Boundary curves of individual items in the distribution of total depressive symptom scores approximate an exponential pattern in a general population

Shinichiro Tomitaka [Corresp., 1, 2] , **Yohei Kawasaki** [2] , **Kazuki Ide** [2] , **Maiko Akutagawa** [2] , **Hiroshi Yamada** [2] , **Toshiaki A Furukawa** [3] , **Yutaka Ono** [4]

[1] Department of Mental Health, Panasonic Health Center, Tokyo, Japan

[2] Department of Drug Evaluation and Informatics, Graduate School of Pharmaceutical Sciences, University of Shizuoka, Shizuoka, Japan

[3] Department of Health Promotion and Human Behavior, Department of Clinical Epidemiolog/ Graduate School of Medicine/ School of Public Health, Kyoto University, Kyoto, Japan

[4] Center for the Development of Cognitive Behavior Therapy Training, Tokyo, Japan

Corresponding Author: Shinichiro Tomitaka
Email address: tomitaka.shinichiro@jp.panasonic.com

Background. Previously, we proposed a model for ordinal scale scoring in which individual thresholds for each item constitute a distribution by each item. This lead us to hypothesize that the boundary curves of each depressive symptom score in the distribution of total depressive symptom scores follow a common mathematical model, which is expressed as the product of the frequency of the total depressive symptom scores and the probability of the cumulative distribution function of each item threshold. To verify this hypothesis, we investigated the boundary curves of the distribution of total depressive symptom scores in a general population.

Methods. Data collected from 21,040 subjects who had completed the Center for Epidemiologic Studies Depression Scale (CES-D) questionnaire as part of a national Japanese survey were analyzed. The CES-D consists of 20 items (16 negative items and 4 positive items). The boundary curves of adjacent item scores in the distribution of total depressive symptom scores for the 16 negative items were analyzed using log-normal scales and curve fitting.

Results. The boundary curves of adjacent item scores for a given symptom approximated a common linear pattern on a log normal scale. Curve fitting showed that an exponential fit had a markedly higher coefficient of determination than either linear or quadratic fits. With negative affect items, the gap between the total score curve and boundary curve continuously increased with increasing total depressive symptom scores on a log-normal scale, whereas the boundary curves of positive affect items, which are not considered manifest variables of the latent trait, did not exhibit such increases in this gap.

Discussion. The results of the present study support the hypothesis that the boundary curves of each depressive symptom score in the distribution of total depressive symptom scores commonly follow the predicted mathematical model, which was verified to approximate an exponential mathematical pattern.

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3 Shinichiro Tomitaka[1, 2], Yohei Kawasaki[2], Kazuki Ide[2], Maiko Akutagawa[2], Hiroshi Yamada[2],

4 Toshiaki A. Furukawa[3], Ono Yutaka[4]

5 1) Department of Mental Health, Panasonic Health Center, Tokyo, Japan

6 2) Department of Drug Evaluation and Informatics, Graduate School of Pharmaceutical Sciences,

7 University of Shizuoka, Shizuoka, Japan

8 3) Department of Health Promotion and Human Behavior, Department of Clinical Epidemiology,

9 Kyoto University Graduate School of Medicine/School of Public Health, Kyoto, Japan

10 4) Center for the Development of Cognitive Behavior Therapy Training, Tokyo, Japan

12 Corresponding author:

13 Shinichiro Tomitaka[1,2]

14 Landic building, Nishi-shinbashi 3-8-3, Minato-ku, Tokyo, 105-003, Japan

15 Email address: tomitaka.shinichiro@jp.panasonic.com

40   mathematical pattern.

41

42   **Introduction**

43   Depression is a major public health concern as one of the leading causes of disease

44   burden worldwide, with an estimated 350 million people of all ages affected around the globe

45   (Moussavi *et al*., 2007). Given that the presence of depressive symptoms is closely linked with

46   clinical levels of depression, there has been great interest in understanding the distribution of

47   depressive symptoms in the general population (Blazer & Kessler, 1994; Kroenke *et al*., 2009).

48   Building a model or formulating a theory based on empirical data is fundamental to

49   advancing scientific practice. However, despite the accumulation of knowledge regarding

50   depressive symptoms, little information is available for the development of a mathematical

51   model that may predict the distribution of depressive symptoms.

52   In developing a model of the distribution of depressive symptoms in the general

53   population, it is first necessary to identify reproducible phenomenon. Several recent studies with

54   large sample sizes have revealed that total depressive symptom scores in the general population

55   approximate an exponential pattern, except at the lowest end of the range of scores. In an

56   analysis of data from nearly 10,000 non-psychotic respondents to the British National Household

57   Psychiatric Morbidity Survey, Melzer et al. observed that an exponential curve provided the best

58   fit for total depressive and neurotic symptom scores on the Revised Clinical Interview Schedule

59   (CIS-R) (Melzer *et al.,* 2002). The authors of the present study have similarly observed that the

60   right tail of the distribution of total depressive symptom scores on the Center for Epidemiologic

61   Studies Depression Scale (CES-D) follows an exponential curve, based on data from a national

62   survey of the Japanese population including data from nearly 25,000 respondents (Tomitaka *et*

63   *al.,* 2015a). A similar study involving a large sample of Japanese employees further supported

64   the exponential pattern of CES-D scores, except at the lowest end of the scale (Tomitaka *et al.,*

65 2016a). We considered the aforementioned phenomenon in the development of our mathematical

66 model of the distribution of depressive symptoms and conducted research regarding the

67 mechanisms responsible for the generation of an exponential distribution as follows.

68   The CES-D allows individual to self-rate the frequency of a variety of depressive

69 symptoms (sadness, fatigue, etc.) on a scale consisting of four possible responses: "rarely (less

70 than 1 day)," "some (1 to 2 days)," "occasionally (3 to 4 days)," and "most of the time (5 to 7

71 days)" (Radloff, 1997). Recently, we have demonstrated that, in a general population, responses

72 to each of the 16 individual items related to negative symptoms of depression on the CES-D tend

73 to exhibit exponential patterns between the "some" and "most" responses, while this pattern is

74 not observed for "rarely" responses. (Tomitaka *et al.*, 2015b). Based on this finding and those of

75 previous reports, we then proposed that the 16 items related to negative depressive symptoms are

76 manifest variables influenced by a unidimensional latent trait of depressive symptoms, which

77 itself follows an exponential distribution (Tomitaka *et al.*, 2015b, 2016b).

