Diminishing returns as a function of disturbance

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According to Spearman's law of diminishing returns (SLODR), IQ test scores are more g saturated among those with low, compared to those with high, ability. The present simulation shows that such difference in saturation can be observed if test scores are affected by a disturbing factor, e.g. low motivation, illness, or linguistic confusion, that varies in magnitude between individuals. More contemporary criteria of SLODR can also be satisfied if test scores are affected by disturbance, especially if the disturbance variable is negatively skewed. This indicates a possible threat against the validity of findings supporting SLODR and points at the importance for researchers to try to eliminate the influence of such disturbing factors from their studies.

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12   Abstract

13 According to Spearman's law of diminishing returns (SLODR), IQ test scores are more *g*

14 saturated among those with low, compared to those with high, ability. The present simulation

15 shows that such difference in saturation can be observed if test scores are affected by a disturbing

16 factor, e.g. low motivation, illness, or linguistic confusion, that varies in magnitude between

17 individuals. More contemporary criteria of SLODR can also be satisfied if test scores are

18 affected by disturbance, especially if the disturbance variable is negatively skewed. This

19 indicates a possible threat against the validity of findings supporting SLODR and points at the

20 importance for researchers to try to eliminate the influence of such disturbing factors from their

21 studies.

23   *Keywords:* differentiation hypothesis; disturbance; performance; simulation; Spearman's law of

24   diminishing returns; validity

26

27   Introduction

28   To measure abilities can be tricky, as performances can be assumed to be affected not only

29   by peoples' abilities but also by various disturbing factors. That is why athletes' performances

30   tend to fluctuate more than their underlying true abilities can be supposed to do. Here the

31   disturbing factors could be such things as bad weather, a sore knee, or lack of motivation.

32   Performances on tests of cognitive or creative abilities, for instance, could also be influenced by

33   disturbing factors, such as low motivation, illness, or linguistic confusion.

34   According to the intelligence-creativity threshold hypothesis, intelligence should have a

35   positive influence on creative potential but only up to a certain level, the threshold is often set at

36   IQ 120, after which a further increase in intelligence should have no additional effect on creative

37   potential. This hypothesis has received some empirical support (e.g. Cho, Te Nijenhuis, Van

38   Vianen, Kim, & Lee, 2010; Jauk, Benedek, Dunst, & Neubauer, 2013; Shi, Wang, Yang, Zhang,

39   & Xu, 2017, but see Kim, 2005, for a non-confirming meta-analysis).

40   However, a simulation by Sorjonen, Ingre, and Melin (2019) indicates a high probability to

41   observe such threshold-like associations if subjects experience varying degrees of disturbance

42   that affects their performances both on the tests of intelligence and on the tests of creativity. A

43   high degree of disturbance will tend to attenuate the association between measured intelligence

44   and creativity among those with high true abilities while it accentuates the association among

45   those with low abilities.

46   Spearman's (1927) "law of diminishing returns" (SLODR), also called the "differentiation

47   hypothesis" (Garrett, 1946), has often been evaluated, and demonstrated, with a method where

48   the g saturation of intelligence test scores, which corresponds to the strength of the correlations

49   between the test scores, has been compared between groups assumed to have high or low average

50   *g*. An often-observed lower degree of *g* saturation in the former group has been taken to indicate

51   that intelligence is more differentiated/specialized among those with high *g*, with some

52   individuals having, for example, especially high verbal or spatial intelligence. A higher *g*

53   saturation among those with assumed low *g*, on the other hand, has been seen to suggest that in

54   this subgroup intelligence is less specialized and more one-dimensional.

55   For example, Deary et al. (1996) claimed support for SLODR after analyzing data from

56   Irish schoolchildren ($N = 10,535$) on eight Differential Aptitude Tests (DAT) subtests, measuring

57   (1) Verbal reasoning, (2) Numerical ability, (3) Abstract reasoning, (4) Clerical speed and

58   accuracy, (5) Mechanical reasoning, (6) Space relations, (7) Spelling, and (8) Language usage.

