A Bayesian approach to construct confidence intervals for comparing the rainfall dispersion in Thailand

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ABSTRACT

Natural disasters such as drought and flooding are the consequence of severe rainfall fluctuation, and rainfall amount data often contain both zero and positive observations, thus making them fit a delta-lognormal distribution. By way of comparison, rainfall dispersion may not be similar in enclosed regions if the topography and the drainage basin are different, so it can be evaluated by the ratio of variances. To estimate this, credible intervals using the highest posterior density based on the normal-gamma prior (HPD-NG) and the method of variance estimates recovery (MOVER) for the ratio of delta-lognormal variances are proposed. Monte Carlo simulation was used to assess the performance of the proposed methods in terms of coverage probability and relative average length. The results of the study reveal that HPD-NG performed very well and was able to meet the requirements in various situations, even with a large difference between the proportions of zeros. However, MOVER is the recommended method for equal small sample sizes. Natural rainfall datasets for the northern and northeastern regions of Thailand are used to illustrate the practical use of the proposed credible intervals.

INTRODUCTION

Natural phenomena can often be random events in statistics, and natural rainfall is an important one because it is directly related to the quality of life, agriculture, economic growth, industry, among others. Rice is the main export product from Thai agriculture, so natural rainfall is a crucial water resource for farmers. However, climate change has incurred prolonged droughts, thereby decreasing agricultural and fishery yields, and conversely, caused violent flooding and health-related issues in Thailand (Marks, 2011). Farmers who consume approximately 70% of the country's water supply are at the forefront of these impacts due to changes in rainfall amount (Marks, 2011). Thus, extreme rainfall variation is one of the main consequences of climate change and can result in drought or flooding, and ineffective water management can exacerbate both situations (Duangdai and Likasiri, 2017).

Thailand is divided into five regions: northern, northeastern, central, eastern, and southern. The north is mountainous and contains the sources of several important rivers, including the Mekong Basin (a principal river system of Thailand). Nan is one of the provinces in the northern region faced with heavy rainfall for around two weeks which caused landslides and flooding during the rainy season (mid-May to mid-October) (Hariraksapitak et al., 2018). Nicely and Counselor (2018) reported that approximately 60 % of the northeastern region's arable land has been turned over to total rice cultivation. Meanwhile, 5-10 % of precipitation above the normal level occurred during May–July 2018 in the northern and northeastern regions, as reported by the Thai Meteorological Department (Nicely and Counselor, 2018). These situations have led us to consider estimating the rainfall fluctuation between the northern and northeastern regions of Thailand in terms of their ratio of variances during July 2018. It is of major importance to estimate the dispersion in weekly rainfall amount between the northern and northeastern regions in Thailand for the benefit of key industries, and the information gained could help in realizing climate change awareness. The information could be advantageous for the Thai government and other

46 related organizations to realize and plan for solving or reducing the risk of environmental issues if a large
47 variation in rainfall is known in advance. The results from normality plots (Figure 1), histograms (Figure
48 2), and applying the Akaike information criterion (AIC) show that the weekly natural rainfall data for
49 both regions fit delta-lognormal distributions. Note that the log-transformed data of the positive rainfall
50 amounts fit a normal distribution which is symmetrical.

51 Aitchison and Brown (1963) first introduced the delta-lognormal distribution for non-negative data
52 containing zero values with the probability $0 < \delta < 1$; positive observations with the remainder of
53 the probability $1 - \delta$ follow a lognormal distribution and the zeros follow a binomial distribution with
54 binomial proportion δ. The delta-lognormal distribution has been fitted for real-world examples in many
55 research areas such as the environment (Owen and DeRouen, 1980; Hasan and Krishnamoorthy, 2018),
56 fishery surveys (Pennington, 1983; Smith, 1988, 1990; Lo et al., 1992; Fletcher, 2008; Wu and Hsieh,
57 2014) and medicine (Zhou and Tu, 1999, 2000; Tian and Wu, 2006; Hasan and Krishnamoorthy, 2018).

58 In statistical inference, one of the parameters of interest is the variance (dispersion in environment)
59 defined as the second central moment; the positive square root of the variance is called the standard
60 deviation (Casella and Berger, 2002). To compare two independent populations, the ratio of their
61 variances is the measurement of the variation between them with no difference resulting in a ratio equal
62 to 1. Confidence intervals (CIs) have been applied to several distributions to estimate the variability in
63 terms of variance and the ratio of variances. For example, Krishnamoorthy et al. (2006) obtained CIs for
64 lognormal variance based on the idea of the generalized confidence interval (GCI) which they applied to
65 the variation in the air lead levels in 15 industrial facilities in a health hazard evaluation. Bebu and Mathew
66 (2008) claimed that GCI was better than a modified signed log-likelihood ratio test for establishing CIs
67 for the ratio of variances for bivariate lognormal distributions even when the sample sizes were small.
68 Niwitpong (2017) showed that GCI performed well for the ratio of variances of lognormal distributions to
69 solve the problems that occur when applying the traditional approach.

70 Casella and Berger (2002) argued that CIs can capture the parameter of interest better than point
71 estimates. Although a few attempts have been conducted on CIs for the ratio of variances for some
72 distributions, research on the delta-lognormal distribution has yet to be carried out. Clearly, there is a
73 need for research that investigates and constructs CIs for the ratio of delta-lognormal variances using the
74 highest posterior density based on normal-gamma prior (HPD-NG) and MOVER. These proposed CIs
75 were compared with the existing HPD-based Jeffreys' (HPD-Jef) and Jeffreys' Rule (HPD-Rul) priors of
76 Harvey and van der Merwe (2012), GCI of Wu and Hsieh (2014) and fiducial GCI (FGCI) of Hasan and
77 Krishnamoorthy (2018).

78 The organization of this paper is as follows. The concepts of all of the CIs for the ratio of variances
79 in delta-lognormal distributions are elaborated in Section 2. The simulation procedure and numerical
80 results are reported in Section 3. In Section 4, the proposed methods are applied to real-world datasets
81 to estimate the rainfall fluctuation between the northern and northeastern regions in Thailand. Last, we
82 present a discussion and conclusions on the study outcomes.

METHODS

84 Let $X_{ij} = (X_{i1}, X_{i2}, ..., X_{in_i})$; $i = 1, 2$ and $j = 1, 2, ..., n_i$ be non-negative random samples draw from a
85 delta-lognormal distribution with parameters μ_i, σ_i^2 and δ_i, stand for $X_{ij} \sim \Delta(\mu_i, \sigma_i^2, \delta_i)$. For $X_{ij} = 0$,
86 the number of having zero $n_{i(0)} \sim B(n_i, \delta_i)$ and $n_i = n_{i(0)} + n_{i(1)}$. For $X_{ij} > 0$, $X_{ij} \sim LN(\mu_i, \sigma_i^2)$ where
87 $Y_{ij} = \ln(X_{ij}) \sim N(\mu_i, \sigma_i^2)$. Aitchison and Brown (1963) defined the population variance of X_{ij} as

$$\omega_i = (1 - \delta_i) \exp(2\mu_i + \sigma_i^2) \left[\exp(\sigma_i^2) - (1 - \delta_i) \right] \tag{1}$$

88 The maximum likelihood estimates of δ_i, μ_i and σ_i^2 are $\hat{\delta}_i = \frac{n_{i(0)}}{n_i}$, $\hat{\mu}_i = \frac{1}{n_{i(1)}} \sum_{j=1}^{n_{i(1)}} y_{ij}$, and $\hat{\sigma}_i^2 = \frac{1}{n_{i(1)}} \sum_{j=1}^{n_{i(1)}} (y_{ij} - \hat{\mu}_i)^2$,
89 respectively. From (1), the ratio in delta-lognormal variances is obtained as

$$\theta = \ln \omega_1 - \ln \omega_2 \tag{2}$$

90 where ω_i is log-transformed as $\ln \omega_i = \ln(1 - \delta_i) + (2\mu_i + \sigma_i^2) + \ln \left[\exp(\sigma_i^2) - (1 - \delta_i) \right]$. The methods
91 for constructing CIs for θ are described as seen below.

GCI

On the ideas of GCI, the generalised pivotal quantity (GPQ) is necessary to satisfy the two requirements of Weerahandi (1993). The GPQ of δ_i based on VST was proposed by Wu and Hsieh (2014) as

$$R_{\delta_i} = \sin^2 \left[\arcsin \sqrt{\hat{\delta}_i} - \frac{W_i}{2\sqrt{n_i}} \right] \tag{3}$$

where $W_i = 2\sqrt{n_i} \left(\arcsin \sqrt{\hat{\delta}_i} - \arcsin \sqrt{\delta_i} \right) \stackrel{d}{\sim} N(0,1)$. The GPQs of μ_i and σ_i^2 are

$$R_{\mu_i} = \hat{\mu}_i - Z_i \sqrt{(n_{i(1)}-1)\hat{\sigma}^2/[n_{i(1)}U_i]}$$
$$R_{\sigma_i^2} = (n_{i(1)}-1)\hat{\sigma}_i^2/U_i \tag{4}$$

which are presented by Krishnamoorthy and Mathew (2003). These GPQs lead to obtain the GPQ of θ as

$$R_\theta = R_{\ln \omega_1} - R_{\ln \omega_2} \tag{5}$$

where $R_{\ln \omega_i} = \ln(1 - R_{\delta_i}) + (2R_{\mu_i} + R_{\sigma_i^2}) + \ln \left[\exp(R_{\sigma_i^2}) - (1 - R_{\delta_i}) \right]$. $[R_\theta(\zeta/2), R_\theta(1 - \zeta/2)]$ becomes the $100(1 - \zeta)\%$GCI for θ. Algorithm 1 shows the steps to compute GCI as seen below.

Algorithm 1:

1). Generate $W_i \sim N(0,1)$, $Z_i \sim N(0,1)$ and $U_i \sim \chi^2_{n_{i(1)}-1}$.

