the WASPAS method are listed below:

Step 1: The input data of the technique is represented in the form of a matrix of alternatives and attributes, which is based on the data received from the expert.

$$\overline{\overline{\mathfrak{C}}}_{C_{ij}} = \begin{bmatrix} \begin{pmatrix} [0.3,0.6]e^{i2\pi([0.1,0.3])}, \\ [0.3,0.4]e^{i2\pi([0.3,0.3])} \end{pmatrix} & \begin{pmatrix} [0.2,0.3]e^{i2\pi([0.2,0.4])}, \\ [0.3,0.5]e^{i2\pi([0.3,0.4])} \end{pmatrix} & \begin{pmatrix} [0.3,0.4]e^{i2\pi([0.1,0.3])}, \\ [0.2,0.2]e^{i2\pi([0.2,0.3])} \end{pmatrix} & \begin{pmatrix} [0.1,0.2]e^{i2\pi([0.2,0.2])}, \\ [0.1,0.1]e^{i2\pi([0.1,0.3])} \end{pmatrix} \\ \begin{pmatrix} [0.2,0.3]e^{i2\pi([0.6,0.7])}, \\ [0.3,0.4]e^{i2\pi([0.2,0.3])} \end{pmatrix} & \begin{pmatrix} [0.3,0.4]e^{i2\pi([0.5,0.6])}, \\ [0.2,0.3]e^{i2\pi([0.1,0.3])} \end{pmatrix} & \begin{pmatrix} [0.4,0.5]e^{i2\pi([0.4,0.5])}, \\ [0.3,0.4]e^{i2\pi([0.2,0.2])} \end{pmatrix} & \begin{pmatrix} [0.5,0.6]e^{i2\pi([0.3,0.4])}, \\ [0.2,0.4]e^{i2\pi([0.2,0.2])} \end{pmatrix} \\ \begin{pmatrix} [0.1,0.3]e^{i2\pi([0.3,0.4])}, \\ [0.1,0.2]e^{i2\pi([0.1,0.2])} \end{pmatrix} & \begin{pmatrix} [0.2,0.4]e^{i2\pi([0.3,0.5])}, \\ [0.2,0.3]e^{i2\pi([0.2,0.2])} \end{pmatrix} & \begin{pmatrix} [0.1,0.3]e^{i2\pi([0.2,0.2])}, \\ [0.3,0.3]e^{i2\pi([0.2,0.3])} \end{pmatrix} & \begin{pmatrix} [0.2,0.2]e^{i2\pi([0.1,0.1])}, \\ [0.3,0.4]e^{i2\pi([0.2,0.3])} \end{pmatrix} \\ \begin{pmatrix} [0.3,0.4]e^{i2\pi([0.3,0.3])}, \\ [0.2,0.2]e^{i2\pi([0.1,0.2])} \end{pmatrix} & \begin{pmatrix} [0.3,0.5]e^{i2\pi([0.3,0.4])}, \\ [0.2,0.2]e^{i2\pi([0.1,0.2])} \end{pmatrix} & \begin{pmatrix} [0.2,0.2]e^{i2\pi([0.2,0.3])}, \\ [0.2,0.2]e^{i2\pi([0.1,0.2])} \end{pmatrix} & \begin{pmatrix} [0.1,0.1]e^{i2\pi([0.1,0.3])}, \\ [0.1,0.2]e^{i2\pi([0.1,0.2])} \end{pmatrix} \end{bmatrix}$$

