Including autapomorphies is important for tip-dating with clocklike data, but not with non-clock data (#13158)

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Including autapomorphies is important for tip-dating with clocklike data, but not with non-clock data

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Tip-dating, where fossils are included as dated terminal taxa in **Bayesian** dating inference, is an increasingly popular method. Data for these studies often come from morphological character matrices originally developed for non-dated, and usually parsimony, analyses. In parsimony, only shared derived characters (synapomorphies) provide grouping information, so many character matrices have an ascertainment bias: they leave out autapomorphies (unique derived character states), which are considered uninformative. There has been no study of the affect of this ascertainment bias in tip-dating, but autapomorphies can be informative in model-based inference. We expected that excluding autapomorphies would shorten the morphological branchlengths of terminal branches, and thus bias downwards the time branchlengths inferred in tip-dating. We tested for this effect using a matrix for Carboniferous-Permian eureptiles where all autapomorphies had been deliberately coded. Surprisingly, date estimates are virtually unchanged when autapomorphies are excluded, although we find large changes in morphological rate estimates and small effects on topological and dating confidence. We hypothesized that the puzzling lack of effect on dating was caused by the non-clock nature of the eureptile data. We confirm this explanation by simulating strict clock and non-clock datasets, showing that autapomorphy exclusion biases dating only for the clocklike case. A theoretical solution to ascertainment bias is computing the ascertainment bias correction $(Mk_{narsinf})$, but we explore this correction in detail, and show that it is computationally impractical for typical datasets with many character states and taxa. Therefore we recommend that palaeontologists collect autapomorphies whenever possible when assembling character matrices.

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| 27 | the eureptile data. We confirm this explanation by simulating strict clock and non-clock |
| 28 | datasets, showing that autapomorphy exclusion biases dating only for the clocklike case. A |
| 29 | theoretical solution to ascertainment bias is computing the ascertainment bias correction |
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| 33 | matrices. |
| 34 | |
| 35 | Keywords |
| 36 | |
| 37 | tip-dating, total-evidence dating, autapomorphies, parsimony, Eureptilia, BEASTmasteR |
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| 40 | |
| 41 | |

42 Introduction

43

| 44 | In parsimony phylogenetic analyses, the only data informative for reconstructing the tree | | | | | |
|-----------------|--|--|--|--|--|--|
| 45 | topology are those with grouping information: potentially shared, derived character states | | | | | |
| 46 | (synapomorphies; (Hennig et al. 1999)). An autapomorphy – a state unique to one Operational | | | | | |
| <mark>47</mark> | Taxonomic Unit (OTU; (Mishler 2005)) contributes one step to any possible topology. | | | | | |
| 48 | Therefore, autapomorphies are routinely excluded from further analysis in cladistics programs | | | | | |
| 49 | (e.g., the TNT <i>xinact</i> and <i>info</i> commands (Goloboff et al. 2008); the PAUP* <i>exclude</i> command | | | | | |
| 50 | (Swofford 2003)), and autapomorphic characters are often not even collected during assembly | | | | | |
| 51 | of a character-taxon matrix. | | | | | |
| 52 | | | | | | |
| 53 | In model-based inference, autapomorphies <i>can</i> be informative (Lewis 2001; Wright & Hillis | | | | | |
| 54 | 2014), because autapomorphies contribute information about the overall rate of change in the | | | | | |
| 55 | character matrix and site-specific rate heterogenetity. In addition, tip-dating analyses might be | | | | | |
| 56 | expected to be particularly sensitive to autapomorphies: all autapomorphies occur on terminal | | | | | |
| 57 | branches by definition, so their exclusion will shorten the morphological branchlengths of | | | | | |
| 58 | terminal branches (and thus presumably their time branchlengths), and perhaps increase | | | | | |
| <mark>59</mark> | estimated branch-wise rate variation. | | | | | |
| 60 | | | | | | |
| 61 | An alternative to inclusion of autapomorphies is ascertainment-bias correction, where the | | | | | |
| 62 | likelihood of unobservable character patterns, L_{unobs} , is calculated, and the likelihood of the | | | | | |
| 63 | observed data is normalized by dividing by 1- L_{unobs} (Felsenstein 1992; Lewis 2001). The two | | | | | |

common corrections are the Markov-*k* model with an ascertainment bias correction for the
unobservability of invariant characters (M*k*-variable-only, or M*k*v; (Lewis 2001)), and Markov-*k*with an ascertainment bias correction for parsimony-uninformative characters, M*k*_{parsinf} (Allman
et al. 2010; Ronquist & Huelsenbeck 2003). These corrections are options in MrBayes and can
be implemented in Beast2 XML, but several studies briefly mention that the scalability and
correctness of M*k*_{parsinf} computations may be problematic (dos Reis et al. 2016; Koch & Holder
2012; Matzke 2016).