59   When dividing the participants into two subgroups, high and low scorers, based on verbal

60   reasoning, the amount of variance in the seven remaining subtests accounted for by a first

61   unrotated principal component equaled 36.2% and 43.6% among the high and low scorers,

62   respectively. When the division was based on numerical ability and space relations the

63   corresponding values were 40.9% and 46.2% for high scorers and 46.2% and 52.9% for low

64   scorers, respectively. Other studies claiming support for SLODR include Detterman and Daniel

65   (1989), Legree, Pifer, and Grafton (1996), Reynolds and Keith (2007), Tucker-Drob (2009), as

66   well as a meta-analysis by Blum and Holling (2017).

67   However, the traditional method of calculating amount of variance accounted for by a first

68   unrotated principal component, or the average intersubtest correlation, in subgroups assumed to

69   differ in *g* has been criticized. It has, for example, been claimed that this method can give

70   confounded results possibly affected by the arbitrary decision how to form the subgroups or by

71   the skewness of test scores (Molenaar, Dolan, Wicherts, & van der Maas, 2010; Murray, Dixon,

72   & Johnson, 2013). According to more contemporary thoughts, ability differentiation, a.k.a.

73   diminishing returns, might be operationalized as (1) a negatively skewed latent g, (2)

74   heteroscedastic subtest residuals, with a larger residual variance among those with high

75   compared to low g, and (3) non-linear g loadings (Hessen & Dolan, 2009; Molenaar, Dolan, &

76   van der Maas, 2011; Molenaar, Dolan, & Verhelst, 2010; Murray et al., 2013; Tucker-Drob,

77   2009). The fulfilment of these criteria can be analyzed with structural equation modelling (SEM)

78   based methods.

79   Researchers using such more contemporary methods/operationalizations have questioned if

80   there really is any convincing support for SLODR. For example, when using a traditional

81   method, Murray et al. (2013) found that support for SLODR was proportional to the average

82   skew of the test battery, i.e. mainly found in positively skewed batteries. When using more

83   contemporary methods, they found heteroscedastic residuals, positively correlated with g, in

84   positively skewed test batteries, while the criterium of non-linear g-loadings, on the contrary,

85   was found in negatively skewed batteries. The criterium of a negatively skewed latent g was

86   never fulfilled. In analyzes of data from young Spanish adults ($N = 588$) on 14 WAIS-III

87   subtests, Molenaar et al. (2011) found some indications of quadratic second-order factor loadings

88   and a negatively skewed second order common factor (g). However, contrary to predictions by

89   SLODR, residual variances tended to decrease with increased g.

90   As mentioned above, simulations have indicated a high probability to observe threshold-

91   like associations, for example between intelligence and creativity, if subjects experience varying

92   degrees of disturbance that affects their performances (Sorjonen et al., 2019). Due to its

93   similarity to the intelligence-creativity threshold hypothesis, with both proposing a stronger

94   association between performances among those with low ability, it is possible that SLODR might

95   be sensitive to the influence of disturbing factors as well. The objective of the present simulation

96   study was to assess this possibility. Although the traditional method to analyze SLODR through

97   degree of *g* saturation in subgroups based on different levels of *g* has been deemed untrustworthy

98   (see above), we still include it in the present study in order to see how it might be affected by

99   disturbance in the measurement of intelligence. We do this due to personal curiosity and because

100 a majority of all studies of SLODR have used this method. They do, for example, seem to

101 constitute most of the studies in Blum and Holling's (2017) meta-analysis. However, anyone

102 wishing not to know how the traditional method is affected by disturbance is advised not to read

103 the sections with the heading "Traditional paradigm" below, nor the first paragraph in the

104 discussion, nor to look at panel A in Figure 1 and Figure 2.