Step 2: Further, we normalize the information in decision matrix by using Eqs. (47)–49, and obtain following resutls:

$$\overline{\overline{\mathfrak{C}}}_{C_{0,j}} = \left\{ \begin{pmatrix} [0.3,0.6]e^{i2\pi([0.2,0.4])}, \\ [0.1,0.1]e^{i2\pi([0.1,0.3])} \end{pmatrix}, \begin{pmatrix} [0.5,0.6]e^{i2\pi([0.6,0.7])}, \\ [0.2,0.3]e^{i2\pi([0.1,0.2])} \end{pmatrix}, \begin{pmatrix} [0.2,0.4]e^{i2\pi([0.3,0.5])}, \\ [0.1,0.2]e^{i2\pi([0.1,0.2])} \end{pmatrix}, \begin{pmatrix} [0.3,0.5]e^{i2\pi([0.3,0.4])}, \\ [0.1,0.2]e^{i2\pi([0.1,0.2])} \end{pmatrix} \right\}$$

$$\overline{\overline{\mathfrak{C}}}_{C_{ij}}^{/} = \begin{bmatrix} \begin{pmatrix} [0.231, 0.375] \\ e^{i2\pi([0.083, 0.214])}, \\ [0.273, 0.364] \\ e^{i2\pi([0.273, 0.231])} \end{pmatrix} & \begin{pmatrix} [0.154, 0.187]\, e^{i2\pi([0.167, 0.285])}, \\ [0.273, 0.455]\, e^{i2\pi([0.273, 0.307])} \end{pmatrix} & \begin{pmatrix} [0.231, 0.25]\, e^{i2\pi([0.083, 0.214])}, \\ [0.182, 0.182]\, e^{i2\pi([0.182, 0.231])} \end{pmatrix} & \begin{pmatrix} [0.077, 0.125] \\ e^{i2\pi([0.167, 0.143])}, \\ [0., 0.]\, e^{i2\pi([0., 0.])} \end{pmatrix} \\ \begin{pmatrix} [0.133, 0.187] \\ e^{i2\pi([0.375, 0.412])}, \\ [0.25, 0.308] \\ e^{i2\pi([0.182, 0.25])} \end{pmatrix} & \begin{pmatrix} [0.2, 0.25]\, e^{i2\pi([0.312, 0.353])}, \\ [0.167, 0.231]\, e^{i2\pi([0.091, 0.25])} \end{pmatrix} & \begin{pmatrix} [0.267, 0.312]\, e^{i2\pi([0.25, 0.294])}, \\ [0.25, 0.308]\, e^{i2\pi([0.182, 0.167])} \end{pmatrix} & \begin{pmatrix} [0.333, 0.375] \\ e^{i2\pi([0.187, 0.2353])}, \\ [0.167, 0.308] \\ e^{i2\pi([0.182, 0.167])} \end{pmatrix} \\ \begin{pmatrix} [0.084, 0.214] \\ e^{i2\pi([0.230, 0.267])}, \\ [0.091, 0.167] \\ e^{i2\pi([0.091, 0.167])} \end{pmatrix} & \begin{pmatrix} [0.167, 0.285]\, e^{i2\pi([0.230, 0.333])}, \\ [0.182, 0.25]\, e^{i2\pi([0.182, 0.167])} \end{pmatrix} & \begin{pmatrix} [0.083, 0.214]\, e^{i2\pi([0.153, 0.133])}, \\ [0.273, 0.25]\, e^{i2\pi([0.182, 0.25])} \end{pmatrix} & \begin{pmatrix} [0.167, 0.142] \\ e^{i2\pi([0.076, 0.067])}, \\ [0.273, 0.333] \\ e^{i2\pi([0.182, 0.25])} \end{pmatrix} \\ \begin{pmatrix} [0.230, 0.267] \\ e^{i2\pi([0.230, 0.214])}, \\ [0.182, 0.167] \\ e^{i2\pi([0.091, 0.167])} \end{pmatrix} & \begin{pmatrix} [0.230, 0.334]\, e^{i2\pi([0.230, 0.285])}, \\ [0.182, 0.167]\, e^{i2\pi([0.091, 0.167])} \end{pmatrix} & \begin{pmatrix} [0.153, 0.133]\, e^{i2\pi([0.153, 0.214])}, \\ [0.182, 0.167]\, e^{i2\pi([0.091, 0.167])} \end{pmatrix} & \begin{pmatrix} [0.076, 0.067] \\ e^{i2\pi([0.076, 0.214])}, \\ [0.091, 0.167] \\ e^{i2\pi([0.091, 0.167])} \end{pmatrix} \end{bmatrix}$$