71

72 The effect inclusion/exclusion of autapomorphies and ascertainment-bias correction has not 73 been studied in a tip-dating context. Datasets appropriate for doing so are rare because they 74 need to systematically collect all autapomorphies, as well as dates for the OTUs. Müller and Reisz (2006) (Müller & Reisz 2006) constructed an all-fossil, morphological matrix of early 75 76 eureptiles and tested the effect of inclusion/exclusion of autapomorphies in undated Bayesian 77 inference, and recommended including autapomorphies. Lee and Palci (2015) discussed the 78 importance of autapomorphies for tip-dating, but did not conduct a test. We obtained dates for Müller and Reisz's taxa, and use the dataset to test the effects of autapomorphy inclusion. 79 80 Surprisingly, no effect on dates was found. This might be due to the non-clocklike nature of the 81 dataset, an explanation we confirm with a simulation study that shows autapomorphy 82 exclusion biases terminal branchlength estimates when the data are highly clocklike, but not in 83 a non-clock dataset. We also examine the $Mk_{parsinf}$ correction and show that it scales poorly for 84 characters with more than two states, limiting its usability.

85

- 86 Methods 87 Data. The morphological matrix was taken from Müller and Reisz (Müller & Reisz 2006). The 88 date ranges for OTUs were derived from the literature, following best practices guidelines 89 (2012). Correlation between time and morphological branchlengths in a TNT parsimony analysis 90 was used as a rough assessment of clocklike behavior. 91 92 Tip-dating eureptiles. Tip-dating in Beast2 (Bouckaert et al. 2014; Drummond & Bouckaert 93 2015) with Birth-Death-Serial Sampling (BDSS) or SA-BDSS (Sampled Ancestors) tree models 94 (Gavryushkina et al. 2015; Gavryushkina et al. 2014) requires a specialized XML input file. To set 95 this up, we used BEASTmasteR (Alexandrou et al. 2013; Matzke 2015; Matzke & Wright 2016), a 96 set of R functions that convert NEXUS character matrices, an Excel file containing tip date 97 ranges, and other priors and settings, into XML. Three different site models were used: Mk, Mkv, and Mk_{parsinf}. The summary Maximum Clade Credibility (MCC) trees were plotted with 95% 98 99 highest posterior densities (HPDs) on inferred node (blue) and tip dates (red) using 100 **BEASTmasteR** functions and custom R scripts. Mean node dates, node 95% HPD widths, 101 posterior probabilities, and rates were compared between pairs of analyses (with/without 102 autapomorphies) for nodes/bipartitions shared between analyses (n=14), with the Wilcoxon 103 signed-rank test (WSRT) for paired samples. Due to the small number of tests, no multiple-test 104 correction was used. 105 106 Simulation. To test whether clocklike behavior is needed to observe effects of autapomorphy
- 107 exclusion on date estimates, a BDSS tree similar in size to the empirical dataset (30 species) was

| 108 | simulated using TreeSim (Stadler 2015). A "strict clock" dataset of 1000 binary characters was | | | | | | |
|------------------|---|--|--|--|--|--|--|
| 109 | simulated on this tree under the Mk model with a rate low enough that a substantial | | | | | | |
| <mark>110</mark> | proportion of the characters were invariant or autapomorphic. A "non-clock" dataset was | | | | | | |
| 111 | produced by reshuffling the time-branchlengths of the simulated tree, and then simulating | | | | | | |
| 112 | another 1000 characters at the same rate. Datasets were filtered to produce variable-only and | | | | | | |
| 113 | parsimony-informative-only datasets, effectively imposing ascertainment bias. Beast2 runs | | | | | | |
| 114 | were conducted on both simulated datasets under Mk, Mkv, and Mk _{parsinf} using the same setup | | | | | | |
| 115 | as for the empirical analysis. All scripts, Beast2 inputs and outputs, and further details of the | | | | | | |
| 116 | analyses are available in Supplemental Material (SM). | | | | | | |
| 117 | | | | | | | |
| 118 | Scalability of the Mk _{parsinf} correction. Although listed as an option in MrBayes for a-over a | | | | | | |
| 119 | decade, surprisingly, $Mk_{parsinf}$ has not been formally described anywhere in the literature, | | | | | | |
| 120 | leading to widespread lack of knowledge of how it works and whether or not it is | | | | | | |
| 121 | computationally feasible on typical datasets. Nor has there been any formal treatment of its | | | | | | |
| 122 | computational scalability. The key issue is the number of unobservable character patterns for a | | | | | | |
| 123 | character with a particular number of states, as the likelihood of each unobservable pattern | | | | | | |
| 124 | must be calculated. While this is feasible for a binary character (which appears to be the | | | | | | |
| 125 | assumption made by MrBayes), for a dataset with many taxa and multistate characters, the | | | | | | |
| 126 | number of unobservable site patterns rapidly climbs into the millions. Appendix 1 contains a | | | | | | |
| 127 | derivation of the number of likelihood calculations required by $Mk_{parsinf}$, and discussion of | | | | | | |
| 128 | computational scalability. | | | | | | |