2). Compute R_{μ_i}, $R_{\sigma_i^2}$, and R_{δ_i}.

3). Compute $R_{\ln \omega_i}$.

4). Compute R_{θ_i}.

5). Repeat 1-4 a number of times (say, $m = 2,500$).

6). For the 2500 times, compute $(100 - \zeta)\%$GCI for θ.

FGCI

The fiducial generalised pivotal quantity (FGPQ) was proved and defined by Hannig et al. (2006). Hannig (2009) claimed that their generalised fiducial recipe has been developed as the GPQ concept, so their ideas are applied with generalised inference directly. Here $X_{ij} \sim \Delta(\mu_i, \sigma_i^2, \delta_i)$ is considered. Hasan and Krishnamoorthy (2018) showed the FGPQs of μ_i and σ_i^2 as

$$T_{\mu_i} = \hat{\mu}_i - Z_i \sqrt{\hat{\sigma}_i^2/(n_{i(1)}U_i)} \tag{6}$$
$$T_{\sigma_i^2} = \hat{\sigma}_i^2/U_i \tag{7}$$

where $Z_i \sim N(0,1)$ and $U_i \sim \chi^2_{n_{i(1)}-1}/(n_{i(1)}-1)$ are independent random variables. Also, the FGPQs of δ_i was developed as $T_{1-\delta_i} \sim beta(n_{i(1)} + 0.5, n_{i(0)} + 0.5)$. By three FGPQs, the FGPQ of $\ln \omega_i$ is defined as $T_{\ln \omega_i} = \ln T_{1-\delta_i} + (2T_{\mu_i} + T_{\sigma_i^2}) + \ln \left[\exp(T_{\sigma_i^2}) - T_{1-\delta_i} \right]$, then the FGPQs of θ can be expressed as

$$T_\theta = T_{\ln \omega_1} - T_{\ln \omega_2} \tag{8}$$

which can establish the $100(1 - \zeta)\%$FGCI for θ that is $[T_\theta(\zeta/2), T_\theta(1 - \zeta/2)]$; $T_\theta(\zeta)$ stands for $(\zeta)100$th percentile of T_θ. Algorithm 2 details the computation of FGCI.

Algorithm 2:

1). Generate $Z_i \sim N(0,1)$ and $U_i \sim \chi^2_{n_{i(1)}-1}$.

2). Compute T_{μ_i}, $T_{\sigma_i^2}$, and $T_{1-\delta_i}$.

3). Compute $T_{\ln \omega_i}$.

4). Compute T_{θ_i}.

5). Repeat 1-4 a number of times (say, $m = 2,500$).

6). For the 2500 times, compute $(100 - \zeta)\%$FGCI for θ.

MOVER interval

This interval is expanded from CIs for delta-lognormal mean of Hasan and Krishnamoorthy (2018) to the ratio of delta-lognormal variances. Given the individual CIs for the parameters of interest γ, the MOVER concept is used to establish a CI for a linear combination of parameters, proposed by Krishnamoorthy and Oral (2017). Let $\gamma = (\gamma_1, \quad \gamma_2, \quad \ldots \quad \gamma_p)$ be a p-dimensional parameter vector. The estimate of γ_i is $\hat{\gamma}_i; i = 1, 2, ..., p$ that are independent. The MOVER interval for $\sum_{i=1}^{p} c_i \gamma_i$ is defined as

$$CI_{MOVER} = \left[\sum_{i=1}^{p} c_i \hat{\gamma}_i - \sqrt{\sum_{i=1}^{p} c_i^2 (\hat{\gamma}_i - l_i^*)^2}, \sum_{i=1}^{p} c_i \hat{\gamma}_i + \sqrt{\sum_{i=1}^{p} c_i^2 (\hat{\gamma}_i - u_i^*)^2} \right] \quad (9)$$

where $l_i^* = l_i$ if $c_i > 0$, u_i if $c_i < 0$ and $u_i^* = u_i$ if $c_i > 0$, l_i if $c_i < 0$. Moreover, (l_i, u_i) stands for the CI for γ_i. Here we let

$$\ln \omega_i = \omega_i^{(1)} + \omega_i^{(2)} + \omega_i^{(3)} = \ln(1 - \delta_i) + (2\mu_i + \sigma_i^2) + \ln \left[\exp(\sigma^2) - (1 - \delta_i) \right] \quad (10)$$

Using $\hat{\mu}_i, \hat{\sigma}_i^2, \hat{\delta}_i$ from a sample, $\ln \hat{\omega}_i = \hat{\omega}_i^{(1)} + \hat{\omega}_i^{(2)} + \hat{\omega}_i^{(3)}$ is obtained, and CIs for individual parameters $\omega_i^{(1)}, \omega_i^{(2)}$ and $\omega_i^{(3)}$ are also constructed. To begin with $\omega_i^{(1)}$, $(100 - \zeta)\%$CI-based Wilson was proposed by Wilson (1927), that is

$$CI_{\omega_i^{(1)}} = [l_{\omega_i^{(1)}}, u_{\omega_i^{(1)}}] = \ln \left[\left\{ (n_{i(1)} + Z_{i,\frac{\zeta}{2}}^2/2) \mp \left(Z_{i,1-\frac{\zeta}{2}} \sqrt{\frac{n_{i(0)}n_{i(1)}}{n_i} + \frac{Z_{i,\frac{\zeta}{2}}^2}{4}} \right) \right\} / (n_i + Z_{i,\frac{\zeta}{2}}^2) \right] \quad (11)$$

where $Z_i = \frac{n_{i(1)} - n_i(1 - \delta_i)}{\sqrt{n_i \delta_i (1 - \delta_i)}} \overset{d}{\sim} N(0,1)$. The $(100 - \zeta)\%$CI for $\omega_i^{(2)}$ is developed from Zou et al. (2009), then

$$CI_{\omega_i^{(2)}} = [l_{\omega_i^{(2)}}, u_{\omega_2^{(1)}}]$$

$$= \left[\hat{\omega}_i^{(2)} - \sqrt{\frac{4\hat{\sigma}_i^2 T_i^2}{n_{i(1)}} + \hat{\sigma}_i^4 \left(1 - \frac{n_{i(1)} - 1}{\chi_{1-\zeta/2, n_{i(1)}-1}^2} \right)^2}, \hat{\omega}_i^{(2)} + \sqrt{\frac{4\hat{\sigma}^{2i} T_i^2}{n_{i(1)}} + \hat{\sigma}_i^4 \left(\frac{n_{i(1)} - 1}{\chi_{\zeta/2, n_{i(1)}-1}^2} - 1 \right)^2} \right] \quad (12)$$

where $T_i = \frac{\hat{\mu}_i - \mu_i}{\sqrt{\sigma_i^2/n_{i(1)}}} \overset{d}{\sim} N(0,1)$ and $\chi_{n_{i(1)}-1}^2$ denoted as chi-square distribution with $n_{i(1)} - 1$ degree of freedom. Next, the $(100 - \zeta)\%$CI for $\omega_i^{(3)}$ is proposed as

$$CI_{\omega_i^{(3)}} = [l_{\omega_i^{(3)}}, u_{\omega_i^{(3)}}] \quad (13)$$

where

$$l_{\omega_i^{(3)}} = \ln \left[\hat{\omega}_i^{(3)} - \sqrt{\exp \left\{ \hat{\sigma}_i^4 \left(1 - \frac{n_{i(1)} - 1}{\chi_{1-\zeta/2, n_{i(1)}-1}^2} \right)^2 \right\} + \left\{ \frac{Z_{i,1-\frac{\zeta}{2}} \sqrt{\frac{n_{i(0)}n_{i(1)}}{n_i} + \frac{Z_{i,\frac{\zeta}{2}}^2}{4}}}{n_i + Z_{i,\frac{\zeta}{2}}^2} \right\}^2} \right]$$

$$u_{\omega_i^{(3)}} = \ln \left[\hat{\omega}_i^{(3)} + \sqrt{\exp \left\{ \hat{\sigma}_i^4 \left(\frac{n_{i(1)} - 1}{\chi_{\zeta/2, n_{i(1)}-1}^2} - 1 \right)^2 \right\} + \left\{ \frac{Z_{i,1-\frac{\zeta}{2}} \sqrt{\frac{n_{i(0)}n_{i(1)}}{n_i} + \frac{Z_{i,\frac{\zeta}{2}}^2}{4}}}{n_i + Z_{i,\frac{\zeta}{2}}^2} \right\}^2} \right]$$

140 As mentioned above, the $(100 - \zeta)\%$ MOVER interval for $\ln \omega_i$ can be written as

$$CI_{\ln \omega_i} = [L_{\ln \omega_i}, U_{\ln \omega_i}] \tag{14}$$

141 where

142 $$L_{\ln \omega_i} = \left(\hat{\omega}_i^{(1)} + \hat{\omega}_i^{(2)} + \hat{\omega}_i^{(3)} \right) - \sqrt{ \left(\hat{\omega}_i^{(1)} - l_{\omega_i^{(1)}} \right)^2 + \left(\hat{\omega}_i^{(2)} - l_{\omega_i^{(2)}} \right)^2 + \left(\hat{\omega}_i^{(3)} - l_{\omega_i^{(3)}} \right)^2 }$$

143 $$U_{\ln \omega_i} = \left(\hat{\omega}_i^{(1)} + \hat{\omega}_i^{(2)} + \hat{\omega}_i^{(3)} \right) + \sqrt{ \left(\hat{\omega}_i^{(1)} - u_{\omega_i^{(1)}} \right)^2 + \left(\hat{\omega}_i^{(2)} - u_{\omega_i^{(2)}} \right)^2 + \left(\hat{\omega}_i^{(3)} - u_{\omega_i^{(3)}} \right)^2 }$$

144 Therefore, the $(100 - \zeta)\%$ MOVER interval for θ is based on Donner and Zou (2012), given by

$$CI_\theta = [L_\theta, U_\theta] \tag{15}$$

145 where

146 $$L_\theta = (\ln \hat{\omega}_1 - \ln \hat{\omega}_2) - \sqrt{ \left(\ln \hat{\omega}_1 - L_{\ln \omega_1} \right)^2 + \left(U_{\ln \omega_2} - \ln \hat{\omega}_2 \right)^2 }$$

147 $$U_\theta = (\ln \hat{\omega}_1 - \ln \hat{\omega}_2) + \sqrt{ \left(U_{\ln \omega_1} - \ln \hat{\omega}_1 \right)^2 + \left(\ln \hat{\omega}_2 - L_{\ln \omega_2} \right)^2 }$$

148 The following detail is used to compute the MOVER interval.

Algorithm 3:
150 *1).* Generate $Z_i \sim N(0,1)$, $T_i \sim N(0,1)$ and $U_i \sim \chi^2_{n_{i(1)}-1}$ are independent.
151 *2).* Compute $CI_{\omega_i^{(1)}}$, $CI_{\omega_i^{(2)}}$ and $CI_{\omega_i^{(3)}}$.
152 *3).* Compute $CI_{\ln \omega_i}$ and CI_θ.
153 *4).* Compute $(100 - \zeta)\%$MOVER for θ.