78   In order to explain the aforementioned findings that total depressive symptom scores

79 approximate an exponential pattern in the general population, we proposed a model of ordinal

80 scales for unidimensional latent traits (Tomitaka *et al.*, in press). This model assumes that

81 individual thresholds for each depressive symptom differ from each other, with each item

82 exhibiting its own unique distribution. When the degree of a participant's latent trait is greater

83 than that of the threshold for the specific depressive symptom, the specific symptom is expected

84 to be present. In this simulation study, we assumed that the threshold for each depressive

85 symptom forms a normal distribution and set the simulated threshold distribution accordingly.

86 The simulation study confirmed that total scores of ordinal scale items approximate a pattern

87 similar to that of the latent trait distribution, suggesting that total scores along an ordinal scale

88   correspond to the latent trait (Tomitaka *et al.*, in press).

89   In general, the most important consideration when evaluating a mathematical model is

90   whether the model is consistent with empirical data. According to our model of ordinal scales,

91   the total score of depressive symptoms, which correspond to the latent trait, is expressed as an

92   exponential distribution, and the distribution of the threshold for each depressive symptom is

93   expressed as a normal distribution. Since the probability of each score for any given item

94   depends on whether the item threshold is smaller than the degree of the individual's latent trait,

95   the cumulative distribution function of each item threshold corresponds to the probability of each

96   item score. Thus, the boundary curve of each item score's frequency for a given degree of total

97   depressive symptom scores can hypothetically be expressed as the product of the frequency of

98   the total depressive symptom scores and the probability of the cumulative distribution function

99   of each item threshold. However, little research has been conducted regarding such a distribution.

100   Thus, in the present study, we investigated the whether the boundary curves, which represent the

101   boundaries of each item score's frequency for a given degree of total depressive symptom scores,

102   followed a common mathematical pattern predicted by the present model. This study used data

103   from a large, cross-sectional national survey of the general Japanese population (Ministry of

104   Health, Labor and Welfare, Statistics and Information Department, 2002).

105   In the present study, we first investigated the boundary curves of adjacent item scores in

106   the distribution of total depressive symptom scores for the 16 negative items. After confirming

107   that the boundary curves of adjacent item scores of each depressive symptom item commonly

108   approximated an exponential pattern, the boundary curves associated with the absence of

109   multiple depressive symptoms were analyzed to determine whether they approximated an

110   exponential pattern. Finally, to ascertain whether the common mathematical pattern of the

111   boundary curves is specific to the 16 negative affect items following the distribution of the

112   unidimensional latent trait of depressive symptoms, we examined whether the boundary curve of

113   4 positive affect items, which do not follow this latent trait distribution, follow the same pattern

114   observed for 16 negative affect items.

115

Materials and Methods

117   The present study utilized data from the Active Survey of Health and Welfare (ASHW)

118   conducted by the Japanese Ministry of Health, Labor, and Welfare in 2000 (Ministry of Health,

119   Labor and Welfare, Statistics and Information Department, 2002). The ASHW is an annual

120   nationwide survey conducted by the Japanese Government to collect data necessary for policy

121   making and health promotion in compliance with the Statistics Act. Legal and ethical approval

122   for the ASWH was provided by the Japanese Ministry of Health, Labor, and Welfare. In 2000,

123   the ASHW examined depressive symptoms among a representative sample from the general

124   Japanese population. To ensure that the sample was adequately representative, survey

125   participants were selected from individuals 12 years and older living across 300 communities in

126   Japan. These communities were selected from 881,851 precincts identified in the 1995 Census

127   using a stratified sampling design. The survey was conducted anonymously, and verbal informed

128   consent was obtained from all participants and legal guardians. The data and methods used by

129   the survey have been described in detail in a previous report (Ministry of Health, Labor and

130   Welfare, Statistics and Information Department, 2002).

131   The questionnaire was returned by 32,729 respondents, though the response rate was not

132   published by the Ministry of Health, Labor, and Welfare and Health. However, the response rates

133   for similar surveys conducted 3 and 4 years earlier were 87.1% and 89.6%, respectively (Kaji *et*

134 *al.*, 2010). Therefore, we assumed that the response rate for the ASWH survey to be greater than

135 80%.

136 The Japanese Ministry of Health, Labor, and Welfare examined our study and permitted

137 us to perform a secondary analysis of the anonymized data from the ASWH in compliance with

138 the Statistics Act. The present study was approved in 2014 by the ethics committee of the

139 Panasonic Health Center (approval number, 2014-1). The authors assert that all procedures

140 contributing to this work were compliant with the ethical standards of the relevant national and

141 institutional committees on human experimentation, and were in accordance with the Helsinki

142 Declaration of 1975 as revised in 2008.

143 We excluded 1,394 respondents owing to the suspect validity of their responses (i.e.,

144 those who answered "rarely" or "most" for all items, regardless of the nature of the item). A total

145 of 9,588 respondents were also excluded from the sample owing to missing information on one

146 or more key study variables (i.e., depressive symptoms, age, sex). The final sample consisted of

147 21,040 respondents between 12 and 98 years of age (ages 12-19; N=2457 [male; n=1269], ages

148 20-29; N=3748 [male; n=1788], ages 30-39; N=3761 [male; n=1783], ages 40-49; N=3629

149 [male; n=1788], ages 50-59; N=3569 [male; n=1800], ages 60-69; N=2253 [male; n=1155], ages

150 70-79; N=1161 [male; n=517], ages 80-89; N=412 [male; n=108], ages 90-98; N=50 [male;

151 n=15]).

152

153 **Measures**

154 Depressive symptoms were assessed using the Japanese version of the CES-D (Shima *et*

155 *al.*, 1985). This 20-item scale assesses the frequency of a variety of depressive symptoms

156 experienced within the previous week [0 = rarely or none of the time (less than 1 day), 1 = some

157   or little of the time (1-2 days), 2 = occasionally or a moderate amount of time (3-4 days), and 3 =

158   most or all of the time (5-7 days)], yielding a total score of 0–60 (Radloff, 1997). Higher scores

159   indicate greater psychological distress. The 20 items of the CES-D are generally grouped into the

160   following four subscales: depressive mood (items 3, 6, 9, 10, 14, 17, and 18); somatic symptoms

161   (items 1, 2, 5, 7, 11, 13, and 20); interpersonal relations (items 15 and 19); and positive affect

162   (items 4, 8, 12, and 16). The positive affect items are reverse-scored. In our previous work, we

163   revealed that the 16 items related to depressive mood, somatic symptoms, and interpersonal

164   relations follow a common mathematical model, while the four items related to positive affect do

165   not, suggesting that these items/symptoms associated with positive affect are not manifest

166   variables of the unidimensional latent trait (Tomitaka *et al*., 2015b).