129

| 130 | Results | | | | | |
|------------------|---|--|--|--|--|--|
| 131 | <i>Tip-dating eureptiles</i> . Fourteen high-posterior bipartitions were inferred in all analyses. MCC | | | | | |
| 132 | trees for two runs are illustrated in Figure 1; for all runs, see Supplemental Figure 1. Summary | | | | | |
| 133 | statistics of key parameters are shown in Table 1. Linear regression of tip age against the root- | | | | | |
| 134 | to-tip distance in a parsimony analysis (the number of morphological steps on all branches | | | | | |
| 135 | leading to a tip) indicated that time and parsimony branchlengths were not correlated. This is | | | | | |
| <mark>136</mark> | evidence that the morphological characters in the eureptile dataset are not evolving in a | | | | | |
| <mark>137</mark> | clocklike manner. | | | | | |
| 138 | | | | | | |
| 139 | Inferred node dates. Estimates of the root age are almost identical between analyses with and | | | | | |
| 140 | without autapomorphies (Table 1). Comparing mean dates for nodes shared across the MCC | | | | | |
| 141 | trees yields no significant differences (WTST, two-sided, $n=14$ shared nodes), with $P=0.359$ for | | | | | |
| 142 | the Mk inference, and P=0.280 for Mkv inferences. | | | | | |
| 143 | | | | | | |
| 144 | Dating uncertainty (HPD widths). Adding data should reduce uncertainty, especially with small | | | | | |
| 145 | morphological datasets. The null hypothesis, that the no-autapomorphies dataset does not | | | | | |
| 146 | have greater HPD widths, was rejected for the Mk inferences (including vs. excluding | | | | | |
| 147 | autapomorphies, 9.20 vs. 9.94, <i>P</i> = 0.023, one-sided WSRT), and a <mark>suggestive</mark> result for the M <i>k</i> v | | | | | |
| 148 | inferences (9.37 vs. 9.66, <i>P</i> =0.105). | | | | | |
| 149 | | | | | | |
| 150 | Posterior probabilities (PPs). PPs were higher for runs including autapomorphies under both the | | | | | |
| 151 | Mk model (including vs. excluding autapomorphies, 0.902 vs. 0.756) and the Mkv model (0.900 | | | | | |

- 152 vs. 0.835). The null hypothesis, that the no-autapomorphies dataset does not have smaller PPs,
- 153 was rejected at a significance level of 0.05 for both the Mk inference (P=0.0095, one-sided
- 154 WSRT) and Mkv inference (P=0.0252).
- 155
- 156 Relaxed clock. The mean of the relaxed clock rate is dramatically affected by inclusion of
- 157 autapomorphies, under both the Mk model (with autapomorphies, rate mean= 0.0782 changes)

158 per site per million years, 95% HPD=[0.015, 0.159]; without: 0.788 [0.0305, 3.982]) and the Mkv

159 model (with: 0.0376 [0.0074, 0.0840]; without: 0.550 [0.0228, 2.655]) (tests in SM). The Mk_{parsinf}

- 160 run of the no-autapomorphies dataset yielded an intermediate clock rate (0.235, 95%
- 161 HPD=[0.0142, 0.664]).
- 162

163 Simulations. Figure 2 shows the simulation procedure and key comparisons. Similar tree

164 topologies were inferred under all datasets, but estimated time-branchlengths differed. When

165 the characters are clocklike and autapomorphies are included, inferred time-branchlengths are

166 highly accurate (2a). However, when autapomorphies are excluded, inferred terminal

167 branchlengths are biased downwards (and accuracy decreases overall). The effect in 2b can also

168 be seen by comparing inference while including vs. excluding autapomorphies, when the

169 characters are clocklike (2c), but this effect disappears for non-clock data (2d).