105

106   Method

107 *Traditional paradigm*

108   Using R 3.5.0 statistical software (R Core Team, 2018) and the psych package (Revelle,

109 2018), data was simulated and analyzed through the following steps (panel A in Figure 1, script

110 available at https://osf.io/7ydp8/): (1) 10,000 virtual subjects were allocated a true *g* score from a

111 random normal distribution ($M = 100$, $SD = 15$); (2) The subjects were allocated 15 test scores

112 ($M = 100$, $SD = 15$) with a defined population correlation with the *g* score. The correlation for

113 the first test score, the selection variable, was set to either 0.1, 0.5, or 0.9 while the correlations

114 for the other 14 variables were drawn from a random uniform distribution between 0.2 and 0.9.

115 Being continuous, these test scores could be seen to correspond to first order factor scores or

116 subtest scores rather than to traditional dichotomous test items and, consequently, the calculated

117 factor loadings correspond to second order factor loadings; (3) The virtual subjects were

118 allocated a disturbance score from a random beta distribution that was either negatively (alpha =

119   9, beta = 1) or positively (alpha = 1, beta = 9) skewed or approximately normally distributed

120   (alpha = 9, beta = 9). These disturbance scores varied between 0 and 1 with a lower value

121   indicating a higher degree of disturbance; (4) The virtual subjects' test scores, including the

122   selection variable, were multiplied by their disturbance score, except in the cases with no effect

123   of disturbance; (5) Based on their value on the selection variable (rescaled to the conventional M

124   = 100, SD = 15), the virtual subjects were divided into 10 subgroups within 20 points on the

125   selection variable. Hence, there was overlap between the subgroups, with some subjects included

126   in more than one group; (6) In each subgroup, the amount of variance in the test scores,

127   excluding the selection variable, that could be explained by a first un-rotated principal

128   component, i.e. g saturation, was calculated. According to SLODR, there should be a negative

129   association between the mean on the selection variable and the degree of g saturation in the

130   subgroups.

131   This procedure was repeated 20 times for each distribution of the disturbance variable and

132   when disturbance had no effect, and for the three levels of correlation between the selection

133   variable (undisturbed) and g. When using one of the test scores for creating the groups and then

134   not including this in the principal component analysis, we follow the procedure by Deary et al.

135   (1996).

136

137   *More contemporary operationalization*

138   As mentioned in the introduction, a more contemporary operationalization of SLODR

139   include the criteria (1) a negatively skewed latent g, (2) heteroscedastic subtest residuals, with a

140   larger residual variance among those with high compared to low g, and (3) non-linear g loadings.

141   Data was simulated similarly as described above, bar the selection variable, with one thousand

142   simulations ($N = 10,000$ in each) for each of the four types of disturbance variables - non-

143   existent, normally distributed, negatively, and positively skewed. Data was analyzed using the

144   lavaan (Rosseel, 2012) and semTools (Jorgensen, Pornprasertmanit, Schoemann, & Rosseel,

145   2018) packages in R.

146   Initially, a one-factor confirmatory factor analysis (CFA) was applied to data (the blue part

147   in panel B in Figure 1). Using the lavPredict function in lavaan, g was estimated for each virtual

148   subject and the skewness, with significance, of this g was calculated with the skew function in

149   semTools. According to SLODR, g should be significantly negatively skewed.

150   Employing the lavPredict function once more, the predicted score, and then the squared

151   difference between the observed and this predicted score, on each of the fifteen tests was

152   calculated for each virtual subject. The individual mean on these squared differences correspond

153   to the individual residual variance, i.e. variance in the test scores that is not accounted for by the

154   individual's g. According to SLODR, there should be a significant positive correlation between g

155   and this residual variance. Following Hessen and Dolan (2009), the logarithm of residual

156   variance was used in this analysis.

157   For the third criterium, the test scores were regressed on a quadratic latent g factor in

158   addition to the linear latent g factor (red part in panel B in Figure 1). The factor loadings of

159   quadratic g were constrained to be the square of the corresponding linear factor loadings. In this

160   way, the quadratic g factor catches the acceleration in the effect of g on the test scores.