167   The boundary curves of adjacent item scores represent the boundaries of the frequencies

168   for response categories, including "rarely," "some," "occasionally," and "most." The boundary

169   curves of adjacent item scores in the distribution of total depressive symptom scores were

170   calculated according to the frequency of each item score and analyzed using a log-normal scale.

171   The fitting curve for exponential, linear, and quadratic models were estimated using the least

172   square method. JMP Version 11 for Windows (SAS Institute, Inc., Cary, NC, USA) was used to

173   calculate descriptive statistics and frequency distribution curves.

174

175   **Results**

176   **Boundary curves of adjacent item scores of individual depressive symptoms**

177   Figure 1 depicts the distribution of total scores for 16 negative items (green lines) and

178   the boundary curves (magenta line, yellow line, and blue line), which divide the distribution of

179   total depressive symptom scores by the adjacent scores of each depressive symptom. The

180 magenta, yellow, and blue lines represent the boundaries between score 0 and score 1, score 1

181 and score 2, and score 2 and score 3, respectively.

182   On a normal scale, the boundary curves of adjacent scores for item 1 (Fig. 1A), item 2

183 (Fig. 1B), and item 3 (Fig. 1C) exhibited a right skewed distribution (upper column of Fig. 1),

184 while they exhibited a linear pattern on a log-normal scale (Fig. 1D-F). The gaps between the

185 total score curve and boundary curves of adjacent scores widened with increasing total

186 depressive symptom scores (lower column of Fig. 1). Analysis of the boundary curves of the

187 remaining 13 negative depressive symptom items (Supplementary Fig. 1-5) revealed the same

188 mathematical pattern as that shown in Fig. 1.

189   Table 1 outlines the empirical constants and coefficient of determination R^2 values for

190 linear, quadratic, and exponential fits, analyzed with respect to the boundary curves of adjacent

191 scores for item 1, item 2, and item 3. Results for the remaining 13 negative depressive symptoms

192 are presented in Supplementary Table 1. In all 48 boundary curves, the coefficients of

193 determination for the exponential fit were higher than those for linear or quadratic fits. The

194 median coefficients of determination for the linear, quadratic, and exponential fits were 0.61,

195 0.91, and 0.97, respectively, suggesting that an exponential distribution best fit the observed

196 boundary curve data.

197

198 **Boundary curves in the absence of multiple symptoms**

199   Boundary curves in the absence of symptoms related to two items (item 1 and item 2,

200 Fig. 2A, D), four items (item 1, item 2, item 3, and item 5; Fig. 2B and 2E), and six items (item 1,

201 item 2, item 3, item 5, item 6, and item 7; Fig. 2C, F) were considered representative of the

202 absence of multiple symptoms. On a normal scale, the boundary curves in the absence of

203 multiple symptoms exhibited a right-skewed distribution (upper column of Fig. 2), while they

204 exhibited a linear pattern on a log-normal scale (lower column of Fig. 2). As indicated by the

205 double-headed arrows, the gap between the total score curve and boundary curve widened as

206 total depressive symptom scores increased.

207   Table 2 outlines the empirical constants and coefficient of determination R^2 values for

208 linear, quadratic, and exponential fits, analyzed with respect to the boundary curves in the

209 absence of multiple symptoms. Curves of fit using the exponential model and the observed data

210 in Figure 2A-C yielded coefficients of determination of 0.99, 0.97, and 0.93, respectively,

211 suggesting that exponential distributions had the best fit with the observed data.

212   The rate parameter of the curve fit for the boundary curve (constant b) decreased as the

213 number of symptoms absent increased (Table 2), consistent with the finding that the slope of the

214 boundary curve on a log-normal scale became steeper as the number of symptoms absent

215 increased. Moreover, the range of total scores narrowed as the number of symptoms absent

216 increased (Fig. 2D-F). The data presented in Fig. 2F indicates that, in the absence of symptoms

217 related to the six abovementioned items, the maximum range of total scores spanned 16 points.

218

219 **Boundary curves of positive affect items in the distribution of total depressive symptom**

220 **scores for 17 depressive symptom items**

221   To ascertain whether the common mathematical pattern of the boundary curves is

222 specific to the 16 depressive symptom items, which are the manifest variables of a

223 unidimensional latent trait, we compared a representative curve (item 1) to the boundary curve of

224   an item related to positive affect (item 4), which is not a manifest variable of unidimensional

225   latent trait. While the boundary curves of both item 1 and item 4 exhibited a common right-

226   skewed distribution (upper column of Fig. 3) on a normal scale, the gap between the total score

227   curve and boundary curves differed between the two (upper column of Fig. 3). For item 1, the

228   gap between the total score and boundary curves exhibited a pattern similar to that observed in

229   Figs. 1 and 2, increasing to a certain point and then decreasing to zero at the end of the range

230   (Fig. 3A). For item 4, however, this gap decreased until the end of the range, beginning with

231   scores of 3 (Fig. 3B).

232   While the boundary curves of both item 1 (Fig. 3C) and item 4 (Fig. 3D) exhibited a

233   linear pattern in the rage of total scores from 3–51 points on a log-normal scale, the gap between

234   the total score curve and boundary curve again exhibited different patterns between items 1 and 4

235   (lower column of Fig. 3). For item 1, the gap between the total score curve and boundary curve

236   widened as total depressive symptom scores increased, similar to the pattern observed in lower

237   column of Fig. 1 and 2 (Fig. 3C). Conversely, for item 4, the gap between the total score curve

238   and boundary curve remained stable (Fig. 4D).