170

- 171 Feasibility of Mk_{parsinf}. Equations in Appendix 1 demonstrate that Mk_{parsinf} can be feasible for 2-
- 172 state characters, and for 3-state characters on small datasets (~10 times slower for our
- 173 dataset), but rapidly becomes computationally impractical as the number of taxa or states

increases. The number of unobservable site patterns for various combinations of numbers of 174

- 175 taxa and character states are shown in Table 2.
- 176
- 177 Discussion

| 178 | Although estimated mean rate parameters for the eureptile dataset dropped dramatically when |
|------------------|---|
| 179 | autapomorphies were included (and somewhat less when ascertainment-bias correction was |
| 180 | used instead), the downstream effects on confidence were small (Table 1; SM), and there was |
| 181 | no detectable effect on date inference. This seems surprising, because the exclusion of |
| 182 | autapomorphies must reduce the number of morphological changes on terminal branches. |
| 183 | However, reflection on the interaction between non-clocklike data, and the flexibility of |
| 184 | relaxed-clock Bayesian tip-dating methods, provides an explanation. If the character data are |
| 185 | non-clocklike, then the method will estimate a high rate of branchwise rate variation, indicating |
| 186 | lack of correlation between time elapsed and morphological branchlength. In this situation, |
| 187 | most of the dating information for the analysis comes from the serial-sampling of fossil tips |
| 188 | rather than morphological branchlengths. If morphological branchlength is not correlated with |
| <mark>189</mark> | time, this remains true whether or not autapomorphies are included, and adding |
| <mark>190</mark> | autapomorphies is not likely to change the dating inference. |
| 191 | |
| 192 | Our simulation results (Figure 2) confirm this explanation. The analysis of the empirical |
| 193 | eureptile dataset is likely similar to the situation shown in Figure 2d: inferred time |

- branchlengths are roughly the same whether or not autapomorphies are included. However, on 194
- a clocklike dataset, exclusion of autapomorphies clearly has an effect (Figure 2b). This suggests 195

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196 that the importance of including autapomorphies in tip-dating analyses depends on whether or 197 not the characters have clocklike behavior. Unfortunately, assessing clocklike behavior will be 198 more difficult when autapomorphies have been ignored or gathered only inconsistently (as is 199 common). 200 201 An alternative to coding autapomorphies is the $Mk_{parsinf}$ model. However, Appendix 1 shows 202 that it scales too poorly to be generally useful for characters with large number of states (Table 203 2; SM). All versions of MrBayes back to at least 3.1.2 allow a "coding=informative" 204 ascertainment bias correction to be specified, but the increase in computation time for a run 205 with a single discrete character is very similar whether the character has 2, 3, 4, or 5 states 206 (tested on MrBayes versions 3.1.2 through 3.2.6, and the 3.2.7 development version 207 downloaded on September 7, 2016; data not shown). This suggests that Mk_{parsinf} may be 208 implemented assuming only binary characters, and may be formally incorrect for multistate 209 characters (as briefly noted by (dos Reis et al. 2016; Matzke 2016)), despite many usages in the 210 literature. However, as most morphological datasets are dominated by binary characters, this 211 issue may have limited impact on inference, and requires further study. 212

213 Conclusion

Our study indicates that the common practice of repurposing character matrices devised for parsimony and undated Bayesian analyses may not be sufficient in the world of Bayesian tipdating. For higher quality datasets (many characters, clocklike behavior), the bias in dating introduced by ignoring autapomorphies may become significant. Additionally, ascertainment

| 218 | bias corrections are at present computationally impractical for many datasets with multistate |
|-----|---|
| 219 | characters. Finally, autapomorphies have additional utility for improving estimates of rates and |
| 220 | rate variation, for species identification, for measuring disparity, and because autapomorphies |
| 221 | may become synapomorphies when new taxa are described. Therefore, we recommend that |
| 222 | autapomorphies be coded and used whenever possible. |
| 223 | |
| 224 | Appendix 1: Derivation of the $Mk_{parsinf}$ ascertainment bias correction, and problems with |
| 225 | scalability |
| 226 | |
| 227 | One potential alternative to our recommendation to code autapomorphies could be to employ |
| 228 | the Mk-Parsimony-Informative model (M $k_{parsinf}$), that is, the Markov-k model with an |
| 229 | ascertainment bias correction for not just the unobservability of invariant characters, but also |
| 230 | for the unobservability of autapomorphic characters (in a dataset that excludes autapomorphic |
| 231 | characters). |
| 232 | |
| 233 | A significant question is whether or not the $Mk_{parsinf}$ model can actually be employed on typical |
| 234 | datasets. <i>MrBayes</i> , since at least version 3.1.2, does allow the $Mk_{parsinf}$ ascertainment bias |
| 235 | correction as an option ("Iset coding=informative"; (Ronquist et al. 2011), p. 146, or |
| 236 | http://mrbayes.sourceforge.net/wiki/index.php/Evolutionary_Models_Implemented_in_MrBay |
| 237 | es_3#Standard_Discrete28Morphology.29_Model), but does not contain an extensive |
| 238 | description of how it works, and the model does not seem to have been formally described in |
| 239 | any publication. Allman et al. (Allman et al. 2010) analyse model identifiability in the M k_{parsinf} |