HPD credible interval
155 The HPD credible interval is a parameter estimate based on the posterior probability in Bayesian frame-
156 work. Box and Tiao (1973) defined the HPD ideas consisting of two requirements: the set of value that
157 contain $100(1 - \zeta)\%$ of the posterior distribution and the property that the density within the region is
158 equal or greater than outside. The posterior density is unimodal and symmetric, the region based on their
159 definition becomes to a equal-sided CI ($\zeta/2$ and $1 - \zeta/2$ percentiles of the posterior density). This is
160 clearly different between equal-sided CI and HPD credible interval if there is a highly skewed in the
161 posterior density. Recently, HPDs based on beta and uniform priors were recommended to constructed for
162 single mean and the difference between two delta-lognormal means proposed by Maneerat et al. (2019).
163 The HPD credible interval is then focused on our study. Recall that $X_{ij} > 0$ be random variables from
164 lognormal distribution with parameter $\kappa = \begin{pmatrix} \mu_1 & \sigma_1^2 & \mu_2 & \sigma_2^2 \end{pmatrix}'$; $Y_{ij} = \ln X_{ij} \sim N(\mu_i, \sigma_i^2)$. The Fisher
165 information matrix of κ is $I(\kappa) = diag \left[\frac{n_{1(1)}}{\sigma_1^2} \quad \frac{n_{1(1)}}{2\sigma_1^4} \quad \frac{n_{2(1)}}{\sigma_2^2} \quad \frac{n_{2(1)}}{2\sigma_2^4} \right]$. For $X_{ij} = 0$, the number of zero values
166 $n_{i(0)}$ be random sample of binomial distribution, denoted as $B(n_i, \delta_i)$. The likelihood function of X_{ij} is

$$P(x|\omega) \propto \prod_{i=1}^{2} \delta_i^{n_{i(0)}} (1 - \delta_i)^{n_{i(1)}} (\sigma_i^2)^{-n_{i(1)}/2} \exp \left\{ -\frac{1}{2\sigma_i^2} \sum_{j=1}^{n_{i(1)}} (\ln x_i - \mu_i)^2 \right\} \tag{16}$$

167 which leads to obtain the Fisher information matrix of $\omega = \begin{pmatrix} \mu_1 & \sigma_1^2 & \delta_1 & \mu_2 & \sigma_2^2 & \delta_2 \end{pmatrix}'$

$$I(\omega) = diag \left[\frac{n_{1(1)}}{\sigma_1^2} \quad \frac{n_{1(1)}}{2\sigma_1^4} \quad \frac{n_1}{\delta_1(1-\delta_1)} \quad \frac{n_{2(1)}}{\sigma_2^2} \quad \frac{n_{2(1)}}{2\sigma_2^4} \quad \frac{n_2}{\delta_2(1-\delta_2)} \right]$$

168 Here $\theta = \ln \omega_1 - \ln \omega_2$ so that the HPD credible intervals based on different priors for θ are described.

Jeffreys' prior

The Jeffreys' prior for ω is defined as

$$P(\omega)_J \propto \prod_{i=1}^{2} P(\kappa)_J P(\delta) \propto \prod_{i=1}^{2} \sigma_i^{-2} [\delta_i(1-\delta_i)]^{-1/2} \tag{17}$$

where $P(\kappa)_J \propto \sqrt{I(\sigma^2)}$ and $P(\delta)_J \propto \sqrt{I(\delta)}$. The prior (17) is combined with its likelihood (16) to obtain the posterior distributions of each parameters σ_i^2, μ_i, and δ_i. Firstly, the posterior density of ω is using Bayes' theorem (Casella and Berger, 2002), given by

$$P(\omega|x)_J \propto \prod_{i=1}^{2} \delta_i^{n_{i(0)}-1/2}(1-\delta_i)^{n_{i(1)}-1/2}\sigma_i^{-(n_{i(1)}+2)} \exp\left\{-\frac{1}{2\sigma_i^2}\left[(n_{i(1)}-1)\hat{\sigma}_i^2 + n_{i(1)}(\hat{\mu}_i-\mu_i)^2\right]\right\} \tag{18}$$

Obtain that

$$P(\sigma_i^2|x)_J \propto \prod_{i=1}^{2} (\sigma_i^2)^{-\left(\frac{n_{i(1)}-1}{2}+1\right)} \exp\left\{-\frac{(n_{i(1)}-1)\hat{\sigma}_i^2}{2\sigma_i^2}\right\} \tag{19}$$

which is the posterior density of $\sigma^2|x$, as inverted gamma distribution, denoted as $IG(a_i,b_i)$; $a_i = r_i/2$ and $b_i = r_i\hat{\sigma}_i^2/2$; $r_i = n_{i(1)} - 1$. Given σ_i^2 and x, one obtains that

$$P(\mu_i|\sigma_i^2,x)_J \propto \prod_{i=1}^{2} \exp\left\{-\frac{n_{i(1)}}{2\sigma_i^2}(\mu_i-\hat{\mu}_i)^2\right\} \tag{20}$$

This is the posterior density of $\mu_i|\sigma_i^2,x$, as normal $N(\hat{\mu}_i, \sigma_i^2/n_{i(1)})$. For δ_i, its posterior was proved as

$$p(\delta_i|x)_J \propto \delta_i^{(n_{i(0)}+1/2)-1}(1-\delta_i)^{(n_{i(1)}+1/2)-1}$$

which is the beta distribution with c_i and d_i, denoted as $beta(c_i,d_i)$; $c_i = n_{i(0)}+1/2$ and $d_i = n_{i(1)}+1/2$.

Jeffreys' Rule prior

According to Harvey and van der Merwe (2012), the difference between Jeffreys and Jeffreys' Rule priors is the posterior densities of σ_i^2 and δ_i, while Jeffreys' Rule prior is defined as

$$P(\omega)_{JR} \propto \prod_{i=1}^{2} \sigma_i^{-3}\delta_i^{-1/2}(1-\delta_i)^{1/2} \tag{21}$$

To obtain the joint posterior of ω denoted as $P(\omega|x)_{JR}$, the Jeffreys' Rule prior (21) is combined with the likelihood (16). For σ_i^2, its posterior has been changed to inverted gamma distribution with shape parameter $s_i/2$ and scale parameter $s_i\hat{\sigma}_i^2/2$; $s_i = n_{i(1)}+1$. Then, the posterior distribution of μ_i given σ_i^2 and x is changed because it depends on the posterior density of σ_i^2. Likewise, the posterior of δ_i becomes $P(\delta|x)_{JR} = beta(n_{i(0)}+1/2, n_{i(1)}+3/2)$. Their posterior distributions represent to estimate own parameter, meanwhile $P(\sigma_i^2|x_{ij})_{JR}$, $P(\mu_i|\sigma_i^2,x_{ij})_{JR}$ and $P(\delta_i|x_{ij})_{JR}$ are independent. As a result, HPD credible interval for θ is computed based on Jeffreys' Rule prior. The steps for establishing HPD-intervals based on two priors are detailed as Algorithm 4.

Normal-gamma prior

DeGroot (1970) defined the conjugate families for random sample of normal distribution. This is a necessary to develop credible interval based on Bayesian approach for the delta-lognormal variance. Using Theorem 1 of the conjugate families for random sample of normal distribution (DeGroot, 1970), assume that $(\mathbf{Y} = \ln X_{ij}; i = 1,2\ j = 1,2,...,n_{i(1)})$ be a random sample from normal distribution with the mean $\mu = (\mu_1,\mu_2)$ and precision $\lambda = (\lambda_1,\lambda_2)$; $\lambda_i = 1/\sigma_i^2$ where $X \sim LN(\mu,\lambda)$. The normal-gamma prior

of $\tau = (\mu, \lambda)'$ is defined the marginal distributions between normal $\mu_i | \lambda_i \sim N(\mu, [k_i \lambda_i]^{-1})$ and gamma $\lambda_i \sim G(\alpha_i, \beta_i)$, given by

$$
\begin{aligned}
P(\tau) &= P(\lambda; \alpha_{(0)}, \beta_{(0)}) P(\mu | \lambda; \mu_{(0)}, k_{(0)} \lambda) \\
&= \prod_{i=1}^{2} \left[\frac{\beta_{i(0)}}{\Gamma(\alpha_{i(0)})} \lambda_i^{\alpha_{i(0)} - 1} \exp\{-\beta_{i(0)} \lambda_i\} \right] \left[\frac{(k_{i(0)} \lambda_i)^{1/2}}{\sqrt{2\pi}} \exp\left\{ -\frac{k_i \lambda_i}{2} (\mu_i - \mu_{i(0)})^2 \right\} \right] \\
&\propto \prod_{i=1}^{2} \lambda^{\alpha_{i(0)} - 1/2} \exp\left\{ \frac{-\lambda_i}{2} \left[2\beta_{i(0)} + k_{i(0)} (\mu_i - \mu_{i(0)})^2 \right] \right\}
\end{aligned}
\tag{22}
$$