Step 3: Compute the WSM and WPM of each alternative according to the *CIVIFAAWA* and *CIVIFAAWG* operators ($\psi = 1$), the specific results are illustrated as follows.

$$WSM_i = \left\{ \begin{matrix} \begin{pmatrix} [0.000028, 0.0011]\, e^{i2\pi([0.000007, 0.00009])}, \\ [0.8096, 0.9050]\, e^{i2\pi([0.8096, 0.8925])} \end{pmatrix}, \begin{pmatrix} [0.0001, 0.0003]\, e^{i2\pi([0.0003, 0.0007])}, \\ [0.828, 0.9292]\, e^{i2\pi([0.6953, 0.828])} \end{pmatrix}, \\ \begin{pmatrix} [0.000007, 0.00009]\, e^{i2\pi([0.00002, 0.00003])}, \\ [0.8096, 0.8907]\, e^{i2\pi([0.6952, 0.8279])} \end{pmatrix}, \begin{pmatrix} [0.00003, 0.00004]\, e^{i2\pi([0.00003, 0.0001])}, \\ [0.6953, 0.7296]\, e^{i2\pi([0.3637, 0.7296])} \end{pmatrix} \end{matrix} \right\}$$

$$WPM_i = \left\{ \begin{matrix} \begin{pmatrix} [0.7291, 0.8651]\, e^{i2\pi([0.5452, 0.8377])}, \\ [0.00005, 0.000160]\, e^{i2\pi([0.000059, 0.00020])} \end{pmatrix}, \begin{pmatrix} [0.8658, 0.9222]\, e^{i2\pi([0.9222, 0.9512])}, \\ [0.00008, 0.0004]\, e^{i2\pi([0.00002, 0.00008])} \end{pmatrix}, \\ \begin{pmatrix} [0.5452, 0.8377]\, e^{i2\pi([0.7292, 0.7847])}, \\ [0.00005, 0.000187]\, e^{i2\pi([0.00002, 0.00008])} \end{pmatrix}, \begin{pmatrix} [0.7292, 0.7847]\, e^{i2\pi([0.7292, 0.8693])}, \\ [0.00002, 0.00003]\, e^{i2\pi([0.000003, 0.00003])} \end{pmatrix} \end{matrix} \right\}$$

Further, we examine the values of the score function, such as:

$$WSM_i = \{-0.8542, -0.82, -0.8058, -0.629\}$$

$$WPM_i = \{0.7442, 0.9152, 0.7241, 0.7781\}$$

Step 4: Acquire the score value of each alternative by using the theory of WSM and WPM information:

$$S_1 = 0.4245, S_2 = 0.5682, S_3 = 0.4181, S_4 = 0.4966$$

For convenience, we assume $^\circ F = 0.2$.

Step 5: Identify the ranking information for evaluating or deriving the best preference.

$$S_2 \geq S_4 \geq S_1 \geq S_3$$

From the above analysis, we obtain the best preferences as S_2.

Application in MADM

The significant commitment of this examination is to apply MADM method under CIVIFS for deciding the optimal scheme from the group of complex interval-valued intuitionistic fuzzy data. To determine the best one, we expounded a dynamic interaction. There are m alternative $\overline{\overline{\mathfrak{C}}}_C = \left\{ \overline{\overline{\mathfrak{C}}}_{C_1}, \overline{\overline{\mathfrak{C}}}_{C_2}, \ldots, \overline{\overline{\mathfrak{C}}}_{C_m} \right\}$ and n criteria looking like

Procedure of decision-making

To achieve the acquirement of the best one, we built the dynamic calculation looking like the accompanying stages:

Stage 1: Construct the CIVIF decision matrix utilizing the CIVIF evaluation information.