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| 240 | context, but only cite Nylander et al. (Nylander et al. 2004) for the model; Nylander et al. in turn |
|------------------|--|
| 241 | cite Ronquist and Huelsenbeck, "in prep.," which appears to be an uncorrected reference to |
| 242 | their cited (Ronquist & Huelsenbeck 2003), the MrBayes 3 publication. |
| 243 | |
| 244 | There is therefore a need to explore $Mk_{parsinf}$ in detail. Ascertainment bias correction works by |
| 245 | enumerating site patterns that are unobservable, calculating their likelihood under the model, |
| 246 | and then re-normalizing the observed data likelihood at each site by dividing by 1-L, where L is |
| 247 | the likelihood of the unobservable site pattern(s). In Felsenstein (Felsenstein 1992), the |
| 248 | unobservable pattern was "no restriction site observed," i.e., a column of all 0s. In the Mkv |
| 249 | model, the unobservable patterns include a column of all 0s, a column of all 1s, etc., up to the |
| 250 | number of character states, n, in a particular character. |
| 251 | |
| 252 | However, the situation becomes much more complicated for the $Mk_{parsinf}$ ascertainment bias |
| 253 | correction (Supplemental Table 1). The difficulty (mentioned briefly in (dos Reis et al. 2016; |
| 254 | Matzke 2016)) is that the number of unobservable site patterns scales very poorly with number |
| 255 | of character states and number of taxa. For example, for a 100 taxon data matrix and a 3-state |
| <mark>256</mark> | character, the following is an unobservable site pattern: a column that consists of all 0s, a single |
| <mark>257</mark> | 1 for taxon 99, and a single 2 for taxon 100. But any other variant of this pattern is also |
| 258 | unobservable: all 0s, except state 1 at position 1, and state 2 at position 2, etc. Additional |
| 259 | unobservable patterns include all 1s except for two taxa and all 2s except for two taxa. Also |
| 260 | unobservable are all patterns that are invariant (all 0s, all 1s, all 2s), and all patterns that are |
| 261 | invariant except for one taxon. |

262

Formally speaking, if *n* is the number of taxa, and *k* is the number of states in a character, then there are k^n possible patterns for that character. We can calculate the number of site patterns that are unobservable under $Mk_{parsinf}$ by first selecting the number of character states, *i*, found in a particular unobservable site pattern. For example, in a character assumed to have *k*=5 states, the number of states found in a particular pattern could be *i*=1 (i.e., an invariant site), *i*=2, ..., *i*=5. For each *i*, there are

$$\binom{k}{i} \tag{1}$$

269 ways to pick which of the *k* character states will be found in found in the site pattern.

270 Conditional on *i* character states in a particular unobservable site pattern, one of them will be

271 the "dominant" state (held by all taxa, except for the autapomorphic taxa), and (i - 1) of the

272 character states will be autapomorphies. There are

$$\begin{pmatrix} i \\ 1 \end{pmatrix} \tag{2}$$

ways to choose which character state is dominant. Conditional on the dominant characterstate, there are

$$\binom{n}{i-1} \tag{3}$$

- 275 ways to choose which taxa will be autapomorphic. Conditional on which taxa are
- autapomorphic, there are $_{i-1}P_{i-1}$ permutations of ways to assign the (i 1) character states to
- 277 the (i 1) autapomorphic taxa. This is calculated
- 278

$$\frac{(i-1)!}{((i-1)-(i-1))!} = \frac{(i-1)!}{0!} = (i-1)!$$
(4)

279

280 Taking the product of equations 1-4 and summing over all *i* yields

$$\sum_{i=1}^{k} \binom{k}{i} \binom{i}{1} \binom{n}{i-1} (i-1)!$$

$$\sum_{i=1}^{k} \binom{k}{i} i! \binom{n}{i-1}$$
(5)

autapomorphic patterns that are unobservable under the Mk_{parsinf} ascertainment bias
correction, for a character with k states. This equation is implemented in the R function *num_unobservable_patterns_ParsInf*, included in the Supplemental Material (and available
online via GitHub Gist, at: <u>https://gist.github.com/nmatzke/8f80723b6e1fc80ed5ac</u>).

Calculating the number of unobservable patterns for a range of numbers of taxa and states
(Supplemental Table 1) shows that, for a 100-taxon morphological matrix, the presence of
characters with 3 states in the matrix will necessitate calculating the likelihood for 30,303
additional site patterns. This is computationally imaginable, although it will substantially slow
the MCMC search for a morphological dataset, which usually has only a few hundred
characters. The presence of a 4-state character requires 4,000,804 unobservable patterns. For
a 6-state character there are over 57 billion.

