Bayesian computation for the common coefficient of variation of delta-lognormal distributions with application to common rainfall dispersion in Thailand

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Rainfall fluctuation makes precipitation and flood prediction difficult. The coefficient of variation can be used to measure rainfall dispersion to produce information for predicting future rainfall, thereby mitigating future disasters. Rainfall data usually consist of positive and true zero values that correspond to a delta-lognormal distribution. Therefore, the coefficient of variation of delta-lognormal distribution is appropriate to measure the rainfall dispersion more than lognormal distribution. In particular, the measurement of the dispersion of precipitation from several areas can be determined by measuring the common coefficient of variation in the rainfall from those areas together. The purpose of this research is to construct confidence intervals for the common coefficient of variation of delta-lognormal distributions based on the concepts of the fiducial generalized confidence interval, Bayesian methodology based on the independent Jeffreys and uniform priors, and the method of variance estimates recovery. The performances of the proposed methods were verified by analyzing their coverage probabilities together with their expected lengths via Monte Carlo simulation. The results show that the equal-tailed Bayesian based on the independent Jeffreys prior was suitable. In addition, it can be used the equal-tailed Bayesian based on the uniform prior as an alternative. Rainfall datasets from Nan, Thailand, were used to demonstrate the performances of the proposed confidence intervals.

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ABSTRACT

Rainfall fluctuation makes precipitation and flood prediction difficult. The coefficient of variation can be used to measure rainfall dispersion to produce information for predicting future rainfall, thereby mitigating future disasters. Rainfall data usually consist of positive and true zero values that correspond to a delta-lognormal distribution. Therefore, the coefficient of variation of delta-lognormal distribution is appropriate to measure the rainfall dispersion more than lognormal distribution. In particular, the measurement of the dispersion of precipitation from several areas can be determined by measuring the common coefficient of variation in the rainfall from those areas together. The purpose of this research is to construct confidence intervals for the common coefficient of variation of delta-lognormal distributions based on the concepts of the fiducial generalized confidence interval, Bayesian methodology based on the independent Jeffreys and uniform priors, and the method of variance estimates recovery. The performances of the proposed methods were verified by analyzing their coverage probabilities together with their expected lengths via Monte Carlo simulation. The results show that the equal-tailed Bayesian based on the independent Jeffreys prior was suitable. In addition, it can be used the equal-tailed Bayesian based on the uniform prior as an alternative. Rainfall datasets from Nan, Thailand, were used to demonstrate the performances of the proposed confidence intervals.

INTRODUCTION

Currently, the Earth's climate is rapidly changing due to the emission of greenhouse gases, sulfate aerosols, and black carbon, as well as changing ecosystems and transportation emissions (Nema et al., 2012). These phenomena are directly increasing the global temperature, warming the oceans, melting the polar ice caps, causing the sea level to rise, and initiating extreme events (NASA, 2020). Southeast Asia is a tropical area that is affected by ocean currents, prevailing winds, and abundant rainfall during the monsoon season (WorldAtlas, 2021). Thailand is located in Southeast Asia, where the climate is influenced by the monsoon winds. Especially, the southwest monsoon together with the Inter-Tropical Convergence Zone and tropical cyclones causes plenty of rain to fall over the country (Thai Meteorological Department, 2015). Large amounts of rainfall cause regular flooding in some areas of the country, thereby leading to damage to property and loss of life. Moreover, Thailand is an agricultural country, and rainfall fluctuation makes it difficult to predict heavy precipitation that may cause loss of or damage to crops. Therefore, it is necessary to measure the dispersion of rainfall in specific areas by using statistical tools such as the coefficient of variation (CV) to enable accurate prediction of future catastrophic events. Furthermore, rainfall data usually comprises positive values that conform to a lognormal distribution, and true zero values, in which the frequency conforms to a binomial distribution. Many researchers have reported that rainfall data follow a delta-lognormal distribution (Fukuchi, 1988; Shimizu, 1993; Yue, 2000; Kong et al., 2012; Maneerat et al., 2019a, 2020a,b; Yosboonruang et al., 2019b, 2020; Yosboonruang and Niwitpong, 2020).

The CV is commonly used to measure the dispersion of data since it is free from the unit of measurement. For statistical inference, many researchers have proposed methods to construct confidence intervals for the CV and functions of the CV (e.g. Pang et al. (2005); Hayter (2015); Nam and Kwon (2017); Yosboonruang et al. (2018, 2019a,b, 2020); Yosboonruang and Niwitpong (2020)). However, the common CV of delta-lognormal distributions has not yet been reported. Thus, we are interested in statistical inference based on the common CV of delta-lognormal distributions as it is useful for measuring the dispersion of several independent data series, especially rainfall data from several independent areas. Therefore, the common CV can be used to illustrate the dispersion of rainfall from the whole of different areas.