Stage 2: Normalize the CIVIF decision matrix. The specific conversion process is shown below when dealing with beneficial data and cost data:

$$D=\begin{cases}\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{R_{jk}},\overline{\overline{\mathcal{M}}}^{U}_{R_{jk}}\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{I_{jk}},\overline{\overline{\mathcal{M}}}^{U}_{I_{jk}}\right]\right)},\left[\overline{\overline{\mathcal{N}}}^{L}_{R_{jk}},\overline{\overline{\mathcal{N}}}^{U}_{R_{jk}}\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{I_{jk}},\overline{\overline{\mathcal{N}}}^{U}_{I_{jk}}\right]\right)}\right) & \textit{for benefit sort of data}\\ \left(\left[\overline{\overline{\mathcal{N}}}^{L}_{R_{jk}},\overline{\overline{\mathcal{N}}}^{U}_{R_{jk}}\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{I_{jk}},\overline{\overline{\mathcal{N}}}^{U}_{I_{jk}}\right]\right)},\left[\overline{\overline{\mathcal{M}}}^{L}_{R_{jk}},\overline{\overline{\mathcal{M}}}^{U}_{R_{jk}}\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{I_{jk}},\overline{\overline{\mathcal{M}}}^{U}_{I_{jk}}\right]\right)}\right) & \textit{for cost sort of data}\end{cases}$$

Stage 3: Utilizing the Eq. (17) (CIVIFAAWA) and Eq. (32) (CIVIFAAWG) to aggregate the information in the decision matrix.

Stage 4: Using Eq. (6) to derive the score information.

Stage 5: Evaluate the ranking information in the availability of score information.

Represented example

The significant finding of this investigation is to break down the explained administrators in the conditions of the MADM methodology. For this, we examined some pragmatic information to decide the practicality and probability of the introduced works.

Clarification of the problem

Permit us to ponder a creation association that expects to enroll a publicizing director for an unfilled post. Here, we consider five competitors $\overline{\overline{\mathfrak{C}}}_{C_j}, j = 1,2,3,4,5$, allocated for extra appraisals, such as: $\overline{\overline{\mathfrak{C}'}}_{C_1}$: Oral presentation capacity; $\overline{\overline{\mathfrak{C}'}}_{C_2}$: History; $\overline{\overline{\mathfrak{C}'}}_{C_3}$: Overall tendency; and $\overline{\overline{\mathfrak{C}'}}_{C_4}$: confidence. For this, we consider weight vectors such as 0.4,0.3,0.2,0.1. The five specialists $\overline{\overline{\mathfrak{C}}}_{C_j}, j = 1,2,3,4,5$ are to oversee vagueness under CIVIF information by utilizing dynamic strategies.

Method under CIVIFAAWA and CIVIFAAWG operators

Determine the useful individual from the gathering of people (Five up-and-comers) by utilizing the MADM procedure under CIVIFAAWA and CIVIFAAWG operators. For obtaining the ideal one, we developed the dynamic calculation looking like the accompanying stages:

Stage 1: Construct the CIVIF decision matrix. The specific data covering the cost and beneficial sorts are listed as Table 2.

Stage 2: Normalize the decision matrix referring to the subsequent conversion process.

$$D=\begin{cases}\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{R_{jk}},\overline{\overline{\mathcal{M}}}^{U}_{R_{jk}}\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{I_{jk}},\overline{\overline{\mathcal{M}}}^{U}_{I_{jk}}\right]\right)},\left[\overline{\overline{\mathcal{N}}}^{L}_{R_{jk}},\overline{\overline{\mathcal{N}}}^{U}_{R_{jk}}\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{I_{jk}},\overline{\overline{\mathcal{N}}}^{U}_{I_{jk}}\right]\right)}\right) & \textit{for benefit sort of data}\\ \left(\left[\overline{\overline{\mathcal{N}}}^{L}_{R_{jk}},\overline{\overline{\mathcal{N}}}^{U}_{R_{jk}}\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{N}}}^{L}_{I_{jk}},\overline{\overline{\mathcal{N}}}^{U}_{I_{jk}}\right]\right)},\left[\overline{\overline{\mathcal{M}}}^{L}_{R_{jk}},\overline{\overline{\mathcal{M}}}^{U}_{R_{jk}}\right]e^{i2\pi\left(\left[\overline{\overline{\mathcal{M}}}^{L}_{I_{jk}},\overline{\overline{\mathcal{M}}}^{U}_{I_{jk}}\right]\right)}\right) & \textit{for cost sort of data}\end{cases}$$

Stage 3: Under the CIVIFAAWA and CIVIFAAWG operators, we obtain the aggregation consequence shown in Table 3 ($\psi = 1$).