In *Beast2*, unobservable site patterns have to be physically listed in the XML input; even with a
script to write out the patterns, users can certainly imagine the difficulty of saving and
manipulating XML files containing millions of unobservable patterns. Inspection of the *MrBayes*code seems to indicate that the Mk_{parsinf} correction assumes binary characters only (which is
computationally feasible; Supplemental Table 1); but this leaves open the question of what
calculation, is being done on characters with more than two states.

300

301 This is problematic, as many researchers (e.g. {Dembo et al. 2016}) are probably under the impression that Mk_{parsinf} ascertainment bias correction works for any number of character 302 303 states. It is possible that this issue is of little significance. After all, most morphological 304 characters are binary. Also, as the number of taxa and character states increases, the fraction of the total number of possible patterns (k^n) that are unobservable (equation 5) decreases 305 306 precipitously (Supplemental Tables 2, 3). Thus, perhaps the likelihood of unobservable sites 307 dwindles to irrelevance. This seems to be the observation made in the MrBayes manual 308 ((Ronquist et al. 2011), pp. 146-147), where the authors state they observed that as the total 309 tree length (sum of branchlengths in terms of number of expected changes per site) increases 310 e.g. above 20-30 taxa, the ascertainment bias correction becomes negligible. However, this 311 may depend greatly on the "true" rates - if they are low, and thus invariant 312 and autapomorphic patterns are some of the most probable patterns, then the likelihood 313 correction from unobservable patterns could be large. As this paper showed, in the case of the 314 25-taxon eureptile dataset, just switching from Mk to Mkv models dropped the mean clock rate 315 estimate by about 1/3 in both the autapomorphies-included and autapomorphies-excluded

| 316 | dataset. This would affect the morphological branchlengths (number of expected changes per |
|--|---|
| 317 | site) in a similar way. As $Mk_{parsinf}$ includes the Mkv correction, this suggests $Mk_{parsinf}$ would have |
| 318 | the same or greater effect. |
| 319 | |
| 320 | Two comments we received from readers of a draft of this manuscript deserve attention. First, |
| 321 | Mike Lee (personal communication) pointed out that the equations above would be somewhat |
| 322 | different if the researchers coding characters excluded not only parsimony-uninformative |
| 323 | characters, but also characters that were "partially uninformative." An example would be the |
| <mark>324</mark> | character pattern 00112 – character states 0 and 1 are potential synapomorphies, but character |
| <mark>325</mark> | state 2 is an autapomorphy. Above, we have focused on the "literal" interpretation of |
| <mark>326</mark> | "parsimony-informative," which we think is the understanding commonly used in the literature |
| | |
| <mark>327</mark> | and in programs. We suggest that an ascertainment bias correction that assumes the |
| 327 328 | and in programs. We suggest that an ascertainment bias correction that assumes the unobservability of invariant, parsimony-uninformative, and partially-parsimony-informative |
| 327 328 329 | and in programs. We suggest that an ascertainment bias correction that assumes the unobservability of invariant, parsimony-uninformative, and partially-parsimony-informative characters should have a new name, perhaps simply "partial-parsinf." |
| 327328329330 | and in programs. We suggest that an ascertainment bias correction that assumes the unobservability of invariant, parsimony-uninformative, and partially-parsimony-informative characters should have a new name, perhaps simply "partial-parsinf." |
| 327 328 329 330 331 | and in programs. We suggest that an ascertainment bias correction that assumes the unobservability of invariant, parsimony-uninformative, and partially-parsimony-informative characters should have a new name, perhaps simply "partial-parsinf." Second, Mark Holder (personal communication) pointed out that the scalability problem is less |
| 327 328 329 330 331 332 | and in programs. We suggest that an ascertainment bias correction that assumes theunobservability of invariant, parsimony-uninformative, and partially-parsimony-informativecharacters should have a new name, perhaps simply "partial-parsinf."Second, Mark Holder (personal communication) pointed out that the scalability problem is lessdetrimental, although still daunting, if it is realized that some patterns will have the same |
| 327 328 329 330 331 332 333 | and in programs. We suggest that an ascertainment bias correction that assumes theunobservability of invariant, parsimony-uninformative, and partially-parsimony-informativecharacters should have a new name, perhaps simply "partial-parsinf."Second, Mark Holder (personal communication) pointed out that the scalability problem is lessdetrimental, although still daunting, if it is realized that some patterns will have the samelikelihood under the Mk model (because it is a symmetric-equal-rates model). For example, the |
| 327 328 329 330 331 332 333 334 | and in programs. We suggest that an ascertainment bias correction that assumes the unobservability of invariant, parsimony-uninformative, and partially-parsimony-informative characters should have a new name, perhaps simply "partial-parsinf." Second, Mark Holder (personal communication) pointed out that the scalability problem is less detrimental, although still daunting, if it is realized that some patterns will have the same likelihood under the Mk model (because it is a symmetric-equal-rates model). For example, the patterns 00112, 00221, 11002, 11220, 22110, and 22001, will all have the same likelihood. |
| 327 328 329 330 331 332 333 334 335 | and in programs. We suggest that an ascertainment bias correction that assumes the unobservability of invariant, parsimony-uninformative, and partially-parsimony-informative characters should have a new name, perhaps simply "partial-parsinf." Second, Mark Holder (personal communication) pointed out that the scalability problem is less detrimental, although still daunting, if it is realized that some patterns will have the same likelihood under the Mk model (because it is a symmetric-equal-rates model). For example, the patterns 00112, 00221, 11002, 11220, 22110, and 22001, will all have the same likelihood. Therefore the log-likelihood can be calculated for one of these patterns, and multiplied by the |
| 327 328 329 330 331 332 333 334 335 336 | and in programs. We suggest that an ascertainment bias correction that assumes the unobservability of invariant, parsimony-uninformative, and partially-parsimony-informative characters should have a new name, perhaps simply "partial-parsinf." Second, Mark Holder (personal communication) pointed out that the scalability problem is less detrimental, although still daunting, if it is realized that some patterns will have the same likelihood under the Mk model (because it is a symmetric-equal-rates model). For example, the patterns 00112, 00221, 11002, 11220, 22110, and 22001, will all have the same likelihood. Therefore the log-likelihood can be calculated for one of these patterns, and multiplied by the number of patterns in that category. This amounts to removing <i>i</i>! from equation (5), and using |