239   The curves of fit according to an exponential model were calculated for data from the

240   total score curve in Fig. 3C ($y = 1823e^{-0.23x}$, $R^2 = 0.97$), the boundary curve in Fig. 3C ($y =$

241    $3327e^{-0.15x}$, $R^2 = 0.98$), the total score curve in Fig. 3D ($y = 3327e^{-0.15x}$, $R^2 = 0.98$), and the

242   boundary curve in Fig. 3D ($y = 580e^{-0.14x}$, $R^2 = 0.97$). Consistent with log-normal scale findings,

243   exponential curve fitting showed that, while exponential distributions fitted the boundary curves

244   of both item 1 and item 4, the rate parameters of the curves of fit for the total score curve and

245 boundary curve were similar for item 4 (-0.15 vs. -0.14) but not for item 1 (-0.23 vs. -0.15).

246 Analysis of the boundary curves of the remaining three items associated with positive affect

247 (Supplementary Fig. 6) revealed the same mathematical pattern as that shown in Fig. 3B and D,

248 suggesting that an increasing gap between the total score and boundary curves on a log-normal

249 scale is specific to the 16 depressive symptom items, which comprise the manifest variables of a

250 unidimensional latent trait.

251

Discussion

252

253   In the present study, we aimed to verify whether boundary curves, which represent the

254 boundaries of each item score for a given degree of total depressive symptom scores, followed a

255 common mathematical pattern predicted by the present model. The main findings of this study

256 are as follows: (1) regardless of item choice, the boundary curves for items associated with

257 negative depressive symptoms approximated an exponential pattern; (2) on a log-normal scale,

258 the gap between the total score and boundary curves for negative affect items, but not for

259 positive affect items, increased as total depressive symptom scores increased; (3) the boundary

260 curves associated with the absence of multiple symptoms commonly approximated an

261 exponential pattern.

262 **Boundary curves for items related to negative depressive symptoms follow an exponential**

263 **pattern**

264   Regardless of item choice, the boundary curves for items associated with negative

265 depressive symptoms exhibited a common linear pattern on a log-normal scale (Fig.1-3). In

266 addition, exponential fits showed markedly higher coefficients of determination than either linear

267 or quadratic fits, suggesting that the boundary curves of negative affect items follow an

268   exponential pattern.

269   Although we hypothesized that the boundary curve of each item score could be

270   expressed as the product of the frequency of the total depressive symptom scores and the

271   probability of the cumulative distribution function of each item threshold, we did not predict that

272   the boundary curves for items associated with negative depressive symptoms approximated an

273   exponential distribution. Mathematical speculations can be made about the conditions that

274   enabled such an exponential pattern. As depicted in Fig. 4A, the frequency of the total depressive

275   symptom scores for the 16 items associated with negative depressive symptoms may be

276   expressed as an exponential distribution, $g(x)$ (green line), as follows for a total score of x:

277    $$g(x) = Ce^{-a\,x} \qquad a>0, \qquad\qquad (1)$$

278   where C is the frequency of total scores for the 16 negative items at a score of zero and a is the

279   parameter of the exponential function for these total scores.

280   The frequency of boundary curves between the absence and presence of each symptoms

281   item is then expressed as $f(x)$ (Fig. 4A, magenta line). The probability of the presence of a given

282   symptom, $P(x)$, is expressed as the ratio of the frequency of that symptom (black two-headed

283   arrow) divided by the frequency of total depressive symptom scores (green line) as follows:

284    $$P(x) = \{g(x)\text{-}f(x)\}/g(x). \qquad\qquad (2)$$

285   Fig. 4B depicts the distribution of each item threshold according to the latent trait.

286   According to the present model, the probability of the presence of a given symptom corresponds

287   to the cumulative distribution function of each item threshold where the degree of the threshold

288   for the given item is greater than the latent trait value of x (shaded area).

289   Assuming a normal distribution for each item threshold, the cumulative distribution

290   function of each item threshold may be expressed using a logistic distribution (Bowling SR et al.,

291   2009). The probability of the presence of a given symptom is then expressed as follows:

292                   $P(x) = 1/(1+e^{-(x-\mu)/s})$,   (3),

293   where μ is the mean, s is a parameter of logistic distribution, and $s>0$.

294   Fig. 6 depicts g(x) and f(x) on a logarithmic scale. When g(x) is expressed in logarithmic

295   terms,

296                   $\log g(x) = -ax + \log C.$ (4)   (Fig. 6, green line)

297   According to equations (1), (2), and (3),

298                   $f(x)=g(x)(1-P(x))= \{Ce^{-(a+1/S)X +\mu/s}\}/(1+e^{-(x-\mu)/s}).$ (5)

299   When f(x) is expressed in logarithmic terms,

300                   $\log f(x)= -(a+1/s)x+ \log C +u/s-\log(1+e^{-(x-\mu)/s}),$ (6)   (Fig. 6, magenta

301   curve)

302   where $\log(1+e^{-(x-\mu)/s})$ decreases to zero with increasing values of x, and $\log(1+e^{-(x-\mu)/s})>0$.

303   When the former portion of $\log$ f(x) is expressed as $\log$ h(x),

304                   $\log h(x) = -(a+1/s)x+ \log C +u/s.$ (7)   (Fig. 6, blue dotted

305   line)

306   As x increases, $\log$ f(x) (magenta curve) approaches $\log$ h(x) (blue dotted line) due to the

307   decrease in the value of $\log(1+e^{-(x-\mu)/s})$. As depicted in Fig. 6, the distance between the blue and

308   red lines (black two-headed arrow) decreases as x increases. If the parameter s is large enough,

309   the distance between the blue and red lines mildly decreases with increasing values of x. Thus,

310   although $\log$ f(x) is not a perfect linear function, it approximates a linear pattern.

311

312   **Increasing gap between the total score and boundary curves for negative symptom items on**

313   **a log-normal scale**

314   The results of the present study reveal that the gap between the total score curve and

315   boundary curve for negative affect items increases as total depressive symptom scores increase

316   on a log-normal scale (Fig. 1-3). In support of the finding, the estimated rate parameters of the

317   exponential fit curves for the total score curve and boundary curve were not similar for negative

318   affect items. The data expressed in Fig. 5 may help to explain these findings. As the total

319   depressive symptom scores increase, log f(x) (magenta line) distances itself from log g(x) (green

320   line) because log f(x) approaches h(x) (blue dotted line), resulting in the consistently increasing

321   gap between the total score curve and boundary curves.