Confidence intervals for the common CV of normal and non-normal distributions have previously been constructed. Gupta et al. (1999) obtained the asymptotic variance of the common CV and then constructed confidence intervals for it based on normal distributions; their results showed its suitability in terms of coverage probability and expected length. Tian (2005) developed a method by using the concept of the generalized confidence interval (GCI) for the common CV. Subsequently, Behboodian and Jafari (2008) used the concept of generalized p-values and GCI to construct a new method and compared with Tian's method (Tian, 2005); the former method performed better in terms of coverage probability and expected length. Ng (2014) constructed confidence intervals for the common CV of lognormal distributions using the generalized variables approach; the performance of the proposed method was similar to Tian's method (Tian, 2005). Liu and Xu (2015) provided a confidence distribution interval method to construct the confidence interval for the common CV of several normal populations. Thangjai and Niwitpong (2017) proposed the adjusted method of variance estimates recovery (MOVER) to construct the confidence interval for the weighted CV of two-parameter exponential distributions and then compared it with GCI and a large sample method; they revealed that the adjusted MOVER was suitable only for a positive value CV and that GCI was the best choice for constructing the confidence intervals for the weighted CV of two-parameter exponential distributions. Recently, Thangjai et al. (2020a) applied the adjusted GCI and a computational method for the confidence interval estimation of the common CV of normal distributions; they compared them with GCI and the adjusted MOVER, the results of which show that the adjusted GCI is appropriate for small samples and the computational method was suitable for large ones. In addition, Thangjai et al. (2020b) extended the computational approach and MOVER to construct the confidence intervals for the common CV of lognormal distributions and compared it with the fiducial GCI (FGCI) and Bayesian approaches; of which the FGCI was the best. Unfortunately, the work of Thangjai et al. (2020b) considered only the positively skewed distribution: lognormal distribution. In this work, we regarded the lognormal distribution that contained true zero values, the delta-lognormal distribution, for the confidence interval construction of the common CV. Therefore, the research by Thangjai et al. (2020b) also needs to continue as rainfall data must be the delta-lognormal distribution.

Over an extended period of time, rainfall data usually conform to a delta-lognormal distribution, which has drawn interest from several researchers to present statistical inference for its parameters. Many researchers have proposed methods to construct confidence intervals for the mean and functions of the mean of delta-lognormal distributions, such as the traditional method, the normal algorithm, the exponential algorithm (Kvanli et al., 1998), bootstrapping, the likelihood ratio, the signed log-likelihood ratio (Zhou and Tu, 2000; Tian, 2005; Tian and Wu, 2006), GCI (Tian, 2005; Chen and Zhou, 2006; Li et al., 2013; Wu and Hsieh, 2014; Hasan and Krishnamoorthy, 2018; Maneerat et al., 2018, 2019b), MOVER (Maneerat et al., 2018, 2019a,b), Aitchison's estimator, a modified Cox's method, a modified Land's method, the profile likelihood interval (Fletcher, 2008; Wu and Hsieh, 2014), FGCI (Li et al., 2013; Hasan and Krishnamoorthy, 2018; Maneerat et al., 2019a), as well as Bayesian approaches (Maneerat et al., 2019a). Moreover, confidence interval estimations for the variance (Maneerat et al., 2020a,b), CV (Yosboonruang et al., 2018, 2019a,b), and functions of the CV (Yosboonruang et al., 2020; Yosboonruang and Niwitpong, 2020) of delta-lognormal distributions have been suggested, including GCI, the modified Fletcher's method, FGCI, MOVER, the Bayesian approach, and bootstrapping.

The aim of this study is to construct new confidence intervals for the common CV of delta-lognormal distributions based on three concepts: FGCI, the Bayesian approach, and MOVER. The performances of the proposed methods were evaluated via their coverage probabilities and expected lengths. The methods for the confidence intervals estimation are presented in the next section. Subsequently, the results and discussion of a simulation study are analyzed, followed by the use of rainfall data to assess the applicability of the proposed methods. Last, conclusions on the study is offered.

METHODS

Let X_{ij}, $i = 1,2,\ldots,k,\ j = 1,2,\ldots,n_i$ be a random variable of size n_i from k delta-lognormal distributions with density function

$$f\left(x_{ij};\mu_i,\sigma_i^2,\delta_i\right) = (1-\delta_i)\,\mathrm{I}_0\left[x_{ij}\right] + \delta_i \frac{1}{x_{ij}\sqrt{2\pi}\sigma_i}\exp\left\{-\frac{1}{2}\left[\frac{\ln\left(x_{ij}\right)-\mu_i}{\sigma_i}\right]^2\right\}\mathrm{I}_{(0,\infty)}\left[x_{ij}\right], \tag{1}$$

where $\mathrm{I}_0\left[x_{ij}\right]$ is an indicator function for which the values are 1 when $x_{ij} = 0$, and 0 otherwise; $\mathrm{I}_{(0,\infty)}\left[x_{ij}\right]$ are equal to 0 and 1 when $x_{ij} = 0$ and $x_{ij} > 0$, respectively; and $\delta_i = P\left(X_{ij} > 0\right)$. This distribution is a combination of lognormal and binomial distributions. The numbers of positive and zero observations are defined as n_{i1} and n_{i0}, respectively, where $n_i = n_{i1} + n_{i0}$. According to Aitchison (1955), the mean and variance of a delta-lognormal distribution are defined as

$$E\left(X_{ij}\right) = \delta_i \exp\left(\mu_i + \frac{\sigma_i^2}{2}\right) \tag{2}$$

and

$$\sigma_i^2 = \delta_i \exp\left(2\mu_i + \sigma_i^2\right)\left[\exp\left(\sigma_i^2\right) - \delta_i\right], \tag{3}$$

respectively. Since the CV computed from σ_i/μ_i, then

$$CV\left(X_{ij}\right) = \eta_i = \left[\frac{\exp\left(\sigma_i^2\right)-\delta_i}{\delta_i}\right]^{\frac{1}{2}}. \tag{4}$$