The likelihood can be written as

$$
\begin{aligned}
P(y; \tau) &= \prod_{i=1}^{2} (2\pi)^{-n_{i(1)}/2} \lambda_i^{n_{i(1)}/2} \exp\left\{ \frac{-\lambda_i}{2} \sum_{i=1}^{n_{i(1)}} (y_{ij} - \mu_i)^2 \right\} \\
&\propto \prod_{i=1}^{2} \lambda_i^{n_{i(1)}/2} \exp\left\{ \frac{-\lambda_i}{2} \left[\sum_{i=1}^{n_{i(1)}} (y_{ij} - \hat{\mu}_i)^2 + n_{i(1)} (\hat{\mu} - \mu_i)^2 \right] \right\}
\end{aligned}
\tag{23}
$$

The posterior distribution of τ can be derived as

$$
\begin{aligned}
P(\tau | y) &= P(y; \tau) P(\tau) \\
&\propto \prod_{i=1}^{2} \lambda_i^{\frac{n_{i(1)}}{2}} \exp\left\{ \frac{-\lambda_i}{2} \left[\sum_{i=1}^{n_{i(1)}} (y_{ij} - \hat{\mu}_i)^2 + n_{i(1)} (\hat{\mu} - \mu_i)^2 \right] \right\} \lambda_i^{\alpha_{i(0)} - \frac{1}{2}} \exp\{-\lambda \beta_{i(0)}\} \\
&\quad \exp\left\{ \frac{-\lambda_i}{2} \left[k_{i(0)} (\mu_i - \mu_{i(0)})^2 \right] \right\} \\
&\propto \prod_{i=1}^{2} \lambda^{\frac{n_{i(1)}}{2} + \alpha_{i(0)} - \frac{1}{2}} \exp\left\{ -\lambda_i \left[\frac{1}{2} \sum_{i=1}^{n_{i(1)}} (y_{ij} - \hat{\mu}_i)^2 + \beta_{i(0)} \right] \right\} \exp\left\{ \frac{-\lambda_i}{2} \left[n_{i(1)} (\hat{\mu} - \mu_i)^2 + k_{i(0)} (\mu_i - \mu_{i(0)})^2 \right] \right\} \\
&\propto \prod_{i=1}^{2} \lambda_i^{\alpha_i^* - \frac{1}{2}} \exp\left\{ -\lambda_i \left[\frac{1}{2} \sum_{i=1}^{n_{i(1)}} (y_{ij} - \hat{\mu}_i)^2 + \beta_{i(0)} \right] \right\} \\
&\quad \exp\left\{ \frac{-\lambda_i}{2} \left[(n_{i(1)} + k_{i(0)}) (\mu_i - \mu_i^*)^2 + \frac{n_{i(1)} k_{i(0)} [\hat{\mu}_i - \mu_{i(0)}]^2}{n_{i(1)} + k_{i(0)}} \right] \right\} \\
&\propto \prod_{i=1}^{2} \lambda_i^{\alpha_i^* - 1} \exp\left\{ -\lambda_i \left[\beta_{i(0)} + \frac{1}{2} \sum_{i=1}^{n_{i(1)}} (y_{ij} - \hat{\mu}_i)^2 + \frac{n_{i(1)} k_{i(0)} [\hat{\mu}_i - \mu_{i(0)}]^2}{2(n_{i(1)} + k_{i(0)})} \right] \right\} \\
&\quad \lambda_i^{\frac{1}{2}} \exp\left\{ \frac{-\lambda_i [n_{i(1)} + k_{i(0)}]}{2} (\mu_i - \mu_i^*)^2 \right\} \\
&\propto \prod_{i=1}^{2} \lambda_i^{\alpha_i^* - 1} \exp\{\lambda_i \beta_i^*\} \lambda_i^{\frac{1}{2}} \exp\left\{ \frac{-\lambda_i k_i^*}{2} (\mu_i - \mu_i^*)^2 \right\}
\end{aligned}
\tag{24}
$$

This implied that $\tau_i | y \sim Normal(\mu_i | \mu_i^*, [k_i^* \lambda]^{-1}) Gamma(\lambda_i | \alpha_i^*, \beta_i^*)$ where $\alpha_i^* = \alpha_{i(0)} + n_{i(1)}/2$, $\beta_i^* = \beta_{i(0)} + \frac{1}{2} \sum_{i=1}^{n_{i(1)}} (y_{ij} - \hat{\mu}_i)^2 + \frac{n_{i(1)} k_{i(0)} [\hat{\mu}_i - \mu_{i(0)}]^2}{2(n_{i(1)} + k_{i(0)})}$, $k_i^* = n_{i(1)} + k_{i(0)}$, and $\mu_i^* = \frac{\mu_{i(0)} k_{i(0)} + n_{i(1)} \hat{\mu}_i}{n_{i(1)} + k_{i(0)}}$. From (22), the normal-gamma prior of τ is defined as

$$
P(\tau) \propto \prod_{i=1}^{2} \lambda_i^{-1}
\tag{25}
$$

202 which is normal-gamma distribution, denoted as $NG(\mu_i, \lambda_i | \mu, k_{i(0)} = 0, \alpha_{i(0)} = -1/2, \beta_{i(0)} = 0$ so that its
203 posterior of τ is derived from (24) that becomes

$$P(\tau|y) \propto \prod_{i=1}^{2} \lambda_i^{\frac{n_{i(1)}-1}{2}-1} \exp\left\{-\frac{\lambda_i}{2}\sum_{j=1}^{n_{i(1)}}(y_{ij}-\hat{\mu}_i)^2\right\} \lambda_i^{\frac{1}{2}} \exp\left\{-\frac{n_{i(1)}\lambda_i}{2}(\mu_i-\mu_i^*)^2\right\} \tag{26}$$

204 This is $NG(\mu_{i,n_{i(1)}}, k_{i,n_{i(1)}}, \alpha_{i,n_{i(1)}}, \beta_{i,n_{i(1)}})$; $\mu_{i,n_{i(1)}} = \hat{\mu}_i$, $k_{i,n_{i(1)}} = n_{i(1)}$, $\alpha_{i,n_{i(1)}} = (n_{i(1)}-1)/2$ and $\beta_{i,n_{i(1)}} =$
205 $\frac{1}{2}\sum_{j=1}^{n_{i(1)}}(y_{ij}-\hat{\mu}_i)^2$. The marginal posterior of λ_i becomes

$$P(\lambda_i|y) \propto \lambda_i^{\alpha_{i,n_{i(1)}}-\frac{1}{2}} \exp\left\{-\lambda_i\beta_{i,n_{i(1)}}\right\} \int \exp\left\{-\frac{\lambda_i k_{i,n_{i(1)}}}{2}(\mu_i-\mu_{i,n_{i(1)}}^*)^2\right\} d\mu_i$$

$$\propto \lambda_i^{\alpha_{i,n_{i(1)}}-\frac{1}{2}} \exp\left\{-\lambda_i\beta_{i,n_{i(1)}}\right\} \sqrt{\frac{2\pi}{\lambda_i k_{i,n_{i(1)}}}} \tag{27}$$

$$\propto \lambda_i^{\alpha_{i,n_{i(1)}}-1} \exp\left\{-\lambda_i\beta_{i,n_{i(1)}}\right\}$$

206 which is $\lambda_i|y \sim G(\lambda_i|\alpha_{i,n_{i(1)}}, \beta_{i,n_{i(1)}})$. This can be implied that $\sigma_i^2|y \sim IG(\sigma_i^2|\alpha_{i,n_{i(1)}}, \beta_{i,n_{i(1)}})$. Let $\rho_i =$
207 $\beta_{i,n_{i(1)}} + \frac{k_{i,n_{i(1)}}}{2}(\mu-\mu_{i,n_{i(1)}})^2$ and $a_i = \lambda_i\rho_i$, then $d\lambda_i = \frac{1}{\rho_i}da_i$. Also, let $b_i = \alpha_{i,n_{i(1)}} + \frac{1}{2}$. From (26), the
208 marginal posterior of $\mu_i|y$ is also obtained as

$$P(\mu_i|\lambda_i,y)) \propto \int \lambda_i^{(\alpha_{i,n_{i(1)}}+\frac{1}{2})-1} \exp\left\{-\lambda_i\left[\beta_{i,n_{i(1)}} + \frac{k_{i,n_{i(1)}}}{2}(\mu-\mu_{i,n_{i(1)}})^2\right]\right\} d\lambda_i$$

$$\propto \int \lambda_i^{b_i-1} \exp\left\{-\lambda_i\rho_i\right\} d\lambda_i$$

$$\propto \int \left(\frac{a_i}{\rho_i}\right)^{b_i-1} \frac{1}{\rho_i} \exp(-a_i) da_i$$

$$\propto \rho_i^{-b_i} \int a_i^{b_i-1} \exp(-a_i) da_i \tag{28}$$

$$\propto \rho_i^{-b_i}$$

$$\propto \left[\beta_{i,n_{i(1)}} + \frac{k_{i,n_{i(1)}}}{2}(\mu_i-\mu_{i,n_{i(1)}})^2\right]^{-(\alpha_{i,n_{i(1)}}+\frac{1}{2})}$$

$$\propto \left[1 + \frac{1}{2\alpha_{i,n_{i(1)}}} \frac{k_{i,n_{i(1)}}\alpha_{i,n_{i(1)}}}{\beta_{i,n_{i(1)}}}(\mu_i-\mu_{i,n_{i(1)}})^2\right]^{-(\alpha_{i,n_{i(1)}}+\frac{1}{2})}$$