322   Conversely, for the item associated with positive affect, the gap between the total score curve

323   and boundary curve did not increase as total depressive symptom scores increased on a log-

324   normal scale (Fig. 3D). Consistent with this finding, the estimated rate parameters of the

325   exponential fit curves for the total score curve and boundary curve were similar for positive

326   affect items. Mathematically, the increasing gap corresponds to the cumulative distribution

327   function of each item threshold, which has a continuous increasing property (Fig. 4). Thus, the

328   increasing gap between the total score curve and boundary curve could be specific to negative

329   affect items, which are considered to be manifest variables of a unidimensional latent trait. In

330   general, it is difficult to differentiate whether given items are manifest variables of the

331   unidimensional latent trait, except when the latent trait follows an exponential distribution

332   (Tomitaka et al, 2016b). Although most studies have utilized a taxometric analysis, which is a

333   statistical technique specifically designed to determine whether a given construct is best

334   conceptualized as discrete latent subcategories or as one continuous latent dimension (Slade et al.

335   2005, Okuma et al, 2009), the consistently increasing gap between the total score curve and

336   boundary curve on a log normal scale may be useful in determining whether or not a given item

337 is a manifest variable of a unidimensional latent trait. The finding that the gap between the total

338 score curve and boundary curve for negative affect items increased as total depressive symptom

339 scores increased on a log-normal scale, irrespective of the item subscale, supports a

340 unidimensional latent trait of depressive symptoms.

341 **Boundary curves in the absence of multiple symptoms**

342 The results of the present study indicate that the boundary curves associated with the

343 absence of multiple symptoms commonly approximate an exponential pattern. In addition, as the

344 number of symptoms absent increases, the slope of the boundary curves decreases on a log-

345 normal scale (Fig. 2D, 2E and 2F). Theoretically, the boundary curves associated with the

346 absence of multiple symptoms depend on whether the largest item threshold is smaller than the

347 degree of latent trait. If the largest item threshold follows a normal distribution, the boundary

348 curves in the absence of multiple symptom items could approximate an exponential distribution.

349 From a clinical standpoint, the finding that the range of total depressive symptom scores is

350 determined by the number of symptoms absent is noteworthy. In the present study, in the

351 absence of six depressive symptoms, the maximum total score was 16 points, which is nearly

352 equivalent to the threshold score for major depression in a Japanese population (Shima et al,1985,

353 Wada K et al. 2009). These findings may be useful for the screening of clinical depression in a

354 general population. Further studies are necessary to establish the relationship between the

355 numbers of symptoms absent and total depressive symptom scores.

356 **Strengths and limitations**

357 The present study had some limitations. First, participants did not undergo the standard

358 psychiatric interview and diagnosis associated with a structured interview. Second, although we

359 evaluated whether an exponential model provided a better fit than a linear or quadratic model, we

360   did not examine the fits of other mathematical models. To the best of our knowledge, no other

361   mathematical models have been proposed for the boundary curves of adjacent item scores other

362   than the exponential model. Future studies can evaluate the comparative fit of other models to

363   our empirical data as reported in our supplementary data. Third, in order to explain the

364   mathematical pattern of boundary curves, we assumed a normal distribution for each item

365   threshold. However, it is not clear whether a normal distribution is most appropriate for the

366   empirical data associated with each item threshold, indicating the need for further studies.

367   Conversely, there is a methodological advantage in the present investigation. The

368   sample was representative of the general Japanese population, which reduced selection bias. In

369   addition, the large sample size ($N = 21{,}040$) enabled us to elucidate patterns in the distributions

370   of depressive symptom items. Finally, the present study provides important information

371   regarding the boundary curves in the distribution of total depressive symptom scores. To the best

372   of our knowledge, mathematical modeling remains poorly developed in the field of psychiatry.

373   While a given individual's depressive symptoms are difficult to predict, large populations may

374   follow a certain mathematical patterns. The present study thus proposes a statistical model for

375   the relationship between the prevalence of each depressive symptom and total depressive

376   symptom scores. The degree to which these results generalize to other ordinal scales, including

377   other depression rating scales, intelligence tests, and personality questionnaires, requires further

378   examination.

379

380   **Conclusions**

381   The results of the present study support the hypothesis that the boundary curves of for

382   scores associated with a given depressive symptom in the distribution of total depressive

383   symptom scores exhibit a common mathematical pattern, raising the possibility that boundary

384   curves for given items may be useful in evaluating whether the specific item is a manifest

385   variable of the unidimensional latent trait.

386

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390

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Coefficient of determination R^2 values for the fit of linear, quadratic, and exponential curves to boundary curve data for 16 depressive symptoms.

Empirical constants are given for linear fitting with Y= a*X + b, for quadratic fitting with Y= a*X^2 + b*X + c, for exponential fitting with Y= a*e^{b*X}. The coefficient of determination R^2 values for exponential fits were higher than those for linear or quadratic fits for all boundary curves.

Boundary curve	Linear			Quadratic				Exponential		
	a	b	R^2	a	b	c	R^2	a	b	R^2
Item 1, score 0-1	-22.7	789	0.37	1.7	-106	1496	0.68	2482	-0.23	0.99
Item 1, score 1-2	-34.1	1227	0.59	1.9	-131	2051	0.89	4369	-0.19	0.98
Item 1, score 2-3	-56.7	1615	0.75	3.1	-171	2339	0.94	5172	-0.18	0.95
Item 2, score 0-1	-28.9	1029	0.50	1.8	-119	1796	0.82	2793	-0.19	0.98
Item 2, score 1-2	-34.8	1263	0.61	1.9	-129	2060	0.90	4321	-0.17	0.98
Item 2, score 2-3	-35.9	1320	0.65	1.8	-127	2091	0.91	4274	-0.16	0.98
Item 3, score 0-1	-30.3	1064	0.49	2.0	-130	1914	0.83	4782	-0.24	0.98
Item 3, score 1-2	-35.2	1273	0.61	1.9	-132	2094	0.91	5366	-0.20	0.96
Item 3, score 2-3	-36.0	1316	0.65	1.8	-128	2097	0.91	5320	-0.18	0.96

Coefficient of determination R^2 values for the fit of linear, quadratic, and exponential curves to boundary curve data in the absence of multiple symptoms.