By using the log-transformation (Yosboonruang et al., 2018), let

$$\varphi_i = \frac{1}{2}\left\{\ln\left[\exp\left(\sigma_i^2\right)-\delta_i\right] - \ln\left(\delta_i\right)\right\}. \tag{5}$$

The unbiased estimators for σ_i^2 and δ_i are $\hat{\sigma}_i^2 = \sum_{j=1}^{n_{i1}}\left[\ln\left(x_{ij}\right)-\hat{\mu}_i\right]^2/\left(n_{i1}-1\right)$ and $\hat{\delta}_i = n_{i1}/n_i$, for $i = 1,2,\ldots,k$, where $\hat{\mu}_i = \sum_{j=1}^{n_{i1}}\ln\left(x_{ij}\right)/n_{i1}$, respectively, then

$$\hat{\varphi}_i = \frac{1}{2}\left\{\ln\left[\exp\left(\hat{\sigma}_i^2\right)-\hat{\delta}_i\right] - \ln\left(\hat{\delta}_i\right)\right\}. \tag{6}$$

The approximately unbiased estimate variance of $\hat{\varphi}_i$ is

$$\hat{V}\left(\hat{\varphi}_i\right) \approx \frac{\left(\hat{b}_i-\hat{a}_i\right)\left(1-\hat{a}_i\hat{b}_i\right)-n_{i1}\left(1-\hat{a}_i\right)^2}{4n_{i1}\left(1-\hat{a}_i\right)^2} + \frac{\hat{\sigma}_i^4}{2\left(n_{i1}-1\right)}, \tag{7}$$

where $\hat{a}_i = \left(1-\hat{\delta}_i\right)^{n_i-1}$ and $\hat{b}_i = 1 + \left(n_i - 1\right)\hat{\delta}_i$. The ordinary form of the common log-transformed CV is given by

$$\tilde{\varphi} = \frac{\sum_{i=1}^{k} w_i\hat{\varphi}_i}{\sum_{i=1}^{k} w_i}, \tag{8}$$

where $w_i = 1/\hat{V}\left(\hat{\varphi}_i\right)$. Accordingly, the common CV is defined as

$$\tilde{\eta} = \exp\left(\tilde{\varphi}\right) = \exp\left(\frac{\sum_{i=1}^{k} w_i\hat{\varphi}_i}{\sum_{i=1}^{k} w_i}\right). \tag{9}$$

In the following section, we propose the methods to establish the confidence intervals for the common CV for delta-lognormal distributions.

FGCI

Let $X_{ij},\ i=1,2,\ldots,k,\ j=1,2,\ldots,n_i$ be a random sample with density function $f(x_{ij};\theta_i,\mu_i)$, where $\theta_i=\left(\delta_i,\sigma_i^2\right)$ are the parameters of interest and μ_i is a nuisance parameter. Let x_{ij} be the observed values of X_{ij}. To construct the FGCI (Weerahandi, 1993; Hannig et al., 2006), the fiducial generalized pivotal quantity (FGPQ) $R(X_{ij};x_{ij},\theta_i,\mu_i)$ is needed to satisfy the following two properties:

1. For a given x_{ij}, the conditional distribution of $R(X_{ij};x_{ij},\theta_i,\mu_i)$ is free of the nuisance parameter.

2. The observed value of $R(X_{ij};x_{ij},\theta_i,\mu_i)$, $r(x_{ij};x_{ij},\theta_i,\mu_i)$, is the parameter of interest.

Given that R_α is the $100\alpha-$th percentile of $R(X_{ij};x_{ij},\theta_i,\mu_i)$, then $\left(R_{\alpha/2},R_{1-\alpha/2}\right)$ becomes the $100(1-\alpha)\%$ two-sided FGCI for θ_i. Therefore, the FGPQs for δ_i and σ_i^2 are necessarily used to construct the confidence interval for common CV $(\tilde{\eta})$.

Consider k individual random samples $X_{i1},X_{i2},\ldots,X_{in_i}$. Following Hannig (2009) and Li et al. (2013), the FGPQ for δ_i is as follows

$$R_{\delta_i}\sim\frac{1}{2}Beta(n_{i1},n_{i0}+1)+\frac{1}{2}Beta(n_{i1}+1,n_{i0}). \tag{10}$$

Similarly, Wu and Hsieh (2014) followed the concept of Krishnamoorthy and Mathew (2003) to find the FGPQ for σ_i^2 defined as

$$R_{\sigma_i^2}=\frac{(n_{i1}-1)\,\hat{\sigma}_i^2}{U_i}, \tag{11}$$

where $U_i\sim\chi^2_{n_{i1}-1}$. To find the FGPQ for $\hat{\varphi}$, we then substitute R_{δ_i} and $R_{\sigma_i^2}$ into Eq (6) as follows:

$$R_{\hat{\varphi}_i}=\frac{1}{2}\left\{\ln\left[\exp\left(R_{\sigma_i^2}\right)-\ln\left(R_{\delta_i}\right)\right]-\ln\left(R_{\delta_i}\right)\right\}. \tag{12}$$

Consequently, the FGPQ for common CV $(\tilde{\eta})$ is

$$R_{\tilde{\eta}}=\exp\left(\frac{\sum_{i=1}^{k}R_{w_i}R_{\hat{\varphi}_i}}{\sum_{i=1}^{k}R_{w_i}}\right), \tag{13}$$

where the FGPQ for an estimated variance of $\hat{\varphi}_i$, for which R_{w_i} is the inverse, is given by

$$R_{\hat{V}(\hat{\varphi}_i)}=\frac{\left(R_{b_i}-R_{a_i}\right)\left(1-R_{a_i}R_{b_i}\right)-n_{i1}\left(1-R_{a_i}\right)^2}{4n_{i1}\left(1-R_{a_i}\right)^2}+\frac{R^2_{\sigma_i^2}}{2\,(n_{i1}-1)}, \tag{14}$$

where $R_{a_i}=\left(1-R_{\delta_i}\right)^{n_i-1}$ and $R_{b_i}=1+(n_i-1)R_{\delta_i}$.