Then, $\mu_i|y \sim t_{df}(\mu_i|\mu_{i,n_{i(1)}}, \beta_{i,n_{i(1)}}/[\alpha_{i,n_{i(1)}}k_{i,n_{i(1)}}])$ where $df = 2\alpha_{i,n_{i(1)}} = n_{i(1)}-1$. For $\delta_{i,n_{i(1)}} = 1-\delta_i$, Jin et al. (2017) have investigated and attempted to find the power-divergence (PD) interval for δ_i estimated by Bayesian credible interval-based beta(d,d) prior. For focusing on this study, the beta(d,d) prior of $\delta_{i,n_{i(1)}}$ is given by

$$P(\delta_{i,n_{i(1)}}) = \frac{\Gamma(2d_i)}{[\Gamma(d_i)]^2} \delta_{i,n_{i(1)}}^{d_i-1}(1-\delta_{i,n_{i(1)}})^{d_i-1} \tag{29}$$

209 where $d_i = (2+z_{\zeta/2}^2)/6$; $z_{\zeta/2}$ be a random sample of standard normal distribution. It is combined with
210 the likelihood function of $\delta_{i,n_{i(1)}}$ such that its posterior of $\delta_{i,n_{i(1)}}$ is then

$$P(\delta_{i,n_{i(1)}}) \propto \frac{\Gamma(2d_i)}{[\Gamma(d_i)]^2} \delta_{i,n_{i(1)}}^{(n_{i(1)}+d_i)-1}(1-\delta_{i,n_{i(1)}})^{(n_{i(0)}+d_i)-1} \tag{30}$$

211 which is beta distribution, denoted as $beta(n_{i(1)}+d_i, n_{i(0)}+d_i)$. The posterior distributions of μ_i and σ_i^2
212 based on NG prior and $\delta_{i,n_{i(1)}}$ based on Jin et al. (2017) are obtained, then HPD-NG credible interval for
213 θ can be computed in Algorithm 4.

214 ***Algorithm 4:***

215 *1).* Generate σ_i^{2*} denoted as the posterior distribution of σ_i^2 based on priors :

216 - Jeffreys' prior: $\sigma_{i(J)}^{2*} \sim IG(r_i/2, r_i\hat{\sigma}_i^2/2); \ r_i = n_{i(1)} - 1.$
217 - Jeffreys' Rule prior: $\sigma_{i(JR)}^{2*} \sim IG(s_i/2, s_i\hat{\sigma}_i^2/2); \ s_i = n_{i(1)} + 1.$
218 - NG prior: $\sigma_{i(NG)}^{2*} \sim IG(\sigma_i^2|\alpha_{i,n_{i(1)}}, \beta_{i,n_{i(1)}}).$

219 *2).* Given σ_i^{2*} and x, generate μ_i^* depends on the following priors:

220 - Jeffreys' prior: $\mu_{i(J)}^* \sim N(\hat{\mu}_i, \sigma_{i(J)}^{2*}/n_{i(1)}).$
221 - Jeffreys' Rule prior: $\mu_{i(JR)}^* \sim N(\hat{\mu}_i, \sigma_{i(JR)}^{2*}/n_{i(1)}).$
222 - NG prior: $\mu_{i(NG)}^* \sim t_{df}(\mu_i|\mu_{i,n_{i(1)}}, \beta_{i,n_{i(1)}}/[\alpha_{i,n_{i(1)}}k_{i,n_{i(1)}}]).$

223 *3).* Generate δ_i^*,

224 - Jeffreys' prior: $\delta_{i(J)}^* \sim beta(n_{i(0)} + 1/2, n_{i(1)} + 1/2).$
225 - Jeffreys' Rule prior: $\delta_{i(JR)}^* \sim beta(n_{i(0)} + 1/2, n_{i(1)} + 3/2).$
226 - NG prior: $\delta_{i(NG),n_{i(1)}}^* \sim beta(n_{i(1)} + d_i, n_{i(0)} + d_i).$

227 *4).* Compute ω_i^*,

228 - Jeffreys' prior: $\omega_{i(J)}^* = (1 - \delta_{i(J)}^*)\exp\left(2\mu_{i(J)}^* + \sigma_{i(J)}^{2*}\right)\left[\exp(\sigma_{i(J)}^{2*}) - (1 - \delta_{i(J)}^*)\right].$
229 - Jeffreys' Rule prior: $\omega_{i(JR)}^* = (1 - \delta_{i(JR)}^*)\exp\left(2\mu_{i(JR)}^* + \sigma_{i(JR)}^{2*}\right)\left[\exp(\sigma_{i(JR)}^{2*}) - (1 - \delta_{i(JR)}^*)\right].$
230 - NG prior: $\omega_{i(NG)}^* = \delta_{i(NG),n_{i(1)}}^*\exp\left(2\mu_{i(NG)}^* + \sigma_{i(NG)}^{2*}\right)\left[\exp(\sigma_{i(NG)}^{2*}) - \delta_{i(NG),n_{i(1)}}^*\right].$

231 *5).* Compute $\theta^* = \ln\omega_1^* - \ln\omega_2^*$ based on three priors.
232 *6).* Repeat 1-5 a number of times (say, $m = 2,500$).
233 *7).* For the 2500 times, compute $(100 - \zeta)\%$HPD interval for θ in each prior.

RESULTS

235 The CIs proposed in this study are HPD-NG and MOVER. The former develops the NG prior to obtain
236 the posterior density, while the latter is an extension of Hasan and Krishnamoorthy's MOVER (Hasan and
237 Krishnamoorthy, 2018). Both were compared with the existing CIs HPD-Jef and HPD-Rul of Harvey and
238 van der Merwe (2012), GCI of Wu and Hsieh (2014), and FGCI of Hasan and Krishnamoorthy (2018).
239 Two simulation studies were conducted to show the aforementioned CI performances under equal and
240 unequal sample sizes in different situations:

241 S1). The probability of additional zero δ_i is varied while $\mu_i = 3$ and $\sigma_i^2 = 1$.
242 S2). The mean μ_i and δ_i are varied while $\sigma_i^2 = 1$.

243   S1 was designed and simulated to be consistent with the weekly rainfall datasets, as can be seen in the
244 next section. The second one was constructed to indicate CI performance when both the mean μ_i and δ_i
245 are changed, i.e. whether the numerical simulation is consistent with S1.
246   The coverage probability (CP) and relative average length (RAL, denoted as the ratio of the average
247 lengths of a proposed CI to HPD-Rul) were used to assess the performances of the CIs using Monte Carlo
248 simulations. For 5,000 simulation runs, GPQs and FGPQs were fixed at 2,500 at a nominal level of 0.95.
249 In a comparison of the methods, the following criteria were used to judge the best-performing CI: a CP
250 closer to or greater than the nominal level and the narrowest RAL of less than 1 and minimal. The steps
251 of the simulation procedure are given in Algorithm 5.

Algorithm 5:

1). Generate $X_{ij} \sim \Delta(\mu_i, \sigma_i^2, \delta_i)$; $i = 1,2$ and $j = 1,2,...,n_i$.

2). Compute $n_{i(0)}$, $n_{i(1)}$, $\hat{\mu}_i$, $\hat{\sigma}_i^2$ and $\hat{\delta}_i$.

3). Construct CIs based on the methods as follows:

- GCI, FGCI and MOVER from Algorithm 1,2 and 3, respectively.
- HPD-Jef, HPD-Rul and HPD-NG from Algorithm 4.

4). Repeat 1-3, a number of times, (say, M=5000). All CIs are obtained.

5). Compute CPs and RALs with all CIs.

The findings are based on the simulation work as follows. For the ratio of variances, the numerical evaluation shows that for the method in the S1 scenario, MOVER provided good performance for a small difference between δ_i with equal small sample sizes (Table 1). Importantly, HPD-NG had good coverage (a CP greater than 0.95) and the shortest CIs for both equal and unequal medium sample sizes as well as a large difference in δ_i for unequal large sample sizes. Meanwhile, HPD-Rul's performance satisfied the criteria for a small difference in δ_i and unequal large sample sizes. The results in Table 2 for case S2 indicate that for a small difference in δ_i, MOVER could meet the requirements for equal small sample sizes, while HPD-NG maintained the given target for both equal and unequal medium sample sizes, and unequal large sample sizes. Moreover, HPD-Rul performed well for a large difference in δ_i and unequal medium-to-large sample sizes. From the evidence of both situations, HPD-Jef gave the best CP in all cases even though its average lengths were mostly wider than HPD-NG. Meanwhile, GCI and FGCI performances provided average lengths that were broader than other methods in all situations.

An empirical application

Duangdai and Likasiri (2017) predicted the global temperature, and forest and seasonal rainfall amount in northern Thailand by various mathematical models. Their results indicate that during 1973–2008, the rainfall fluctuation during the rainy season was less than the summer rainfall, although the forest cover was higher in the summer than in the rainy season. Approximately 60 % of the total arable land in the northeast has been turned over to rice cultivation (Nicely and Counselor, 2018). Importantly, both the northern and northeastern regions of Thailand are important agricultural areas where the planted rice is rain-fed and is the most valuable crop in the Lower Mekong Basin (Zhang et al., 2014). These findings led us to focus on the rainfall amounts in the northern and northeastern areas in Thailand, especially regarding rainfall variation in the rainy season. Importantly, a comparison of the rainfall in the northern and northeastern regions in terms of the ratio of variances was investigated and estimated with our proposed CIs. There were 272 substations in total for both regions to record rainfall measurements by the Thai Meteorological Center. The 272 rainfall observed values contained 40 of 62 (64.52%) and 145 of 210 (69.05%) positive records in the north (62 substations) and northeast (210 substations) areas during 2-8 July 2018, respectively. The remainder were zero observations for both sets.