Table 2 **CIVIF data.**

	$\overline{\overline{\mathfrak{C}}}_{C_1}'$	$\overline{\overline{\mathfrak{C}}}_{C_2}'$
$\overline{\overline{\mathfrak{C}}}_{C_1}$	$\left([0.4,0.5]e^{i2\pi([0.3,0.4])},[0.2,0.3]e^{i2\pi([0.2,0.4])}\right)$	$\left([0.41,0.51]e^{i2\pi([0.31,0.41])},[0.21,0.31]e^{i2\pi([0.21,0.41])}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_2}$	$\left([0.3,0.4]e^{i2\pi([0.0,0.1])},[0.0,0.1]e^{i2\pi([0.0,0.1])}\right)$	$\left([0.31,0.41]e^{i2\pi([0.01,0.11])},[0.01,0.11]e^{i2\pi([0.01,0.11])}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_3}$	$\left([0.4,0.5]e^{i2\pi([0.1,0.2])},[0.3,0.4]e^{i2\pi([0.2,0.3])}\right)$	$\left([0.41,0.51]e^{i2\pi([0.11,0.21])},[0.31,0.41]e^{i2\pi([0.21,0.31])}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_4}$	$\left([0.2,0.3]e^{i2\pi([0.4,0.5])},[0.0,0.1]e^{i2\pi([0.1,0.2])}\right)$	$\left([0.21,0.31]e^{i2\pi([0.41,0.51])},[0.01,0.11]e^{i2\pi([0.11,0.21])}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_5}$	$\left([0.4,0.5]e^{i2\pi([0.4,0.5])},[0.0,0.1]e^{i2\pi([0.2,0.3])}\right)$	$\left([0.41,0.51]e^{i2\pi([0.41,0.51])},[0.01,0.11]e^{i2\pi([0.21,0.31])}\right)$
	$\overline{\overline{\mathfrak{C}}}_{C_3}'$	$\overline{\overline{\mathfrak{C}}}_{C_4}'$
$\overline{\overline{\mathfrak{C}}}_{C_1}$	$\left([0.42,0.52]e^{i2\pi([0.32,0.42])},[0.22,0.32]e^{i2\pi([0.22,0.42])}\right)$	$\left([0.43,0.53]e^{i2\pi([0.33,0.43])},[0.23,0.33]e^{i2\pi([0.23,0.43])}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_2}$	$\left([0.32,0.42]e^{i2\pi([0.02,0.12])},[0.02,0.12]e^{i2\pi([0.02,0.12])}\right)$	$\left([0.33,0.43]e^{i2\pi([0.03,0.13])},[0.03,0.13]e^{i2\pi([0.03,0.13])}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_3}$	$\left([0.42,0.52]e^{i2\pi([0.12,0.22])},[0.32,0.42]e^{i2\pi([0.22,0.32])}\right)$	$\left([0.43,0.53]e^{i2\pi([0.13,0.23])},[0.33,0.43]e^{i2\pi([0.23,0.33])}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_4}$	$\left([0.22,0.32]e^{i2\pi([0.42,0.52])},[0.02,0.12]e^{i2\pi([0.12,0.22])}\right)$	$\left([0.23,0.33]e^{i2\pi([0.43,0.53])},[0.03,0.13]e^{i2\pi([0.13,0.23])}\right)$
$\overline{\overline{\mathfrak{C}}}_{C_5}$	$\left([0.42,0.52]e^{i2\pi([0.42,0.52])},[0.02,0.12]e^{i2\pi([0.22,0.32])}\right)$	$\left([0.43,0.53]e^{i2\pi([0.43,0.53])},[0.03,0.13]e^{i2\pi([0.23,0.33])}\right)$