Therefore, $R_{\tilde{\eta}}$ is used to construct the confidence interval for $\tilde{\eta}$. Accordingly, the $100(1-\alpha)\%$ two-sided confidence interval for $\tilde{\eta}$ based on FGCI is $\left(R_{\tilde{\eta}}(\alpha/2),R_{\tilde{\eta}}(1-\alpha/2)\right)$, which denote the $\alpha/2$th and $(1-\alpha/2)$th percentiles of $R_{\tilde{\eta}}$.

Algorithm 1

(For $i=1$ to M)
    Generate $x_{ij},\ i=1,2,\ldots,k,\ j=1,2,\ldots,n_i$ from a delta-lognormal distribution.
    Compute $\hat{\hat{\delta}}_i$ and $\hat{\sigma}_i^2$.
    (For $j=1$ to K)
        Generate $U_i\sim\chi^2_{n_{i1}-1}$, $Beta(n_{i1},n_{i0}+1)$, and $Beta(n_{i1}+1,n_{i0})$.
        Compute $R_{\sigma_i^2}$, R_{δ_i}, $R_{\hat{\varphi}_i}$, and $R_{\tilde{\eta}}$.
    (End j loop)
    Compute the $100(1-\alpha/2)\%$ confidence interval for $\tilde{\eta}$.
(End i loop)

Bayesian methods

Since random samples $X_{ij},\ i=1,2,\ldots,k,\ j=1,2,\ldots,n_i$ have a delta-lognormal distribution with unknown parameters $\phi=\left(\delta_i^*,\mu_i,\sigma_i^2\right)$, where $\delta_i^*=1-\delta_i$, the likelihood function of k-individual random samples can be expressed as

$$L(\phi \mid x_{ij}) \propto \prod_{i=1}^{k} (\delta_i^*)^{n_{i0}}\, \delta_i^{n_{i1}} \left(\sigma_i^2\right)^{-\frac{n_{i1}}{2}} \exp\left\{-\frac{1}{2\sigma_i^2}\sum_{j=1}^{n_{i1}}\left[\ln(x_{ij})-\mu_i\right]^2\right\}. \tag{15}$$

Subsequently, the Fisher information matrix of ϕ derived from the partial derivative of the log-likelihood function with respect to $\delta_i,\ \mu_i$, and σ_i^2 is

$$I(\phi)=\operatorname{diag}\left[\frac{n_1}{\delta_1^*\delta_1}\quad \frac{n_1\delta_1}{\sigma_1^2}\quad \frac{n_1\delta_1}{2\left(\sigma_1^2\right)^2}\quad \cdots\quad \cdots\quad \cdots\quad \frac{n_k}{\delta_k^*\delta_k}\quad \frac{n_k\delta_k}{\sigma_k^2}\quad \frac{n_k\delta_k}{2\left(\sigma_k^2\right)^2}\right]. \tag{16}$$

In the present study, the Bayesian method is used to construct the equal-tailed confidence interval and the credible interval for the common CV. In the following section, we propose the independent Jeffreys and uniform priors.

The Bayesian method using the independent Jeffreys prior

It is accepted that the Jeffreys' prior of unknown parameter ϕ is derived from the square root of the determinant of the Fisher information matrix $I(\phi)$, defined as $p(\phi)=\sqrt{|I(\phi)|}$. Since the parameters of interest $\vartheta=\left(\delta_i^*,\sigma_i^2\right)$, the independent Jeffreys prior for δ_i and σ_i^2 are $p(\delta_i)\propto(\delta_i^*)^{-\frac{1}{2}}\,\delta_i^{-\frac{1}{2}}$ and $p\left(\sigma_i^2\right)\propto 1/\sigma_i^2$ (Harvey and van der Merwe, 2012), respectively. Since δ_i^* and σ_i^2 are independent, then the independent Jeffreys prior for a delta-lognormal distribution is $p(\vartheta)\propto\prod_{i=1}^{k}\sigma_i^{-2}(\delta_i^*)^{-\frac{1}{2}}\,\delta_i^{-\frac{1}{2}}$. Therefore, the joint posterior density of ϕ can be written as

$$\begin{aligned} p(\phi \mid x_{ij}) =& \prod_{i=1}^{k}\frac{1}{Beta\left(n_{i0}+\frac{1}{2},n_{i1}+\frac{1}{2}\right)}(\delta_i^*)^{\left(n_{i0}+\frac{1}{2}\right)-1}\,\delta_i^{\left(n_{i1}+\frac{1}{2}\right)-1}\frac{1}{\sqrt{2\pi}\frac{\sigma_i}{\sqrt{n_{i1}}}}\exp\left[-\frac{1}{2\frac{\sigma_i^2}{n_{i1}}}\left(\mu_i-\hat{\mu}_i\right)^2\right] \\ &\times\frac{\left[\frac{(n_{i1}-1)\hat{\sigma}_i^2}{2}\right]^{\frac{n_{i1}-1}{2}}}{\Gamma\left(\frac{n_{i1}-1}{2}\right)}\left(\sigma_i^2\right)^{-\frac{n_{i1}-1}{2}-1}\exp\left[-\frac{\frac{(n_{i1}-1)\hat{\sigma}_i^2}{2}}{\sigma_i^2}\right], \end{aligned} \tag{17}$$