161   According to SLODR, the addition of quadratic g to the model should result in significant

162   improvement of model fit. Moreover, the estimated value on the quadratic g factor should

163   become increasingly more negative with an increase in g (= diminishing returns), i.e. there

164   should be a significant negative correlation between g and g^2.

165

166   Results

167   *Traditional paradigm*

168   In accordance with SLODR, a negative association between the mean value on the

169   selection variable in subgroups and the amount of variance in the test scores that could be

170   explained by a first un-rotated principal component, i.e. *g* saturation, was apparent, except when

171   disturbance was not allowed to affect the test scores (Figure 2). When the selection variable

172   (undisturbed) was strongly correlated with the true *g* score (panel C) this association was

173   accentuated, but we also see a general decrease in the amount of explained variance compared to

174   when the correlation was weaker (panels A and B). This is expected, because in the former case

175   the subjects are more homogeneous in their *g* and there is less room for a common factor to

176   explain variance in the test scores. With a positively skewed disturbance factor the selection

177   variable also became positively skewed, with few subjects at the high end of the scale, and here

178   we see a high degree of instability in the calculated amount of explained variance.

179

180   *More contemporary operationalization*

181   In Table 1, we see that with no disturbance affecting test scores, the probability to fulfill

182   the criteria of a negatively skewed *g* and a positive correlation between *g* and residual variance

183   are close to what can be expected given a significance level of .05. Fulfillment of the criterium of

184   non-linear *g* loadings, which requires both a significant improvement in model fit when adding a

185   quadratic *g* factor to the model and a significant negative correlation between *g* and g^2 (see the

186   Method section), is less probable.

187   With a negatively skewed disturbance variable affecting test scores, all of the

188   contemporary criteria are fulfilled in 81% of the cases, and without a requirement for a negative

189   correlation between g and g^2 we would see virtually perfect fulfillment. With a normally

190   distributed or a positively skewed disturbance variable, the probability for a positive correlation

191   between g and residual variance and for a negative correlation between g and g^2 are extremely

192   high. However, here we always see, contrary to SLODR criteria, a positively skewed g (Table 1).

193   The effect of a negatively skewed disturbance variable is illustrated in Figure 3. In this

194   example, true g is perfectly normally distributed (skewness = -0.001, p = 0.982, panel A) and has

195   a weak positive linear association with the true score on one of the fifteen tests (r = 0.131, p <

196   0.001, panel B). However, if the test scores are affected by a negatively skewed (skewness = -

197   1.521, p < 0.001) disturbance variable, the estimated g becomes negatively skewed (skewness = -

198   0.365, p < 0.001, panel C). Now we also see slightly heteroscedastic residuals that increase with

199   estimated g (correlation between g and the logarithm of squared deviations from the linear

200   prediction line in panel D = 0.036, p < 0.001, note that the correlation between g and the

201   logarithm of individual residual variance over all fifteen test scores is substantially stronger, r =

202   0.218, p < 0.001) and a quadratic association between estimated g and observed test score (B = -

203   1.266 and p < 0.001 for the quadratic regression term, panel D).

204

205   Discussion

206   The present simulation shows that if test scores are affected by disturbance – which could

207   be, for example, low motivation, illness, or linguistic confusion – that vary in magnitude between

208   individuals, we will, similarly as studies using a traditional paradigm to assess SLODR, tend to

209   see a higher degree of factor saturation among those with a low score on an associated selection

210   variable compared to those with a high score. This is true irrespective of the distribution of the

211   disturbance variable. Consequently, the present findings could be added to the growing number

212   of warnings that a positive result when using the traditional paradigm cannot be trusted to

213   indicate a true SLODR-effect.