Table 3 **Aggregation information matric.**

	CIVIFAAWA operator	*CIVIFAAWG operator*
$\overline{\overline{\mathfrak{C}}}_{C_1}$	$\begin{pmatrix}[0.2048,0.2665]e^{i2\pi([0.1489,0.2048])},\\ [0.5075,0.6012]e^{i2\pi([0.6012,0.6789])}\end{pmatrix}$	$\begin{pmatrix}[0.6789,0.7464]e^{i2\pi([0.6012,0.6789])},\\ [0.0973,0.1489]e^{i2\pi([0.1489,0.2048])}\end{pmatrix}$
$\overline{\overline{\mathfrak{C}}}_{C_2}$	$\begin{pmatrix}[0.1489,0.2048]e^{i2\pi([0.0048,0.0494])},\\ [0.1029,0.3828]e^{i2\pi([0.1029,0.3828])}\end{pmatrix}$	$\begin{pmatrix}[0.6012,0.6789]e^{i2\pi([0.1029,0.3828])},\\ [0.0048,0.0494]e^{i2\pi([0.0048,0.0494])}\end{pmatrix}$
$\overline{\overline{\mathfrak{C}}}_{C_3}$	$\begin{pmatrix}[0.2048,0.2665]e^{i2\pi([0.0494,0.0973])},\\ [0.6012,0.6789]e^{i2\pi([0.5075,0.6012])}\end{pmatrix}$	$\begin{pmatrix}[0.6789,0.7464]e^{i2\pi([0.3828,0.5075])},\\ [0.1489,0.2048]e^{i2\pi([0.0973,0.1489])}\end{pmatrix}$
$\overline{\overline{\mathfrak{C}}}_{C_4}$	$\begin{pmatrix}[0.0973,0.1489]e^{i2\pi([0.2048,0.2665])},\\ [0.1029,0.3828]e^{i2\pi([0.3828,0.5075])}\end{pmatrix}$	$\begin{pmatrix}[0.5075,0.6012]e^{i2\pi([0.6789,0.7464])},\\ [0.0048,0.0494]e^{i2\pi([0.0494,0.0973])}\end{pmatrix}$
$\overline{\overline{\mathfrak{C}}}_{C_5}$	$\begin{pmatrix}[0.2048,0.2665]e^{i2\pi([0.2048,0.2665])},\\ [0.1029,0.3828]e^{i2\pi([0.1029,0.3828])}\end{pmatrix}$	$\begin{pmatrix}[0.6789,0.7464]e^{i2\pi([0.6789,0.7464])},\\ [0.0048,0.0494]e^{i2\pi([0.0048,0.0494])}\end{pmatrix}$

Stage 4: Here, we compute the score values of the aggregated information in Stage 3, see Table 4.

Stage 5: Obtain the ranking information based on the score values, the detailed result is stated in Table 5.

According to the theory of CIFAAWA operator and CIFAAWG operator, the best optimal is $\overline{\overline{\mathfrak{C}}}_{C_5}$.

Table 4 Score values.

	CIVIFAAWA operator	*CIVIFAAWG operator*
$\overline{\overline{\mathfrak{C}_{C_1}}}$	−0.3909	0.5263
$\overline{\overline{\mathfrak{C}_{C_2}}}$	−0.1409	0.4143
$\overline{\overline{\mathfrak{C}_{C_3}}}$	−0.4427	0.4289
$\overline{\overline{\mathfrak{C}_{C_4}}}$	−0.1646	0.5833
$\overline{\overline{\mathfrak{C}_{C_5}}}$	−0.0072	0.6855

Table 5 Ranking lists.

CIFAAWA operator	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
CIFAAWG operator	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$

Table 6 Represented the stability of the proposed work.