Empirical constants are given for linear fitting with Y= a*X + b, quadratic fitting with Y= a*X^2 + bX + c, and exponential fitting with Y= a*e^{b*X}. The coefficient of determination R^2 values for exponential fits were higher than those for linear or quadratic fits for boundary curves in the absence of multiple symptoms.

Boundary curve	Linear			Quadratic				Exponential		
	a	b	R^2	a	b	c	R^2	a	b	R^2
2 items	-19.7	679	*0.31*	1.5	-96	1329	*0.61*	2410	-0.26	*0.99*
4 items	-15.0	508	*0.21*	1.3	-80	1062	*0.45*	2320	-0.40	*0.97*
6 items	-11.7	394	*0.14*	1.1	-65	849	*0.33*	2317	-0.57	*0.94*

Figure 1

Distributions of the total scores of 16 items (green lines) and the boundary curves of the adjacent score for each item.

Boundary curves of item 1 (A, D), item 2 (B, E) and item 3 (C, F) with a normal scale and a log-normal scale are presented, respectively. Magenta, yellow, and blue lines represent the boundary curves between score 0 and score 1, score1 and score 2, and score 2 and score 3 of Likert scale scores (0-1-2-3), respectively.
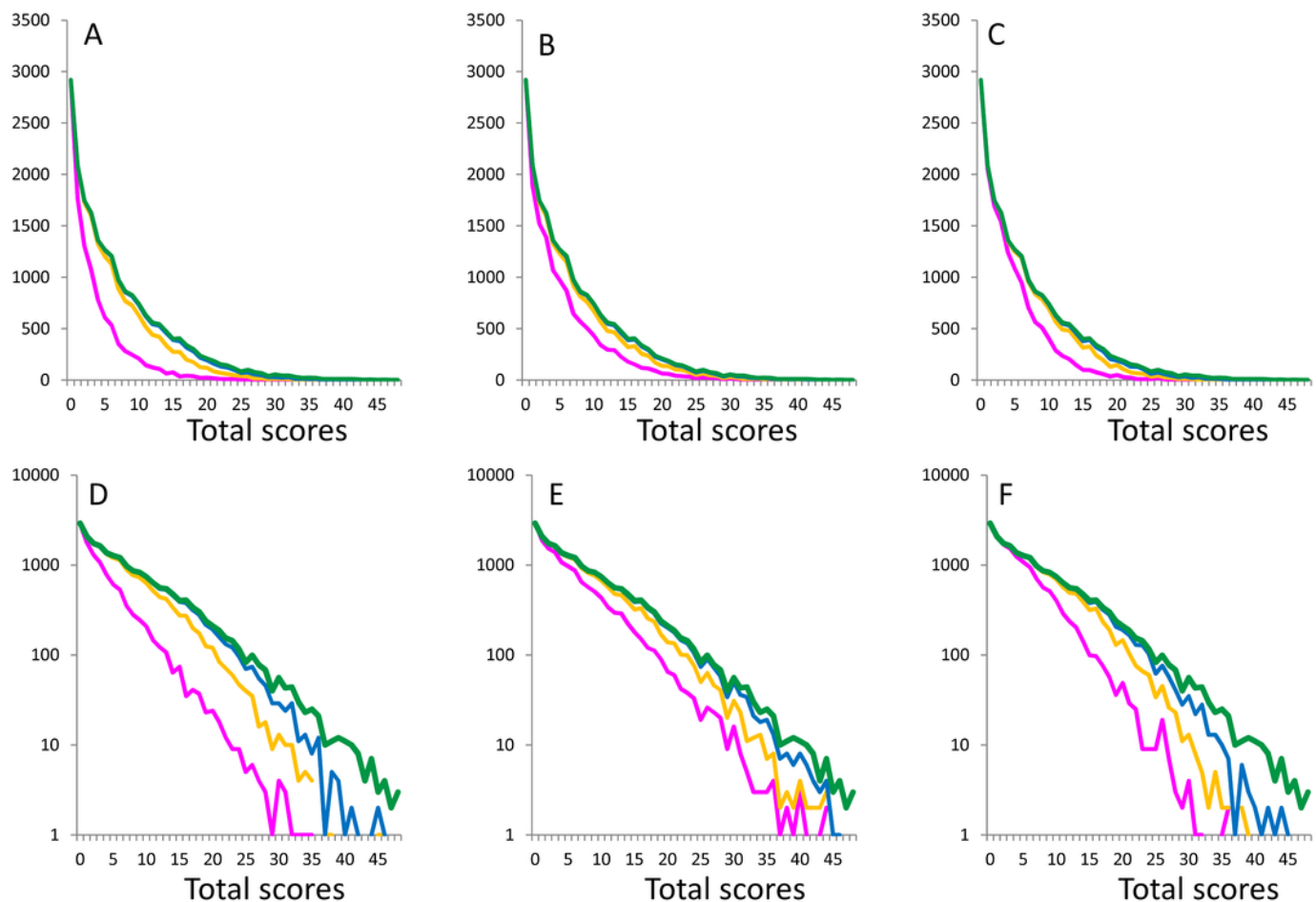
Figure 2

Boundary curves (magenta lines) in absence of two, four, and six items.