214   When evaluating the effect of disturbance using more contemporary criteria, results in

215   accordance with SLODR were seen mainly with a negatively skewed disturbance variable. With

216   a normally distributed or a positively skewed disturbance variable, two of three criteria, namely

217   heteroscedastic residual variance and quadratic factor loadings, were usually fulfilled, while the

218   skewness of estimated g was in the opposite direction compared to SLODR criteria. Hence, one

219   of the advantages of these contemporary criteria seems to be that one of them, negative skewness

220   of estimated g, offers protection against type 1-errors, unless test scores have been affected by

221   negatively skewed disturbance, while the traditional method seems to deliver false support for

222   SLODR irrespective of the distribution of the disturbance variable. However, negatively skewed

223   disturbance would mean that most subjects receive test scores close to their true values while a

224   few perform well below their true ability, and we believe that such lack of validity among a

225   minority of participants can be quite realistic in many test situations.

226   According to Murray et al. (2013), the empirical support for SLODR can be called into

227   question as it is based mainly on studies employing traditional paradigms, which cannot

228   discriminate between a true SLODR-effect and effects of subtest characteristics, e.g. skewness.

229   The present simulation throws some additional pessimism into the pot. If using more

230   contemporary methods and finding support for SLODR in the form of a negatively skewed latent

231   g, heteroscedastic residuals, and quadratic factor loadings, one cannot be sure if this is due to a

232   true SLODR-effect or the influence of some negatively skewed disturbance factor that varies in

233   degree between study participants and that affects their test scores.

234   We do not, of course, claim to have proven that all findings in accordance with SLODR are

235   due to the influence of negatively skewed disturbance. We merely point at the possibility that

236   this could have given SLODR a helping hand in some studies. This points at the importance for

237   researchers to try to eliminate, as far as possible, the influence of all such disturbing factors from

238   their studies. However, we can probably never be absolutely sure that we are measuring all of the

239   participants' true abilities rather than merely their performances.

240   Still, even if findings that support SLODR would be due to the influence of disturbance,

241   this does not necessarily mean that SLODR is just an artifact. For example, according to

242   Detterman's (1993) explanation of SLODR, as well as the more recent process-overlap theory

243   (POT, Kovacs & Conway, 2016, 2019), the "disturbing factor" could be a deficit in some

244   important general cognitive process which restrains the functioning of the whole system.

245   Without this deficit, test scores are mainly due to more specific processes and, consequently, less

246   correlated and g saturated. If Detterman's hypothesis and POT are correct, and if studies using

247   more contemporary criteria start supporting SLODR (it does not look very promising so far), one

248   of the contributions of the present simulation would be to predict that most people experience

249   only low degrees of this deficit while a few experiences a very high level.

250

251   *Conclusions*

252   The present simulation indicates that findings that falsely seem to support Spearman's law

253   of diminishing returns (SLODR) can be obtained if test scores are affected by a disturbance

254   variable that varies in magnitude between respondents. This is true for contemporary

255   operationalizations of SLODR and even more so if using a traditional paradigm to analyze data.

256

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323

325   Figure Captions

326   Figure 1. (A) Steps in the simulation (solid lines) and analyzed parameters (dashed lines) of the

327   traditional paradigm. Test scores and a selection variable (sv) are affected by true g (tg) but the

328   effect is attenuated by disturbance that varies in magnitude between individuals. Amount of

329   variance in the test scores explained by the first un-rotated principal component (ev) is calculated

330   separately in the subgroups defined by the selection variable. Note: 14 test scores, besides the

331   selection variable, were used in the actual simulation. (B) Illustration of the analyses when

332   evaluating more contemporary criteria of SLODR. Test scores are regressed on either one single

333   common g factor (blue part) or on one linear and one quadratic g factor (blue + red part). In the

334   latter case, factor loadings for the quadratic factor are constrained to equal the square of the

335   corresponding linear factor loadings. Note: 15 test scores were used in the actual simulation.