To examine for normality, a Q-Q plot of the log-transformed positive rainfall amounts for the two areas are plotted in Figure 1. Histograms also confirmed the fitted distributions of the northern and northeastern region rainfall records, as shown in Figure 2. Moreover, Table 3 reports the AIC results to check the fitted distribution of the positive rainfall observations for both areas. The results show that the positive rainfall for both had lognormal distributions, while the fact that the records contained zero values implies that both sets fit delta-lognormal distributions. The approximation of rainfall dispersion ratio between north and northeast areas is $\theta = -1.674$ [ratio $= \exp(-1.674) = 0.187$] where the variance $(\omega_1, \omega_2) = (448.34, 2390.93)$; the basic statistics are $(n_1, n_2) = (62, 210)$; $(\hat{\mu}_1, \hat{\mu}_2) = (2.541, 2.576)$; $(\hat{\sigma}_1^2, \hat{\sigma}_2^2) = (0.886, 1.576)$ and $(\hat{\delta}_1, \hat{\delta}_2) = (0.355, 0.309)$. From the results, the 95%CIs for $\exp(\theta)$ given in Table 4 indicate that HPD-NG and MOVER were better for situation S1 [$(\mu_1, \mu_2) = (3,3)$; $(\sigma_1^2, \sigma_2^2) = (1,1)$ and $(\delta_1, \delta_2) = (0.3, 0.3)$].

As mentioned previously, the results can be interpreted as the rainfall variability in the northern region being less than the northeastern one during 2-8 July 2018, which implies that growing rice in the northeastern region was probably more affected than in the northern region because the former's rain variation was larger. It is possible that this information could influence approximately 60 % of the total arable area in the northeast. Note that Nicely and Counselor (2018) estimated 70 % of rice production during the marketing year 2018-2019 cultivated under desirable weather conditions. This analysis might

Table 1. CP and RAL performances of 95%CIs for θ: $(\mu_1,\mu_2)=(3,3)$; $(\sigma_1^2,\sigma_2^2)=(1,1)$

(n_1,n_2)	(δ_1,δ_2)	CP						RAL					
		HPD-Jef	HPD-Rul	HPD-NG	GCI	FGCI	MOVER	HPD-Jef	HPD-Rul	HPD-NG	GCI	FGCI	MOVER
(15,15)	(0.1,0.1)	99.8	99.7	99.8	99.7	99.6	94.4	1.095	[*]	1.152	1.109	1.108	**0.796**
	(0.1,0.2)	99.8	99.7	99.8	99.5	99.5	94.7	1.104	[*]	1.179	1.122	1.120	**0.819**
	(0.1,0.3)	99.8	99.4	99.8	99.3	99.3	94.0	1.115	[*]	1.206	1.135	1.134	**0.847**
	(0.1,0.4)	99.8	99.4	99.9	99.4	99.4	94.3	1.137	[*]	1.273	1.166	1.166	**0.934**
	(0.1,0.5)	99.9	99.6	99.9	99.6	99.6	95.5	1.160	**[*]**	1.344	1.201	1.200	1.045
	(0.2,0.1)	99.8	99.6	99.8	99.5	99.5	93.5	1.105	**[*]**	1.179	1.121	1.120	-
	(0.2,0.2)	99.9	99.6	99.9	99.6	99.6	94.0	1.114	[*]	1.205	1.131	1.130	**0.836**
	(0.2,0.3)	99.7	99.4	99.8	99.3	99.3	94.8	1.122	[*]	1.228	1.139	1.139	**0.856**
	(0.2,0.4)	99.9	99.5	99.9	99.5	99.4	94.3	1.146	[*]	1.296	1.171	1.171	**0.943**
	(0.2,0.5)	99.8	99.3	99.9	99.3	99.4	94.5	1.170	**[*]**	1.373	1.209	1.208	1.072
	(0.3,0.1)	99.8	99.5	99.8	99.4	99.4	94.2	1.114	[*]	1.204	1.133	1.131	**0.845**
	(0.3,0.2)	99.7	99.6	99.8	99.5	99.5	94.5	1.123	[*]	1.230	1.142	1.141	**0.861**
	(0.3,0.3)	99.8	99.2	99.8	99.2	99.2	94.2	1.131	[*]	1.253	1.151	1.149	**0.877**
	(0.3,0.4)	99.8	99.5	99.9	99.3	99.3	95.2	1.153	[*]	1.320	1.179	1.178	**0.967**
	(0.3,0.5)	99.9	99.5	99.9	99.5	99.5	95.8	1.177	**[*]**	1.393	1.210	1.210	1.070
(50,50)	(0.1,0.1)	97.3	97.2	95.5	96.9	97.0	91.2	1.019	[*]	**0.935**	1.027	1.026	-
	(0.1,0.2)	97.6	97.2	95.4	97.2	97.2	90.7	1.021	[*]	**0.936**	1.029	1.028	-
	(0.1,0.3)	96.9	96.5	94.6	96.7	96.5	90.6	1.022	[*]	**0.939**	1.031	1.030	-
	(0.1,0.4)	97.2	96.7	94.8	96.5	96.7	89.9	1.024	[*]	**0.944**	1.034	1.034	-
	(0.1,0.5)	96.8	95.9	94.4	95.8	96.1	89.4	1.029	[*]	**0.955**	1.041	1.041	-
	(0.2,0.1)	97.2	96.6	94.9	96.6	96.6	90.5	1.020	[*]	**0.936**	1.028	1.027	-
	(0.2,0.2)	97.4	96.8	95.1	96.6	96.7	90.4	1.022	[*]	**0.938**	1.029	1.029	-
	(0.2,0.3)	96.4	96.1	94.1	95.8	95.9	89.2	1.023	[*]	**0.941**	1.032	1.032	-
	(0.2,0.4)	96.8	96.3	94.4	96.1	96.0	90.2	1.026	[*]	**0.946**	1.035	1.035	-
	(0.2,0.5)	96.8	96.5	95.1	96.4	96.5	90.1	1.030	[*]	**0.956**	1.041	1.041	-
	(0.3,0.1)	97.4	96.9	95.4	96.8	96.9	90.6	1.022	[*]	**0.940**	1.031	1.030	-
	(0.3,0.2)	96.9	96.5	94.5	96.5	96.3	88.5	1.024	[*]	**0.942**	1.032	1.032	-
	(0.3,0.3)	96.6	96.0	94.1	95.8	95.9	89.0	1.025	[*]	**0.945**	1.034	1.034	-
	(0.3,0.4)	97.1	96.6	95.2	96.8	96.7	89.9	1.027	[*]	**0.950**	1.036	1.035	-
	(0.3,0.5)	96.9	96.4	94.7	96.1	96.1	89.7	1.032	[*]	**0.960**	1.042	1.042	-
(30,50)	(0.1,0.1)	98.1	97.8	96.8	97.7	97.8	90.8	1.030	[*]	**0.964**	1.042	1.042	-
	(0.1,0.2)	98.3	97.9	96.7	97.8	97.7	90.6	1.030	[*]	**0.966**	1.043	1.042	-
	(0.1,0.3)	97.8	97.7	96.2	97.1	97.1	89.3	1.032	[*]	**0.968**	1.044	1.042	-
	(0.1,0.4)	97.7	97.4	96.0	97.1	97.1	90.1	1.033	[*]	**0.972**	1.043	1.042	-
	(0.1,0.5)	97.6	97.2	96.3	97.0	97.1	89.7	1.036	[*]	**0.981**	1.046	1.046	-
	(0.2,0.1)	98.0	97.7	96.6	97.4	97.5	90.9	1.033	[*]	**0.971**	1.047	1.046	-
	(0.2,0.2)	97.8	97.4	96.3	97.3	97.3	90.2	1.034	[*]	**0.972**	1.047	1.046	-
	(0.2,0.3)	97.9	97.4	96.4	97.2	97.2	89.9	1.034	[*]	**0.974**	1.046	1.046	-
	(0.2,0.4)	98.1	97.6	96.4	97.3	97.3	89.6	1.035	[*]	**0.978**	1.047	1.046	-
	(0.2,0.5)	97.6	97.5	96.2	97.0	96.9	89.5	1.039	[*]	**0.987**	1.050	1.049	-
	(0.3,0.1)	98.1	97.6	96.4	97.3	97.3	89.8	1.036	[*]	**0.981**	1.054	1.053	-
	(0.3,0.2)	97.7	97.1	96.0	96.7	96.9	89.6	1.037	[*]	**0.981**	1.053	1.053	-
	(0.3,0.3)	97.6	97.1	95.6	96.7	96.8	89.6	1.038	[*]	**0.984**	1.053	1.052	-
	(0.3,0.4)	97.4	97.0	95.8	96.7	96.7	88.8	1.039	[*]	**0.988**	1.053	1.052	-
	(0.3,0.5)	97.7	97.3	96.2	97.1	97.0	89.6	1.043	[*]	**0.997**	1.056	1.055	-
(50,100)	(0.1,0.1)	96.7	96.4	93.9	96.2	96.3	91.7	1.016	[*]	0.922	1.026	1.026	-
	(0.1,0.2)	96.6	96.2	94.0	96.0	96.2	91.4	1.016	[*]	0.921	1.025	1.025	-
	(0.1,0.3)	96.7	96.2	93.8	96.1	96.1	91.1	1.016	[*]	0.921	1.025	1.025	-
	(0.1,0.4)	96.7	96.6	94.2	96.3	96.4	91.3	1.016	[*]	**0.921**	1.025	1.025	-
	(0.1,0.5)	96.8	96.5	94.5	96.3	96.3	90.9	1.019	[*]	**0.923**	1.026	1.025	-
	(0.2,0.1)	96.7	96.4	94.1	96.1	96.3	91.1	1.017	[*]	**0.923**	1.028	1.028	-
	(0.2,0.2)	96.7	96.3	93.7	95.9	96.0	90.4	1.017	[*]	0.923	1.029	1.027	-
	(0.2,0.3)	96.4	96.1	93.9	96.0	95.9	90.5	1.017	**[*]**	0.922	1.028	1.027	-
	(0.2,0.4)	96.8	96.6	93.7	96.4	96.4	90.7	1.018	**[*]**	0.923	1.028	1.027	-
	(0.2,0.5)	96.4	96.1	93.7	96.1	96.1	90.2	1.018	**[*]**	0.924	1.028	1.027	-
	(0.3,0.1)	96.0	96.0	93.0	95.5	95.4	90.9	1.019	**[*]**	0.926	1.033	1.033	-
	(0.3,0.2)	96.0	95.7	93.0	95.3	95.4	90.2	1.019	**[*]**	0.926	1.032	1.031	-
	(0.3,0.3)	96.4	95.7	93.7	95.8	95.8	90.5	1.019	**[*]**	0.926	1.032	1.031	-
	(0.3,0.4)	96.4	95.7	93.4	95.9	95.9	90.6	1.020	**[*]**	0.926	1.031	1.031	-
	(0.3,0.5)	96.7	96.3	94.2	96.2	96.3	90.7	1.021	**[*]**	0.928	1.030	1.031	-

Remark: [*] stands for HPD-Rul satisfies the criteria.