Boundary curves of two items (item 1 and item 2; A, D), four items (item 1, item 2, item 3, and item 5; B, E), and six items (item 1, item 2, item 3, item 5, item 6, and item 7; C, F) with normal scales and log-normal scales are presented, respectively.
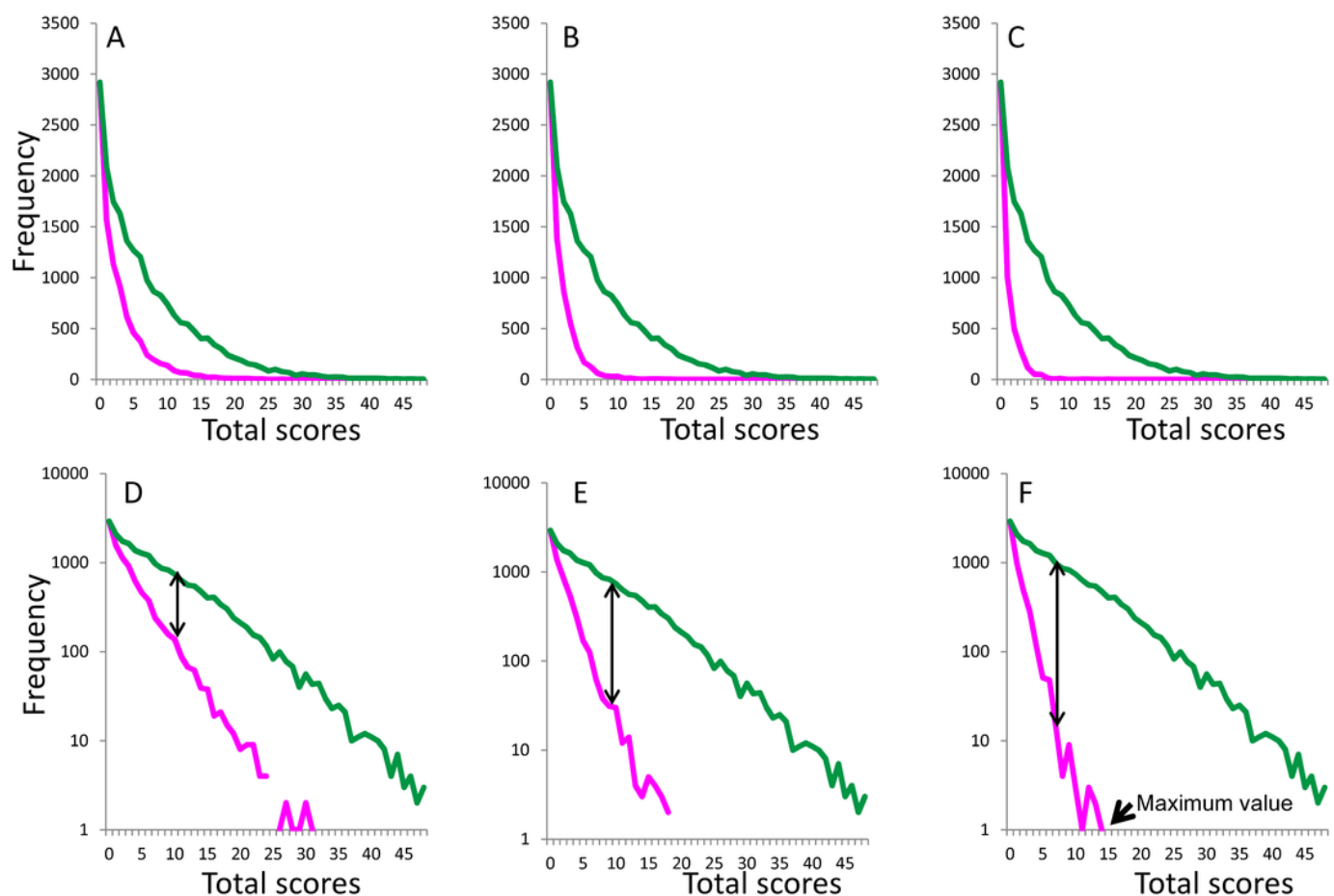
Figure 3

Boundary curves (magenta lines) of a negative depressive symptom item and a positive affect symptom item in the distribution of total depressive symptom scores for 17 depressive symptom items.

Boundary curves of item 1 (A, C) and item 4 (B, D) with a normal scale and a log-normal scale are presented, respectively.
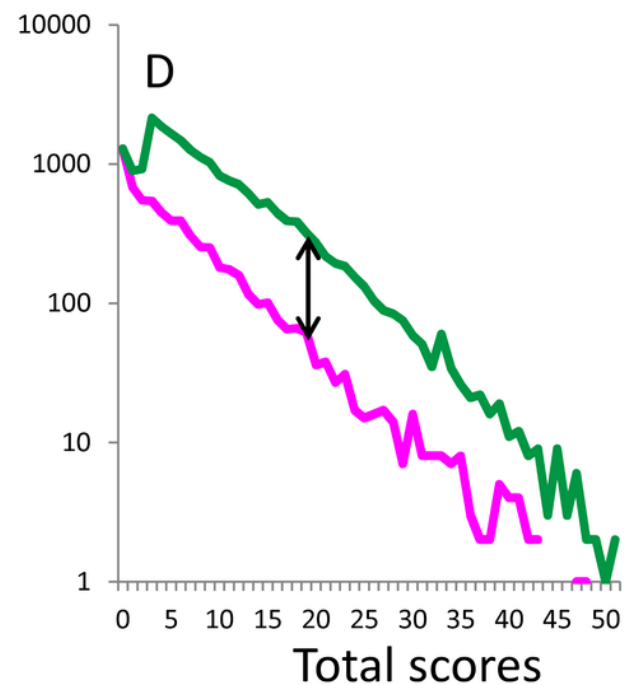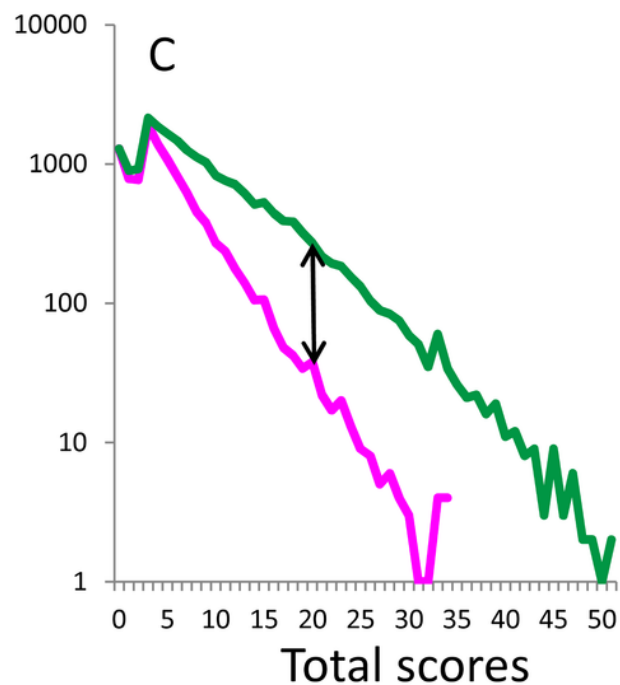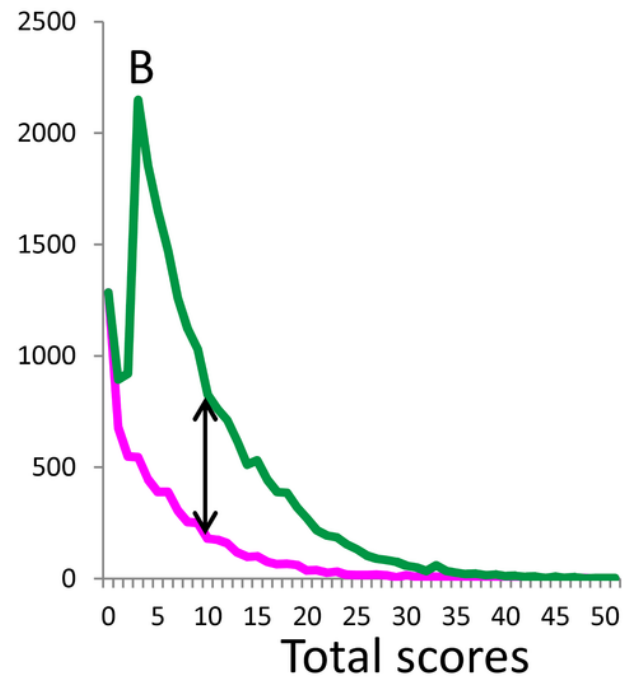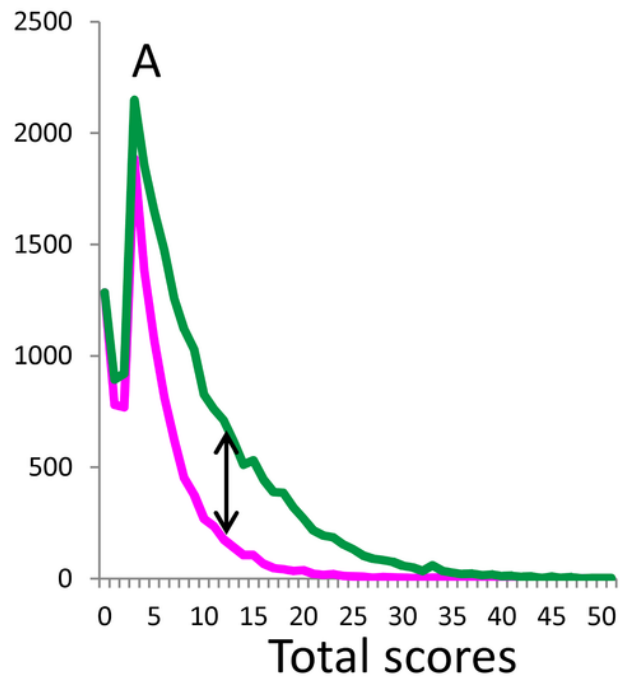
Figure 4