336

337   Figure 2. Amount of variance in 14 tests explained by the first un-rotated principal component

338   as a function of the mean value on the selection variable in the subgroups when the disturbance

339   variable is either negatively, normally, or positively distributed or if there is no effect of

340   disturbance on test scores. Separately for three different degrees of correlation between g and the

341   selection variable (before disturbance), namely (A) 0.1, (B) 0.5, and (C) 0.9. The subgroups

342   always have a maximum range of 20 points on the selection variable. 20 simulations (adjoined

343   by a line) for each distribution of the disturbance variable in each panel and with $N = 10,000$ in

344   each.

345

346   Figure 3. Illustration of the effect of a negatively skewed disturbance variable. (A) A perfectly

347   normally distributed true g has (B) a weak linear association with the true score on one of the

348 fifteen tests. When test scores are affected by a negatively skewed disturbance variable, we

349 observe (C) a negatively skewed estimated g that has (D) a quadratic association with the

350 observed test score. We also observe heteroscedastic residuals that increase with increased g (D).

351 $N = 10,000$ in each panel, black line = best fitting linear association, red line = best fitting

352 quadratic association.

353

Figure 1

Steps in the simulation and analyses.

(A) Steps in the simulation (solid lines) and analyzed parameters (dashed lines) of the traditional paradigm. Test scores and a selection variable (sv) are affected by true g (tg) but the effect is attenuated by disturbance that varies in magnitude between individuals. Amount of variance in the test scores explained by the first un-rotated principal component (ev) is calculated separately in the subgroups defined by the selection variable. Note: 14 test scores, besides the selection variable, were used in the actual simulation. (B) Illustration of the analyses when evaluating more contemporary criteria of SLODR. Test scores are regressed on either one single common g factor (blue part) or on one linear and one quadratic g factor (blue + red part). In the latter case, factor loadings for the quadratic factor are constrained to equal the square of the corresponding linear factor loadings. Note: 15 test scores were used in the actual simulation.