where $\hat{\mu}_i=\sum_{j=1}^{n_{i1}}\ln(x_{ij})/n_{i1}$, and $\hat{\sigma}_i^2=\sum_{j=1}^{n_{i1}}\left[\ln(x_{ij})-\hat{\mu}_i\right]^2/(n_{i1}-1)$. This leads to the posterior density of δ_i^* given by

$$p(\delta_i^* \mid x_{ij}) \propto \prod_{i=1}^{k}\frac{1}{Beta\left(n_{i0}+\frac{1}{2},n_{i1}+\frac{1}{2}\right)}(\delta_i^*)^{\left(n_{i0}+\frac{1}{2}\right)-1}\,\delta_i^{\left(n_{i1}+\frac{1}{2}\right)-1}, \tag{18}$$

which is a beta distribution with parameters $n_{i0}+1/2$ and $n_{i1}+1/2$, denoted by $\delta_i^* \mid x_{ij}\sim Beta\left(n_{i0}+1/2,n_{i1}+1/2\right)$. Similarly, the posterior density of σ_i^2 can be derived as

$$p\left(\sigma_i^2 \mid x_{ij}\right) \propto \frac{\left[\frac{(n_{i1}-1)\hat{\sigma}_i^2}{2}\right]^{\frac{n_{i1}-1}{2}}}{\Gamma\left(\frac{n_{i1}-1}{2}\right)}\left(\sigma_i^2\right)^{-\frac{n_{i1}-1}{2}-1}\exp\left[-\frac{\frac{(n_{i1}-1)\hat{\sigma}_i^2}{2}}{\sigma_i^2}\right], \tag{19}$$

which is in the general form of an inverse gamma distribution denoted by $\sigma_i^2 \mid x_{ij}\sim Inv-Gamma\left[(n_{i1}-1)/2,(n_{i1}-1)\hat{\sigma}_i^2/2\right]$.

The Bayesian method using the uniform prior

Because all possible values are equally likely *a priori* for the uniform prior, then it is a constant function of *a priori* probability (Stone, 2013; O'Reilly and Mars, 2015). According to Bolstad and Curran (2016), the uniform prior for δ_i^* and σ_i^2 are proportional to 1, which can be defined as $p(\delta_i^*)\propto 1$ and $p\left(\sigma_i^2\right)\propto 1$, respectively. It is well-known that δ_i^* is independent of σ_i^2, thereby the uniform prior for the parameters of interest for a delta-lognormal distribution is $p\left(\delta_i^*, \sigma_i^2\right) \propto 1$. Accordingly, the joint posterior density function is defined as

$$p(\phi \mid x_{ij}) = \prod_{i=1}^{k} \frac{1}{Beta(n_{i0}+1, n_{i1}+1)} (\delta_i^*)^{n_{i0}} \delta_i^{n_{i1}} \frac{1}{\sqrt{2\pi}\frac{\sigma_i}{\sqrt{n_{i1}}}} \exp\left[-\frac{1}{2\frac{\sigma_i^2}{n_{i1}}}(\mu_i - \hat{\mu}_i)^2\right] \\ \times \frac{\left[\frac{(n_{i1}-2)\hat{\sigma}_i^2}{2}\right]^{\frac{n_{i1}-2}{2}}}{\Gamma\left(\frac{n_{i1}-2}{2}\right)} \left(\sigma_i^2\right)^{-\frac{n_{i1}-2}{2}-1} \exp\left[-\frac{\frac{(n_{i1}-2)\hat{\sigma}_i^2}{2}}{\sigma_i^2}\right], \tag{20}$$

where $\hat{\mu}_i = \sum_{j=1}^{n_{i1}} \ln(x_{ij})/n_{i1}$, and $\hat{\sigma}_i^2 = \sum_{j=1}^{n_{i1}} [\ln(x_{ij}) - \hat{\mu}_i]^2/(n_{i1}-1)$. We can derived the posterior density of δ_i^* as

$$p(\delta_i^* \mid x_{ij}) \propto \prod_{i=1}^{k} \frac{1}{Beta(n_{i0}+1, n_{i1}+1)} (\delta_i^*)^{n_{i0}} \delta_i^{n_{i1}}, \tag{21}$$

which is consequently a density function of a beta distribution, i.e. $\delta_i^* \mid x_{ij} \sim Beta(n_{i0}+1, n_{i1}+1)$. For σ_i^2, the posterior density has an inverse gamma distribution with respective shape and scale parameters $(n_{i1}-2)/2$ and $(n_{i1}-2)\hat{\sigma}_i^2/2$ which expressed as

$$p\left(\sigma_i^2 \mid x_{ij}\right) \propto \prod_{i=1}^{k} \frac{\left(\frac{(n_{i1}-2)\hat{\sigma}_i^2}{2}\right)^{\frac{n_{i1}-2}{2}}}{\Gamma\left(\frac{n_{i1}-2}{2}\right)} \left(\sigma_i^2\right)^{-\frac{n_{i1}-2}{2}-1} \exp\left(-\frac{\frac{(n_{i1}-2)\hat{\sigma}_i^2}{2}}{\sigma_i^2}\right). \tag{22}$$

Subsequently, we construct the confidence intervals and the credible intervals for the common CV by substituting the posterior densities of δ_i^* and σ_i^2 from the independent Jeffreys and uniform priors into Eqs (6), (7), and (9).