Table 2. CP and RAL performances of 95%CIs for θ: $(\sigma_1^2, \sigma_2^2) = (1,1)$

(n_1,n_2)	(δ_1,δ_2)	(μ_1,μ_2)	CP						RAL					
			HPD-Jef	HPD-Rul	HPD-NG	GCI	FGCI	MOVER	HPD-Jef	HPD-Rul	HPD-NG	GCI	FGCI	MOVER
(15,15)	(0.1,0.1)	(0,0)	99.8	99.5	99.8	99.5	99.5	93.7	1.096	[*]	1.154	1.112	1.110	0.797
		(0,0.3)	99.8	99.4	99.6	99.4	99.3	93.7	1.096	[*]	1.155	1.111	1.108	0.798
		(0,0.5)	99.9	99.6	99.9	99.7	99.7	93.9	1.095	[*]	1.153	1.111	1.108	0.796
		(0,0.7)	99.7	99.5	99.7	99.6	99.6	94.4	1.095	[*]	1.153	1.111	1.109	0.798
		(0,0.9)	99.8	99.6	99.7	99.4	99.5	93.8	1.096	[*]	1.154	1.111	1.110	0.797
	(0.2,0.2)	(0,0)	99.7	99.4	99.8	99.4	99.4	93.9	1.114	[*]	1.205	1.132	1.130	**0.839**
		(0,0.3)	99.9	99.6	99.9	99.6	99.7	94.6	1.113	[*]	1.205	1.130	1.129	**0.835**
		(0,0.5)	99.9	99.5	99.8	99.5	99.6	94.5	1.115	[*]	1.206	1.131	1.129	**0.835**
		(0,0.7)	99.8	99.7	99.9	99.6	99.5	94.6	1.114	[*]	1.205	1.132	1.130	**0.838**
		(0,0.9)	99.8	99.6	99.9	99.5	99.6	94.5	1.114	[*]	1.205	1.131	1.130	**0.836**
	(0.4,0.4)	(0,0)	99.7	99.2	99.9	99.2	99.3	94.6	1.174	[*]	1.382	1.200	1.199	1.038
		(0,0.3)	99.8	99.4	99.9	99.2	99.2	94.7	1.173	[*]	1.380	1.201	1.199	1.033
		(0,0.5)	99.9	99.5	99.9	99.5	99.5	95.2	1.175	[*]	1.384	1.202	1.200	1.033
		(0,0.7)	99.9	99.4	99.9	99.5	99.5	95.1	1.174	[*]	1.383	1.198	1.199	1.029
		(0,0.9)	99.9	99.5	99.9	99.5	99.5	95.5	1.174	[*]	1.383	1.201	1.199	1.038
(30,30)	(0.1,0.1)	(0,0)	98.8	98.3	97.7	98.3	98.4	90.8	1.037	[*]	**0.986**	1.046	1.044	-
		(0,0.3)	98.8	98.4	97.7	98.3	98.2	90.8	1.037	[*]	**0.986**	1.047	1.045	-
		(0,0.5)	98.9	98.6	97.9	98.3	98.4	91.0	1.037	[*]	**0.986**	1.047	1.045	-
		(0,0.7)	98.7	98.2	97.7	98.1	98.0	90.5	1.037	[*]	**0.987**	1.047	1.045	-
		(0,0.9)	98.8	98.4	98.0	98.5	98.4	91.6	1.036	[*]	**0.986**	1.046	1.045	-
	(0.2,0.2)	(0,0)	98.4	98.2	97.7	98.2	98.2	90.3	1.041	[*]	**0.997**	1.052	1.051	-
		(0,0.3)	98.4	98.2	97.6	98.1	98.0	90.3	1.041	[*]	**0.997**	1.052	1.051	-
		(0,0.5)	98.2	97.8	97.3	97.7	97.6	90.0	1.042	[*]	**0.997**	1.052	1.050	-
		(0,0.7)	98.4	98.0	97.5	97.7	97.8	89.9	1.040	[*]	**0.998**	1.051	1.051	-
		(0,0.9)	98.1	98.0	97.4	97.7	97.8	89.9	1.041	[*]	**0.998**	1.051	1.051	-
	(0.4,0.4)	(0,0)	98.1	97.8	97.3	97.4	97.3	89.6	1.058	[*]	1.042	1.070	1.071	-
		(0,0.3)	98.2	97.8	97.6	97.7	97.7	90.3	1.060	[*]	1.042	1.072	1.072	-
		(0,0.5)	98.2	97.7	97.3	97.4	97.4	89.9	1.058	[*]	1.042	1.072	1.071	-
		(0,0.7)	98.4	97.7	97.4	97.5	97.5	89.5	1.059	[*]	1.041	1.071	1.070	-
		(0,0.9)	98.2	97.7	97.2	97.4	97.3	89.8	1.058	[*]	1.041	1.070	1.071	-
(50,50)	(0.1,0.1)	(0,0)	97.1	96.8	95.1	96.7	96.8	90.1	1.020	[*]	**0.936**	1.028	1.027	-
		(0,0.3)	97.3	96.9	95.3	97.0	96.8	91.0	1.020	[*]	**0.935**	1.028	1.027	-
		(0,0.5)	97.7	97.2	95.3	97.1	97.1	90.5	1.019	[*]	**0.934**	1.027	1.026	-
		(0,0.7)	97.4	97.1	95.2	97.0	96.9	90.9	1.019	[*]	**0.935**	1.027	1.026	-
		(0,0.9)	96.8	96.4	94.8	96.2	96.3	89.9	1.020	[*]	**0.936**	1.028	1.027	-
	(0.2,0.2)	(0,0)	96.4	96.0	94.3	96.1	96.0	89.5	1.021	[*]	**0.938**	1.029	1.028	-
		(0,0.3)	97.1	96.7	94.6	96.5	96.6	89.8	1.022	[*]	**0.938**	1.031	1.030	-
		(0,0.5)	96.8	96.4	94.4	96.2	96.2	89.8	1.022	[*]	**0.938**	1.030	1.029	-
		(0,0.7)	96.9	96.5	94.5	96.3	96.3	90.3	1.023	[*]	**0.938**	1.030	1.029	-
		(0,0.9)	96.8	96.4	94.5	96.2	96.2	89.9	1.022	[*]	**0.937**	1.029	1.029	-
	(0.4,0.4)	(0,0)	96.6	96.3	94.4	96.0	95.9	89.1	1.029	[*]	**0.954**	1.038	1.038	-
		(0,0.3)	96.8	96.4	94.4	96.0	96.1	88.9	1.029	[*]	**0.954**	1.038	1.038	-
		(0,0.5)	96.9	96.4	94.6	95.9	96.2	89.3	1.029	[*]	**0.954**	1.038	1.038	-
		(0,0.7)	96.7	96.0	94.3	95.7	95.8	89.2	1.029	[*]	**0.955**	1.038	1.038	-
		(0,0.9)	97.0	96.3	94.7	96.1	96.1	89.2	1.030	[*]	**0.955**	1.039	1.038	-
(30,50)	(0.1,0.1)	(0,0)	98.4	98.0	96.8	97.8	97.7	90.8	1.029	[*]	**0.965**	1.042	1.041	-
		(0,0.3)	98.3	98.1	97.0	97.8	97.9	90.5	1.029	[*]	**0.965**	1.042	1.041	-
		(0,0.5)	98.5	98.1	97.0	97.7	97.7	91.2	1.030	[*]	**0.965**	1.043	1.041	-
		(0,0.7)	98.2	97.8	96.5	97.5	97.4	89.9	1.029	[*]	**0.964**	1.042	1.040	-
		(0,0.9)	98.1	97.8	96.7	97.7	97.7	90.0	1.030	[*]	**0.964**	1.042	1.041	-
	(0.2,0.2)	(0,0)	98.1	97.7	96.7	97.5	97.5	90.1	1.033	[*]	**0.972**	1.048	1.046	-
		(0,0.3)	97.8	97.5	96.3	97.3	97.3	90.2	1.033	[*]	**0.972**	1.047	1.046	-
		(0,0.5)	98.1	97.8	96.8	97.4	97.4	90.5	1.033	[*]	**0.972**	1.047	1.046	-
		(0,0.7)	98.1	97.6	96.3	97.3	97.3	90.1	1.033	[*]	**0.971**	1.046	1.045	-
		(0,0.9)	97.8	97.2	96.0	97.0	97.0	89.6	1.034	[*]	**0.972**	1.047	1.047	-
	(0.4,0.4)	(0,0)	97.4	97.1	96.3	96.7	96.9	89.6	1.046	[*]	1.003	1.063	1.062	-
		(0,0.3)	97.6	97.3	96.4	96.8	96.8	89.7	1.046	[*]	1.004	1.063	1.062	-
		(0,0.5)	98.2	97.4	96.6	97.2	97.1	89.5	1.046	[*]	1.003	1.062	1.062	-
		(0,0.7)	97.5	97.1	95.9	96.6	96.6	88.7	1.046	[*]	1.004	1.063	1.062	-
		(0,0.9)	97.6	96.9	96.0	96.6	96.7	88.5	1.046	[*]	1.004	1.063	1.063	-
(50,100)	(0.1,0.1)	(0,0)	96.7	96.3	94.1	96.4	96.2	91.6	1.016	[*]	**0.921**	1.026	1.025	-
		(0,0.3)	96.7	96.5	94.3	96.3	96.3	92.3	1.016	[*]	**0.922**	1.026	1.025	-
		(0,0.5)	96.7	96.5	93.9	96.3	96.1	92.0	1.015	[*]	**0.921**	1.026	1.025	-
		(0,0.7)	96.5	96.3	93.8	96.0	96.0	92.0	1.016	[*]	**0.922**	1.026	1.025	-
		(0,0.9)	97.1	96.7	94.2	96.6	96.5	92.1	1.016	[*]	**0.921**	1.025	1.025	-
	(0.2,0.2)	(0,0)	96.0	95.5	93.2	95.5	95.4	90.4	1.017	[*]	0.923	1.029	1.028	-
		(0,0.3)	96.7	96.5	94.0	96.5	96.4	91.3	1.017	[*]	0.922	1.029	1.028	-
		(0,0.5)	96.3	95.9	93.5	95.8	95.7	90.8	1.018	[*]	0.923	1.029	1.028	-
		(0,0.7)	96.6	96.3	93.8	96.1	96.1	91.0	1.016	[*]	0.923	1.028	1.027	-
		(0,0.9)	96.7	96.1	93.7	96.2	96.2	91.2	1.018	[*]	0.923	1.029	1.028	-
	(0.4,0.4)	(0,0)	96.1	95.5	93.2	95.4	95.4	89.5	1.023	[*]	0.932	1.035	1.035	-
		(0,0.3)	96.4	95.8	93.0	95.8	95.7	89.3	1.023	[*]	0.933	1.036	1.036	-
		(0,0.5)	96.3	95.6	93.2	95.7	95.6	89.3	1.023	[*]	0.933	1.036	1.036	-
		(0,0.7)	96.3	96.0	93.6	96.0	95.9	90.1	1.023	[*]	0.932	1.035	1.036	-
		(0,0.9)	96.2	95.8	93.7	95.8	95.9	89.4	1.024	[*]	0.933	1.036	1.037	-