Distribution of the total depressive symptom scores of 16 negative items, boundary curves in the absence or presence of each symptom item (magenta line) and the distribution of each item threshold.

(A) The probability of a specific symptom is expressed as the ratio of the frequency of the presence of a specific item (black double-headed arrow) to the frequency of total depressive symptoms (green line). (B) The probability of a specific symptom with the total score of X also corresponded to the cumulative distribution function of each item threshold (shaded area).
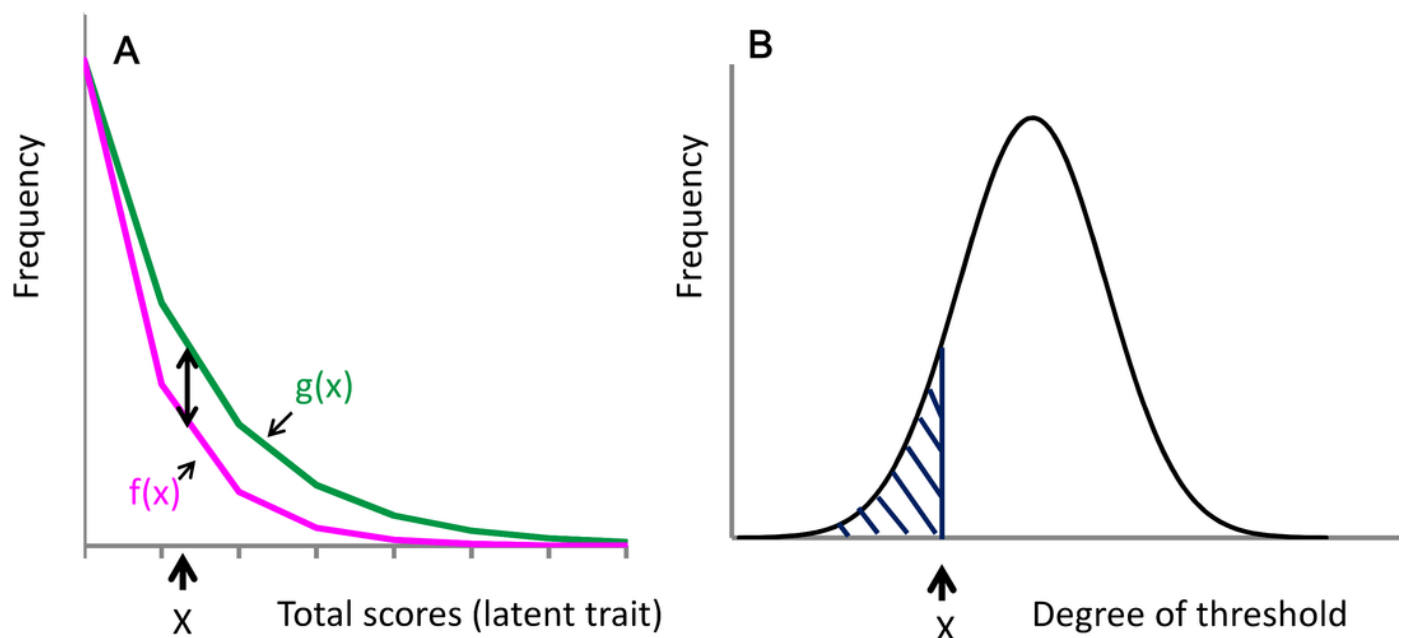
Figure 5

Distribution of the total depressive symptom scores of 16 negative items (green line) and boundary curves in the absence or presence of each symptom item (magenta line) in a single logarithmic chart.

The blue dotted line is log h(x). The distance between the blue line and magenta line (double arrow) decreased from $\log(1+e^{-\mu)/s})$ approached zero with x.