- 338 for its Alignment class, but we have not tested it in combination with the
- 339 ascertained/excludefrom/excludeto options in the XML.
- 340
- 341 Equation (5) applies to unordered characters, where any autapomorphies will be parsimony-

342 uninformative. If it is instead assumed that the characters are ordered, then any pattern with

- 343 more than two states will be parsimony-informative. For example, the pattern 011112 would
- 344 be parsimony-uninformative for an unordered character, but parsimony-informative for an
- ordered character, because bipartitions grouping states (0,1) and (1,2) would be favoured over
- 346 trees grouping (0,2). Thus, the number of unobservable patterns (assuming the researchers
- 347 doing the character scoring had this in mind when building their matrix) is much reduced, since

348 only patterns with 1 or 2 character states are unobservable. The equation is:

349

$$\sum_{i=1}^{2} \binom{k}{i} \binom{i}{1} \binom{n}{i-1} (i-1)!$$
(6)

350

351 The unobservable pattern counts for an ordered character are shown in Supplemental Table 4,352 and fractions in Supplemental Table 5.

353

Resolution of the discussion about when and where $Mk_{parsinf}$ is functional, useful, or

355 unnecessary may be difficult, as it depends in part on gnarly philosophical questions about

356 what the "complete" morphology matrix would look like (how many invariant morphological

357 characters are "truly" observable in any particular clade?). This is closely tied to another

| 358 | difficult question: what is the "true" morphological rate for "all" of the morphology? We can | | | | | |
|-----|---|--|--|--|--|--|
| 359 | briefly suggest that probably such questions are almost unanswerable in the abstract, and that | | | | | |
| 360 | any meaningful statements about rates and completeness must be made with reference to | | | | | |
| 361 | some method of character collection. It certainly appears that these problems should be | | | | | |
| 362 | studied more carefully than can be done here. Unless these issues are resolved, however, it | | | | | |
| 363 | may be that including all codeable autapomorphies, and using the Mkv ascertainment bias | | | | | |
| 364 | correction, is the best option. | | | | | |
| 365 | | | | | | |
| 366 | | | | | | |
| 367 | Data accessibility | | | | | |
| 368 | All data, code, and Beast2 inputs and outputs are available in this article's Dryad data | | | | | |
| 369 | repository: http://datadryad.org/review?doi=doi:10.5061/dryad.8q4c8 | | | | | |
| 370 | | | | | | |
| 371 | Competing interests | | | | | |
| 372 | We have no competing interests. | | | | | |
| 373 | | | | | | |
| 374 | Authors' Contributions | | | | | |
| 375 | RBI conceived of the idea, obtained the dataset, and assembled the tip dates and | | | | | |
| 376 | documentation from the literature. NJM wrote BEASTmasteR, conducted the computational | | | | | |
| 377 | analyses and drafted the manuscript. Both authors planned the study, and edited and corrected | | | | | |
| 378 | the manuscript. | | | | | |



379

380

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384



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Figure 1(on next page)

Comparison of the tip-dated phylogenies of early eureptiles inferred when excluding or including autapomorphies.