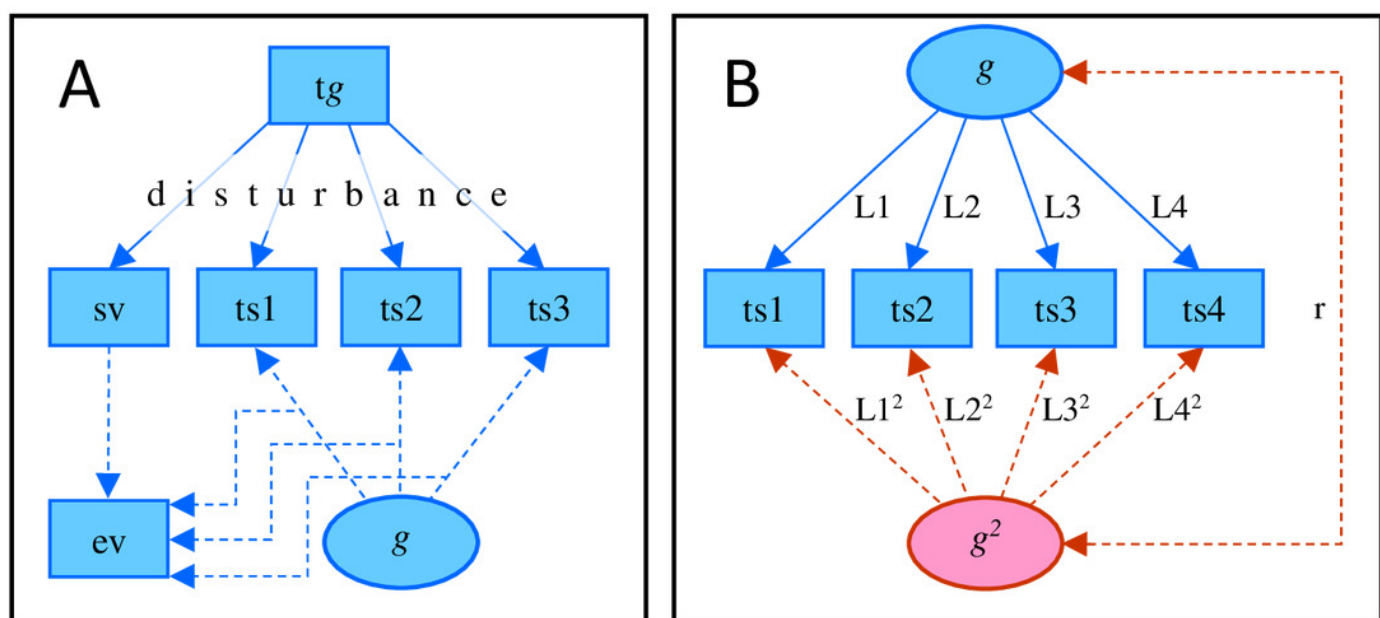



Figure 2

Explained variance as a function of mean intelligence.

Amount of variance in 14 tests explained by the first un-rotated principal component as a function of the mean value on the selection variable in the subgroups when the disturbance variable is either negatively, normally, or positively distributed or if there is no effect of disturbance on test scores. Separately for three different degrees of correlation between g and the selection variable (before disturbance), namely (A) 0.1, (B) 0.5, and (C) 0.9. The subgroups always have a maximum range of 20 points on the selection variable. 20 simulations (adjoined by a line) for each distribution of the disturbance variable in each panel and with N = 10,000 in each.

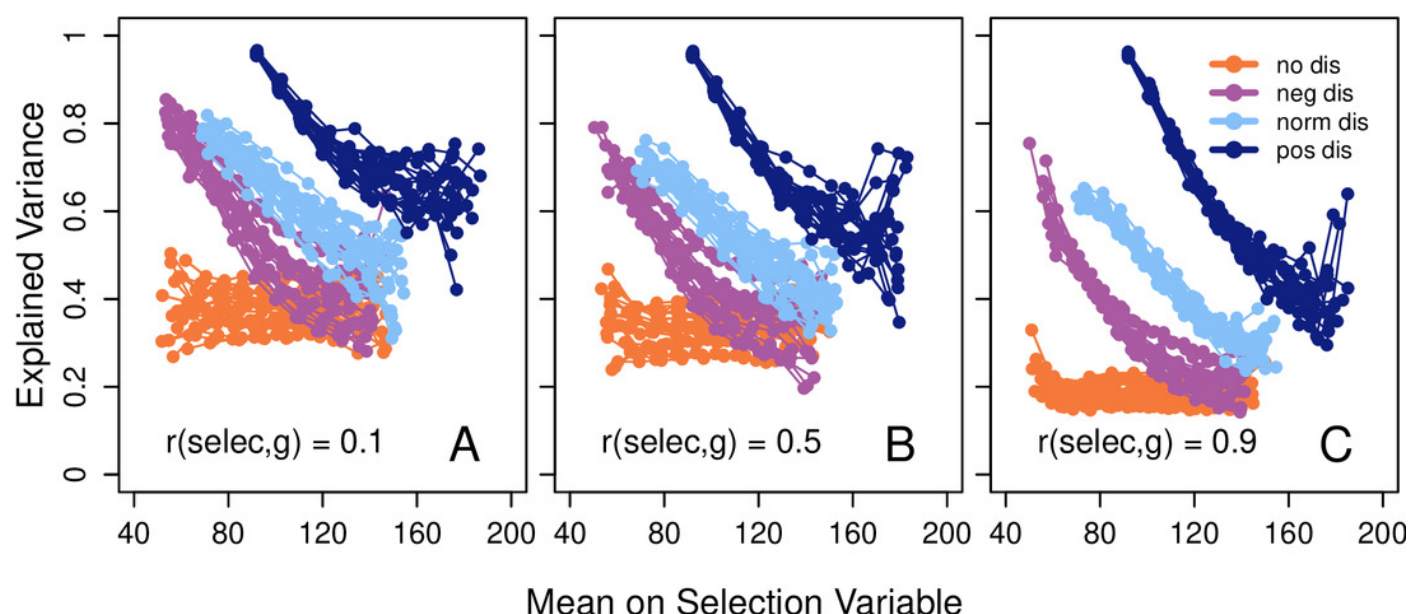
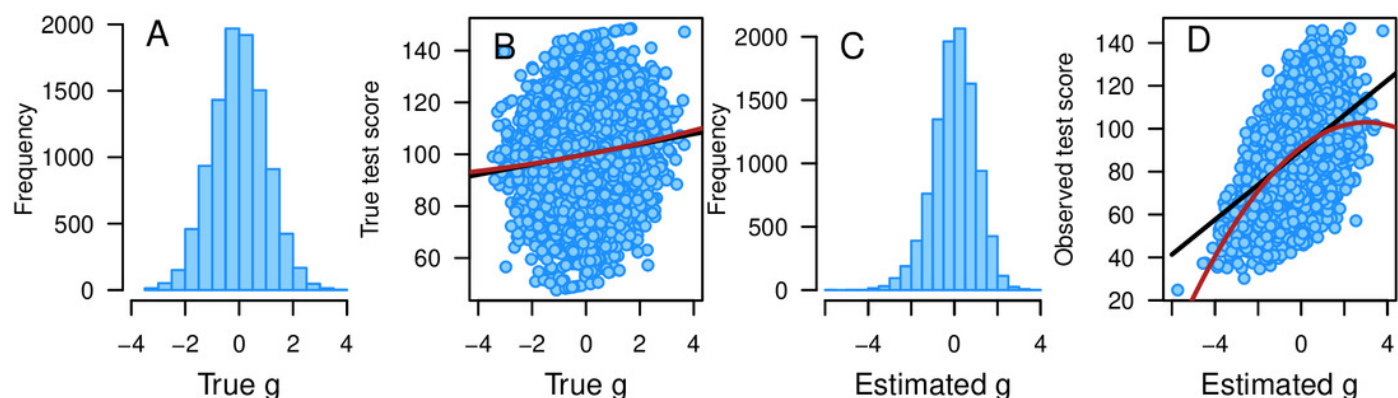