Algorithm 2
(For $i = 1$ to M)
  Generate x_{ij}, $i = 1, 2, \ldots, k$, $j = 1, 2, \ldots, n_i$ from a delta-lognormal distribution.
  Compute $\hat{\delta}_i$ and $\hat{\sigma}_i^2$.
  (For $j = 1$ to K)
    Generate the posterior densities of $\delta_i^* \mid x_{ij}$.
      1. Independent Jeffreys prior: $\delta_i^* \mid x_{ij} \sim Beta\left(n_{i0}+\frac{1}{2}, n_{i1}+\frac{1}{2}\right)$.
      2. Uniform prior: $\delta_i^* \mid x_{ij} \sim Beta(n_{i0}+1, n_{i1}+1)$.
    Generate the posterior densities of $\sigma_i^2 \mid x_{ij}$.
      1. Independent Jeffreys prior: $\sigma_i^2 \mid x_{ij} \sim Inv-Gamma\left[\frac{n_{i1}-1}{2}, \frac{(n_{i1}-1)\hat{\sigma}_i^2}{2}\right]$.
      2. Uniform prior: $\sigma_i^2 \mid x_{ij} \sim Inv-Gamma\left[\frac{n_{i1}-2}{2}, \frac{(n_{i1}-2)\hat{\sigma}_i^2}{2}\right]$.
    Compute $\hat{\varphi}_i$, $\hat{V}(\hat{\varphi}_i)$, and $\tilde{\eta}$.
  (End j loop)
  Compute the $100(1-\alpha/2)\%$ confidence intervals and credible intervals for $\tilde{\eta}$.
(End i loop)

MOVER

Following the method of Zou and Donner (2008), let φ_1 and φ_2 be the parameter of interest and then let $\hat{\varphi}_1$ and $\hat{\varphi}_2$ be the independent estimators of φ_1 and φ_2, respectively. Furthermore, the lower and upper confidence limits for φ_1+φ_2 are

$$CI_{\varphi_1+\varphi_2} = \left[L_{\varphi_1+\varphi_2}, U_{\varphi_1+\varphi_2}\right] = \hat{\varphi}_1 + \hat{\varphi}_2 \pm z_{\alpha/2}\sqrt{\widehat{Var}(\hat{\varphi}_1) + \widehat{Var}(\hat{\varphi}_2)}, \tag{23}$$

Subsequently, let l_i and u_i, for $i = 1, 2$, be the lower and upper bounds of the confidence interval for φ_i, respectively. Since l_i and u_i provide the possible parameter values, then $l_1 + l_2$ is close to $L_{\varphi_1+\varphi_2}$ and

u_1+u_2 is close to $U_{\varphi_1+\varphi_2}$. To obtain the lower limit $L_{\varphi_1+\varphi_2}$, the estimated variance of $\hat{\varphi}_i$ at $\varphi_i = l_i$ is given by

$$\widehat{Var}\left(\hat{\varphi}_{l_i}\right) = \frac{\left(\hat{\varphi}_i - l_i\right)^2}{z^2_{\alpha/2}}. \tag{24}$$

Similarly, to obtain the upper limit $U_{\varphi_1+\varphi_2}$, the estimated variance of $\hat{\varphi}_i$ at $\varphi_i = u_i$ is given by

$$\widehat{Var}\left(\hat{\varphi}_{u_i}\right) = \frac{\left(u_i - \hat{\varphi}_i\right)^2}{z^2_{\alpha/2}}. \tag{25}$$

Next, by substituting $\widehat{Var}\left(\hat{\varphi}_{l_i}\right)$ and $\widehat{Var}\left(\hat{\varphi}_{u_i}\right)$ into Eq (23), we obtain

$$L_{\varphi_1+\varphi_2} = \hat{\varphi}_1 + \hat{\varphi}_2 - \sqrt{\left(\hat{\varphi}_1 - l_1\right)^2 + \left(\hat{\varphi}_2 - l_2\right)^2} \tag{26}$$

and

$$U_{\varphi_1+\varphi_2} = \hat{\varphi}_1 + \hat{\varphi}_2 - \sqrt{\left(u_1 - \hat{\varphi}_1\right)^2 + \left(u_2 - \hat{\varphi}_2\right)^2}. \tag{27}$$

Thereby, the unbiased estimate variance of $\hat{\varphi}_i$ at $\varphi_i = l_i$ and $\varphi_i = u_i$ can be expressed as

$$\widehat{Var}\left(\hat{\varphi}_i\right) = \frac{1}{2}\left[\frac{\left(\hat{\varphi}_i - l_i\right)^2}{z^2_{\alpha/2}} + \frac{\left(u_i - \hat{\varphi}_i\right)^2}{z^2_{\alpha/2}}\right],\ i = 1,2. \tag{28}$$