Remark: [*] stands for HPD-Rul satisfies the criteria.
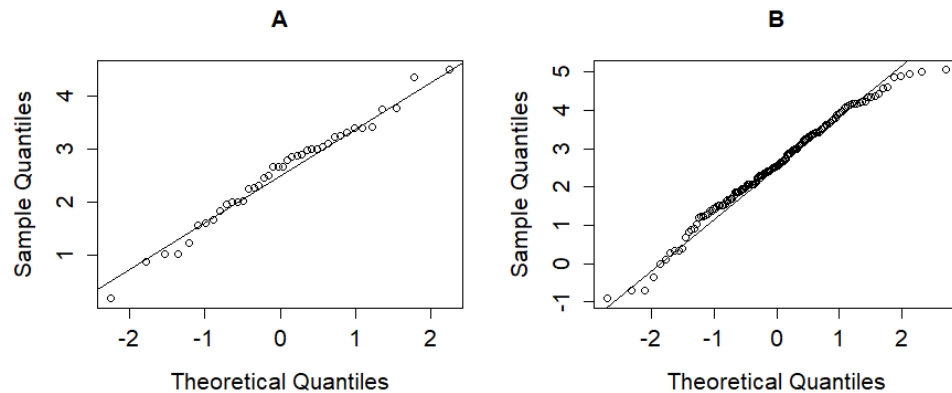
Figure 1. Q-Q plots of log-transformation of non-zero records in (A) northern (B) northeastern areas
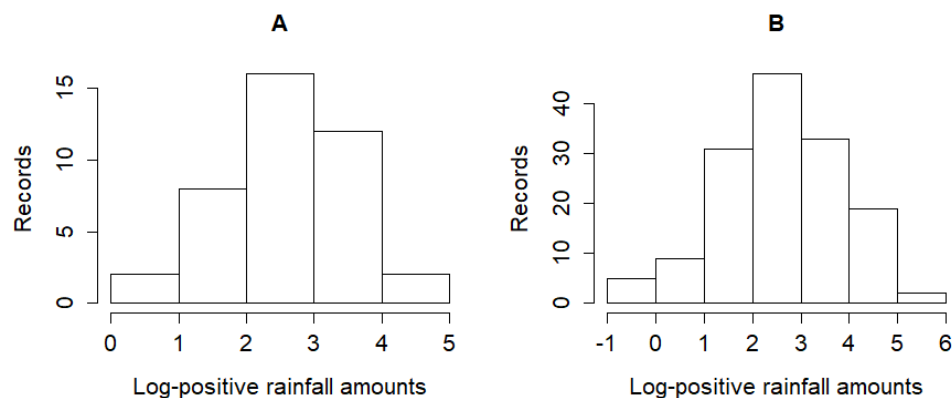
Figure 2. Histogram plots of log-transformation of weekly positive rainfall records in (A) northern (B) northeastern areas

305 provide useful information to the Royal Thai Government to carry out effective water management. Our
306 findings made a few realizations about natural disasters due to climate change during the rainy season
307 in the northern and northeastern Thailand as well. The results are also in agreement with the simulation
308 study ones in Table 1.

DISCUSSION
309

310 The HPD-NG was proposed for constructing confidence intervals for comparing the rainfall dispersion in
311 north and northeast regions. How to select the prior in this situation is that we found the CP performance
312 of HPD depend on the posterior densities of σ^2 and δ in the previous study. To our knowledge, we
313 believed that μ and σ^2 might be suitable for random variables of the normal and gamma distributions,
314 respectively. This becomes the normal-gamma prior of (μ, σ^2). For δ, it was motivated by Jeffreys' prior,
315 while a beta distribution was also developed and recommended by Jin et al. (2017) so that beta (d,d)
316 becomes a prior of δ in this study.

Table 3. Results of AIC for the nonzero rainfall records in north and northeast areas

Regions	AIC						
	Exponential	Weibull	Lognormal	Normal	t-distribution	Cauchy	Logistic
Northern	316.677	317.046	**314.927**	348.812	332.373	336.093	337.678
Northeastern	1232.797	1231.199	**1227.468**	1411.599	1332.576	1331.557	1376.586

Table 4. 95%CIs for the weekly rainfall ratio between north and northeast regions

Methods	95%CIs for $\exp(\theta)$ Lower	Upper	Length
HPD-Jef	0.0455	0.9255	0.8800
HPD-Rul	0.0477	0.9239	0.8762
HPD-NG	0.0546	0.7793	0.7247
GCI	0.0464	0.9860	0.9396
FGCI	0.0652	1.1484	1.0832
MOVER	0.0512	0.5896	0.5384

317   The difference between HPD-Jef and HPD-Rul is the posterior densities of σ^2 and δ, although it can
318 be seen from the results of the simulation study that HPD-Rul's outcomes were agreement with Harvey
319 and van der Merwe (2012) when focusing on the ratio of delta-lognormal means. This implies that both
320 HPD performances are dependent on the posterior of σ^2 and δ. For the proposed HPD-NG, its prior of
321 $\tau = (\mu, \lambda); \lambda = 1/\sigma^2$ is the inverse of the Jeffreys' prior, leading us to obtain the posterior distributions of
322 μ and σ^2 derived from the NG prior under the assumption that the mean μ has a normal distribution and
323 the precision λ has a gamma distribution. After that, the posterior of σ^2 has a inverse-gamma distribution
324 with its parameters $(\alpha_{i,n_{i(1)}}, \beta_{i,n_{i(1)}})$. Importantly, the parameter $\beta_{i,n_{i(1)}}$ is different from the inverse-gamma
325 based one on Jeffreys' prior, resulting in a different posterior of μ as well. Likewise, the prior of δ was
326 found and recommended by Jin et al. (2017). For these reasons, the HPD-NG prior was developed to
327 estimate the ratio between delta-lognormal variances. However, more research on this topic needs to be
328 conduct to find an appropriate prior to obtain a better performance. Furthermore, MOVER could maintain
329 its performance to satisfy the target criteria even with a small difference between the binomial proportions
330 and small sample sizes. It is possible that an interval estimate for δ based on Wilson's interval satisfies
331 the criteria for small to moderate sample sizes, as confirmed by Donner and Zou (2011).

CONCLUSIONS

333 The purpose of this study was to develop CIs, namely HPD-NG and MOVER, for the ratio of delta-
334 lognormal variances through Monte Carlo simulations. By way of comparison, both were examined to
335 report their performance with the existing methods: HPD-Jef, HPD-Rul, GCI and FGCI. These proposed
336 CIs were applied to estimate and compare the rainfall fluctuation between the northern and northeastern
337 regions in Thailand in terms of the ratio of variances.
338   The findings of this study indicate that HPD-NG is the best and most recommended CI in situations
339 S1 (with varied δ) for both equal and unequal medium sample sizes, and for a large difference between
340 the binomial proportions for unequal large sample sizes. Both MOVER and HPD-Rul are the next best
341 CIs recommended for a small difference between the binomial proportions with different sample sizes:
342 MOVER for equal small sample sizes and HPD-Rul for unequal large sample sizes. For situation S2
343 where δ and μ are both varied, HPD-NG delivered the best performance for a small binomial proportion
344 and both equal and unequal medium sample sizes as well as a small proportion of zeros for unequal large
345 sample sizes. MOVER is also recommended for situations similar to situation S1, while HPD-Rul is
346 recommended for a large proportion of zeros and unequal medium-to-large sample sizes. Although the
347 CPs of HPD-Jef, GCI and FGCI results satisfied the criteria, their average lengths were wider than our
348 proposed methods in all situations.

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