Figure 3

Illustration of the effect of disturbance.

Illustration of the effect of a negatively skewed disturbance variable. (A) A perfectly normally distributed true g has (B) a weak linear association with the true score on one of the fifteen tests. When test scores are affected by a negatively skewed disturbance variable, we observe (C) a negatively skewed estimated g that has (D) a quadratic association with the observed test score. We also observe heteroscedastic residuals that increase with increased g (D). $N = 10,000$ in each panel, black line = best fitting linear association, red line = best fitting quadratic association.

Fulfillment of contemporary SLODR criteria.

Proportions of a significantly skewed common factor (g), correlation between g and residual variance, quadratic factor loadings, correlation between g and the quadratic term (g^2), and fulfilment of all crucial criteria as functions of the distribution of the disturbance variable.

Table 1. Proportions of a significantly skewed common factor (g), correlation between g and residual variance, quadratic factor loadings, correlation between g and the quadratic term (g^2), and fulfilment of all crucial criteria as functions of the distribution of the disturbance variable.

	Disturbance			
Criteria	No	Negative	Normal	Positive
Positive skew	0.030	0.000	1.000	1.000
Negative skew	0.027	1.000	0.000	0.000
$r(g,$ res.$)^a > 0$	0.021	1.000	1.000	1.000
$r(g,$ res.$)^a < 0$	0.032	0.000	0.000	0.000
$p(g^2)^b < 0.05$	0.031	0.997	0.998	0.945
$r(g, g^2)^c > 0$	0.010	0.000	0.000	0.000
$r(g, g^2)^c < 0$	0.012	0.811	0.974	0.940
All^d	0.000	0.811	0.000	0.000

Note: 1000 simulations in each column, [a] correlation between g and the logarithm of residual variance, [b] significance when adding a quadratic term to the factor model, [c] correlation between g and the quadratic term, [d] fulfilment of all crucial criteria (in bold).