Parameter	*Operator*	*Score values*	*Ranking values*
$\psi = 1$	CIVIFAAWA operator	$-0.3909, -0.1409, -0.4427, -0.1646, -0.0072$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
	CIVIFAAWG operator	$0.5263, 0.4143, 0.4289, 0.5833, 0.6855$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
$\psi = 5$	CIVIFAAWA operator	$-0.3901, -0.1261, -0.4416, -0.1571, 0.0065$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
	CIVIFAAWG operator	$0.5255, 0.4049, 0.4278, 0.5811, 0.6826$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
$\psi = 11$	CIVIFAAWA operator	$-0.3888, -0.1169, -0.4401, -0.1518, 0.0151$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
	CIVIFAAWG operator	$0.5242, 0.3989, 0.4262, 0.5791, 0.6803$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
$\psi = 51$	CIVIFAAWA operator	$-0.3818, -0.1049, -0.4329, -0.141, 0.0274$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
	CIVIFAAWG operator	$0.5176, 0.3901, 0.4185, 0.5724, 0.6748$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
$\psi = 101$	CIVIFAAWA operator	$-0.3773, -0.1013, -0.4289, -0.1363, 0.0323$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$
	CIVIFAAWG operator	$0.5138, 0.3876, 0.4142, 0.5696, 0.6726$	$\overline{\overline{\mathfrak{C}_{C_5}}} > \overline{\overline{\mathfrak{C}_{C_2}}} > \overline{\overline{\mathfrak{C}_{C_4}}} > \overline{\overline{\mathfrak{C}_{C_1}}} > \overline{\overline{\mathfrak{C}_{C_3}}}$

Influence of parameter

Here, we discuss the stability and influence of the derived operators based on the different values of parameters ψ. Therefore, by using the information in Table 2 and various parameter values, we obtain the subsequent consequence listed in Table 6.

We have gotten the consistent advantageous ideal $\overline{\overline{\mathfrak{C}_{C_5}}}$ based on diverse operators by utilizing the particular upsides of the boundary. This result shows that our calculation model has a good stability.

Table 7 Comparison information for Table 1.

Methods	Score values	Ranking values
Xu (2007)	× × × × × × × × × ×	× × × × × × × × × ×
Xu & Yager (2006)	× × × × × × × × × ×	× × × × × × × × × ×
Wang & Liu (2012)	× × × × × × × × × ×	× × × × × × × × × ×
Wang & Liu (2011)	× × × × × × × × × ×	× × × × × × × × × ×
Huang (2014)	× × × × × × × × × ×	× × × × × × × × × ×
Seikh and Mandal (2021)	× × × × × × × × × ×	× × × × × × × × × ×
Garg & Rani (2019a) and *Garg & Rani (2019b)*	$0.4174, 0.3054, 0.3199, 0.4744, 0.5766$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_3}$
CIVIFAAWA operator	$-0.3909, -0.1409, -0.4427, -0.1646, -0.0072$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_3}$
CIVIFAAWG operator	$0.5263, 0.4143, 0.4289, 0.5833, 0.6855$	$\overline{\overline{\mathfrak{C}}}_{C_5} > \overline{\overline{\mathfrak{C}}}_{C_2} > \overline{\overline{\mathfrak{C}}}_{C_4} > \overline{\overline{\mathfrak{C}}}_{C_1} > \overline{\overline{\mathfrak{C}}}_{C_3}$

Notes.
" ×" denotes it is unsuitble to calculate the score values.

Comparative analysis

Here, our main theme to evaluate the comparison between proposed method with few existing analyses to show the stability and effectiveness of the proposed method. For this, we use various existing operators such as Aggregation operators (AOs) (*Xu, 2007*), geometric AOs (*Xu & Yager, 2006*), information AOs (*Wang & Liu, 2012*), Einstein geometric AOs (*Wang & Liu, 2012*), Hamacher AOs (Huang, 2014), Dombi AOs (*Seikh & Mandal, 2021*) under the IFSs, and AOs *Garg & Rani (2019a)* and *Garg & Rani (2019b)* based on CIVIFSs. The comparative analysis is stated in Table 7 for the data in Table 2.