Figure 1. Comparison of the tip-dated phylogenies of early eureptiles inferred when excluding (a) or including (b) autapomorphies, under Mkv ascertainment bias correction. Numbers are posterior probabilities. Bars represent the 95% HPD.

a. Mkv, autapomorphies excluded



b. Mkv, autapomorphies included



Labidosaurikos Labidosaurus Captorhinus aquti Captorhinus laticeps Saurorictus Protocaptorhinus Rhiodenticulatus Romeria texana Concordia Thuringothyris Paleothyris Brouffia Coelostegus Procolophonidae Millerettidae Mesosauridae Caseidae Protorothvris Anthracodromeus Cephalerpeton Araeoscelis Petrolacosaurus Hvlonomus Seymouriamorpha Diadectomorpha



Figure 2(on next page)

Simulation procedure and results.

Figure 2. Simulation procedure (top) and results (bottom, a-d). The lack of an effect of excluding autapomorphies on dating in the empirical eureptile result is similar to the result on non-clock data shown in 2d.





Table 1(on next page)

Comparison of summary statistics from the five Beast2 runs.

Table 1. Comparison of summary statistics from the five Beast2 runs using "best-practices"tip dates.

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 Table 1. Comparison of summary statistics from the five Beast2 runs using "best-practices" tip dates.

| Run # | 1 | 2 | 3 | 4 | 5 |
|-----------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| Data | Including autapomorphies | | Excluding autapomorphies | | |
| Model | Mk | Mkv | Mk | Mkv | Mk-parsinf |
| Ln Posterior | -1393.4 | -1362.2 | -1154.2 | -1144.9 | -1134.4 |
| ESS | 1801 | 1485 | 1801 | 1801 | 1801 |
| Root age | 332.6 [330.2, 335.3] | 332.5 [330.0, 335.1] | 332.6 [330.1, 335.1] | 332.6 [330.1, 335.1] | 332.6 [330.0, 335.1] |
| Birth | 0.360 [0.0355, 1.316] | 0.424 [0.0405, 1.708] | 0.342 [0.0463, 1.221] | 0.381 [0.0402, 1.377] | 0.564 [0.0444, 2.841] |
| Death | 0.336 [9.17e-5, 1.315] | 0.3995 [1.13e-4, 1.723] | 0.318 [4.97e-6, 1.220] | 0.357 [2.57e-4, 1.391] | 0.541 [6.37e-4, 2.843] |
| Sampling | 0.0271 [7.90e-4, 0.0626] | 0.0264 [0.00104, 0.0650] | 0.0271 [8.85e-4, 0.063] | 0.0261 [9.96e-4, 0.0634] | 0.0256 [7.66e-4, 0.0643] |
| Clock rate mean | 0.0782 [0.015, 0.159] | 0.0376 [0.0074, 0.0840] | 0.788 [0.0305, 3.982] | 0.550 [0.0228, 2.655] | 0.235 [0.0142, 0.664] |
| Clock rate SD | 1.747 [1.201, 2.399] | 1.712 [1.111, 2.309] | 2.436 [1.572, 3.477] | 2.341 [1.488, 3.379] | 2.079 [1.318, 2.984] |



Table 2(on next page)

Number of patterns unobservable under parsimony-informative ascertainment bias.

Table 2. Number of patterns that are unobservable under the $Mk_{parsinf}$ ascertainment bias correction.

| - | | | | | | |
|-----------|------|-------------|---------|------------|------------|-------------|
| | | # states: 2 | 3 | 4 | 5 | 6 |
| # of taxa | 4 | 10 | 63 | 292 | 1045 | 3006 |
| | 5 | 12 | 93 | 544 | 2505 | 9276 |
| | 10 | 22 | 333 | 4084 | 42505 | 381546 |
| | 20 | 42 | 1263 | 32164 | 730005 | 15085086 |
| | 50 | 102 | 7653 | 500404 | 30062505 | 1698527706 |
| | 100 | 202 | 30303 | 4000804 | 490250005 | 57089105406 |
| | 200 | 402 | 120603 | 32001604 | 7921000005 | 1.87E+12 |
| | 500 | 1002 | 751503 | 500004004 | 3.11E+11 | 1.86E+14 |
| | 1000 | 2002 | 3003003 | 4000008004 | 4.99E+12 | 5.97E+15 |

Table 2. Number of patterns that are unobservable in the $Mk_{parsinf}$ model.