When this concept is extended to k parameters, the lower and upper confidence limits for $\upsilon = \sum_{i=1}^{k} \varphi_i$ are given by

$$L_\upsilon = \upsilon - \sqrt{\left(\hat{\varphi}_1 - l_1\right)^2 + \left(\hat{\varphi}_2 - l_2\right)^2 + \ldots + \left(\hat{\varphi}_k - l_k\right)^2} \tag{29}$$

and

$$U_\upsilon = \upsilon + \sqrt{\left(u_1 - \hat{\varphi}_1\right)^2 + \left(u_2 - \hat{\varphi}_2\right)^2 + \ldots + \left(u_k - \hat{\varphi}_k\right)^2}. \tag{30}$$

According to Krishnamoorthy and Oral (2017) and recall the common log-transformed CV from Eq (8), the upper and lower confidence limits for σ_i^2 and δ_i are required to construct the confidence interval for the common CV of delta-lognormal distributions. Since the estimate of σ_i^2 is

$$\hat{\sigma}_i^2 = \frac{1}{n_{i1}-1}\sum_{j=1}^{n_{i1}}\left[\ln\left(x_{ij}\right) - \hat{\mu}_i\right]^2, \tag{31}$$

where $\left(n_{i1}-1\right)\hat{\sigma}_i^2/\sigma_i^2 \sim \chi^2_{n_{i1}-1}$, the $100\left(1-\alpha\right)\%$ confidence interval for σ_i^2 is derived as

$$CI_{\sigma_i^2} = \left(l_{\sigma_i^2}, u_{\sigma_i^2}\right) = \left[\frac{\left(n_{i1}-1\right)\hat{\sigma}_i^2}{\chi^2_{1-\alpha/2,n_{i1}-1}}, \frac{\left(n_{i1}-1\right)\hat{\sigma}_i^2}{\chi^2_{\alpha/2,n_{i1}-1}}\right]. \tag{32}$$

To construct the confidence interval for δ_i, the concept of the variance stabilizing transformation proposed by DasGupta (2008) and Wu and Hsieh (2014) was used. Therefore, the confidence interval for δ_i is given by

$$CI_{\delta_i} = \left(l_{\delta_i}, u_{\delta_i}\right) = \sin^2\left(\arcsin\sqrt{\hat{\delta}_i} \pm \frac{1}{2\sqrt{n_i}} Z_{i(1-\alpha/2)}\right). \tag{33}$$

Since $\varphi_i = \left\{\ln\left[\exp\left(\sigma_i^2\right) - \delta_i\right] - \ln\left(\delta_i\right)\right\}/2$, then let

$$l_i = \frac{1}{2}\left\{\ln\left[\exp\left(l_{\sigma_i^2}\right) - l_{\delta_i}\right] - \ln\left(l_{\delta_i}\right)\right\} \tag{34}$$

and

$$u_i = \frac{1}{2}\left\{\ln\left[\exp\left(u_{\sigma_i^2}\right) - u_{\delta_i}\right] - \ln\left(u_{\delta_i}\right)\right\}, \tag{35}$$

for $i = 1, 2, \ldots, k$. Therefore, the $100(1-\alpha)\,\%$ confidence interval for $\tilde{\eta}$ based on MOVER can be written as

$$CI_{\tilde{\eta}}^{m} = \left[L_{\tilde{\eta}}^{m}, U_{\tilde{\eta}}^{m}\right], \tag{36}$$

where

$$L_{\tilde{\eta}}^{m} = \exp\left(\tilde{\varphi} - \sqrt{\sum_{i=1}^{k} w_i^2\left(\hat{\varphi}_i - l_i\right)^2 / \sum_{i=1}^{k} w_i^2}\right) \tag{37}$$

and

$$U_{\tilde{\eta}}^{m} = \exp\left(\tilde{\varphi} + \sqrt{\sum_{i=1}^{k} w_i^2\left(u_i - \hat{\varphi}_i\right)^2 / \sum_{i=1}^{k} w_i^2}\right). \tag{38}$$

Algorithm 3
(For $i = 1$ to M)
  Generate x_{ij}, $i = 1, 2, \ldots, k$, $j = 1, 2, \ldots, n_i$ from a delta-lognormal distribution.
  Compute $\hat{\delta}_i$ and $\hat{\sigma}_i^2$.
  Compute $\hat{\varphi}_i$ and $\hat{V}(\hat{\varphi}_i)$.
  Compute $l_{\sigma_i^2}$, $u_{\sigma_i^2}$, l_{δ_i}, u_{δ_i}, l_i, u_i.
  Compute the $100(1-\alpha/2)\,\%$ confidence interval for $\tilde{\eta}$.
(End i loop)

RESULTS AND DISCUSSION

Monte Carlo simulation studies

The R statistical program via Monte Carlo simulation was used to compute the coverage probabilities and expected lengths for evaluating the performances of the proposed confidence intervals, including FGCI, the Bayesian approach based on the independent Jeffreys and uniform priors, and MOVER. The criteria for choosing the best performing confidence interval were coverage probabilities equal to or greater than the nominal confidence level of 0.95 and the shortest expected length. To generate the data, we set the number of populations as $k = 3, 5, 10$; sample sizes as $n_1 = n_2 = \ldots = n_k = n = 25, 50, 100$; probabilities of non-zero values as $\delta_1 = \delta_2 = \ldots = \delta_k = \delta = 0.2, 0.5, 0.8$; and variances as $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2 = \sigma^2 = 0.5, 1.0, 2.0$. For each combination of parameters, 10,000 simulation runs were generated together with 2,000 replications for FGCI and the Bayesian approaches by applying Algorithms 1 and 2, respectively.