Under the various kinds of operators, we have gotten a completely consistent optimal judgment $\overline{\overline{\mathfrak{C}}}_{C_5}$ by utilizing the particular upsides of the boundary. The best optimal is $\overline{\overline{\mathfrak{C}}}_{C_5}$ according to the theory which was proposed by *Garg & Rani (2019a)* and *Garg & Rani (2019b)* based on CIVIFSs and proposed operators. Further, the derived theory of Aggregation operators (AOs) (*Xu, 2007*), geometric AOs (*Xu & Yager, 2006*), information AOs (*Wang & Liu, 2012*), Einstein geometric AOs (*Wang & Liu, 2012*), Hamacher AOs (*Huang, 2014*), Dombi AOs (*Seikh & Mandal, 2021*) under the IFSs have been failed, due to various limitations, because these operators or information ware proposed based on FSs, IFSs, IVIFSs, CFSs, and CIFSs which are the particular cases of the proposed information and hence they are not able to evaluate our suggestion information (CIVIF values).

Therefore, it can be inferred that the presented information and MADM model are very valuable and dominant for handling awkward information.

CONCLUSION

In this manuscript, we combined four main theories such as CIVIF information, Aczel-Alsina operational laws, averaging/geometric aggregation operators, and the WASPAS technique. Furthermore, the theory of CIVIF information is the modified version of the FSs, IFSs, CFSs, CIFSs, and IVIFSs, because these are the special cases of the invented theory. Further, we derived the theory of aggregation operators based on Aczel-Alsina operational laws for CIVIF information. The theory of the WAPSAS technique is also proposed based on Aczel-Alsina aggregation operators for CIVIF information. The key influence of this assessment is debated below: (1) We initiated the Aczel-Alsina operational laws and their related results; (2) We initiated the principle of CIVIFAAWA, CIVIFAAOWA, CIVIFAAHA, CIVIFAAWG, CIVIFAAOWG, and CIVIFAAHG operators, and illustrated their well-known properties and results; (3) We derived the WASPAS method for CIVIFSs and evaluated their main steps with the help of some numerical examples; (4) We demonstrated the MADM strategy under the invented works; (5) We expressed the supremacy and dominancy of the invented works with the help of sensitive analysis and geometrical shown of the explored works.

In the future, we concentrate to derive some new ideas such as complex fuzzy superior Mandelbrot sets, complex intuitionistic fuzzy mandelbrot set, and their extension, and we try to utilize them in the field of artificial intelligence, machine learning, game theory, neural networks, and clustering analysis to improve or enhance the quality of the presented information.

ADDITIONAL INFORMATION AND DECLARATIONS

Funding

This article is funded by the Qinglan Project of Jiangsu Province, Jiangsu Famous Teacher's Studio of Dual Qualification for Vocational Education and Intelligent Equipment Manufacturing. The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

Grant Disclosures

The following grant information was disclosed by the authors:
Qinglan Project of Jiangsu Province, Jiangsu Famous Teacher's Studio of Dual Qualification for Vocational Education and Intelligent Equipment Manufacturing.

Competing Interests

The authors declare there are no competing interests.

Author Contributions

- Haojun Fang conceived and designed the experiments, authored or reviewed drafts of the article, and approved the final draft.
- Tahir Mahmood analyzed the data, performed the computation work, prepared figures and/or tables, and approved the final draft.
- Zeeshan Ali performed the experiments, analyzed the data, prepared figures and/or tables, and approved the final draft.
- Shouzhen Zeng performed the experiments, authored or reviewed drafts of the article, and approved the final draft.
- Yun Jin conceived and designed the experiments, performed the computation work, authored or reviewed drafts of the article, and approved the final draft.

Data Availability

The following information was supplied regarding data availability:

No raw data or code are used in the paper.

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