The coverage probabilities and expected lengths of the 95% confidence and credible intervals for the common CV of delta-lognormal distributions for all k are reported in Tables 1 - 3. In addition, the coverage probabilities and expected lengths of the proposed methods for various sample sizes, probabilities of non-zero values, and variances are shown in Figs 1 - 6, respectively. The results for k parameters show that the coverage probabilities of the equal-tailed Bayesian based on the independent Jeffreys and uniform priors were consistency close to the nominal confidence level in almost all cases, while the other methods were close to or greater than the nominal confidence level for some cases. Furthermore, in terms of the expected lengths, the equal-tailed based on independent Jeffreys prior were shorter than the uniform prior for all cases. In addition, the expected lengths of the Bayesian credible interval based on the independent Jeffreys prior were shorter than the others in almost every case when $\sigma_i^2 = 0.5$. For all k and sample sizes together with $\sigma_i^2 = 1, 2$, the expected lengths of the equal-tailed Bayesian based on the independent Jeffreys prior were the shortest when $\delta_i = 0.2, 0.5$, while MOVER had the shortest expected lengths for $\delta_i = 0.8$. For FGCI, the coverage probabilities and their expected lengths were very wide for all cases in which it is not reasonable for the construction of confidence interval. However, the equal-tailed Bayesian based on the independent Jeffreys prior is suitable for constructing the confidence interval for the common CV in delta-lognormal distribution since the coverage probabilities were close to the target for almost all cases, although the expected lengths were not always shorter than the other methods in some cases.

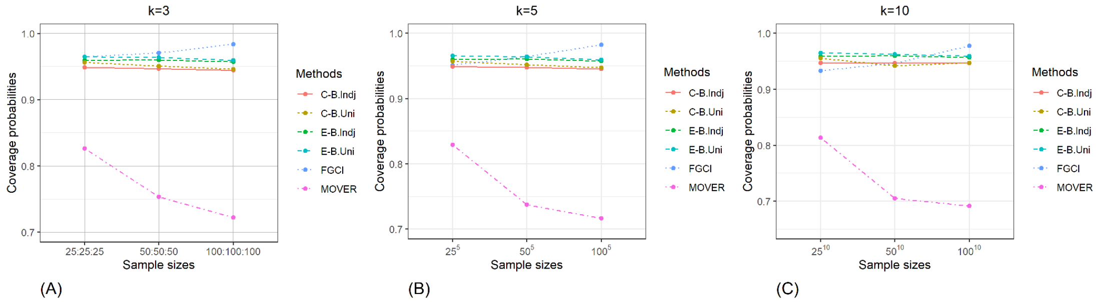

Figure 1. Comparison of the coverage probabilities of the proposed methods according to sample sizes for (A) $k = 3$ (B) $k = 5$ (C) $k = 10$.

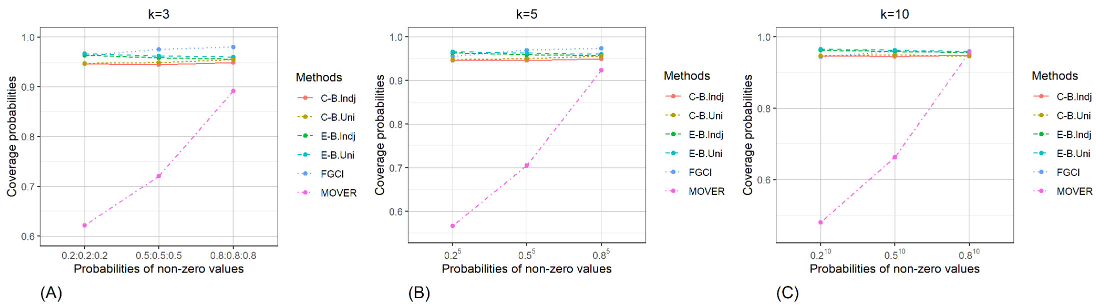

Figure 2. Comparison of the coverage probabilities of the proposed methods according to probabilities of non-zero values for (A) $k = 3$ (B) $k = 5$ (C) $k = 10$.

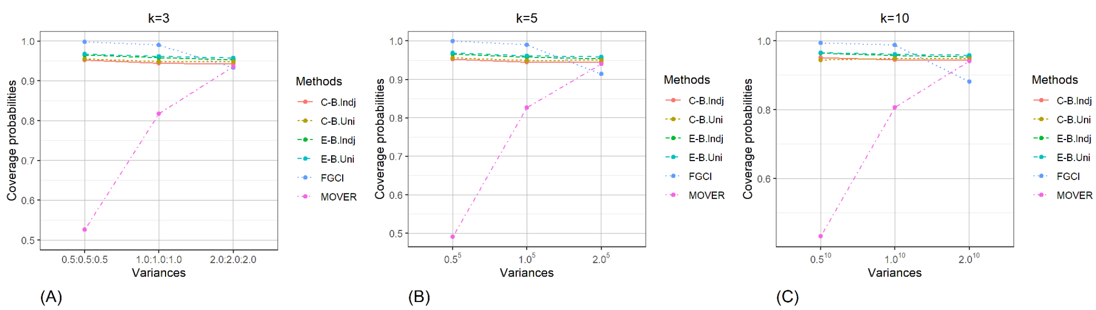

Figure 3. Comparison of the coverage probabilities of the proposed methods according to variances for (A) $k = 3$ (B) $k = 5$ (C) $k = 10$.

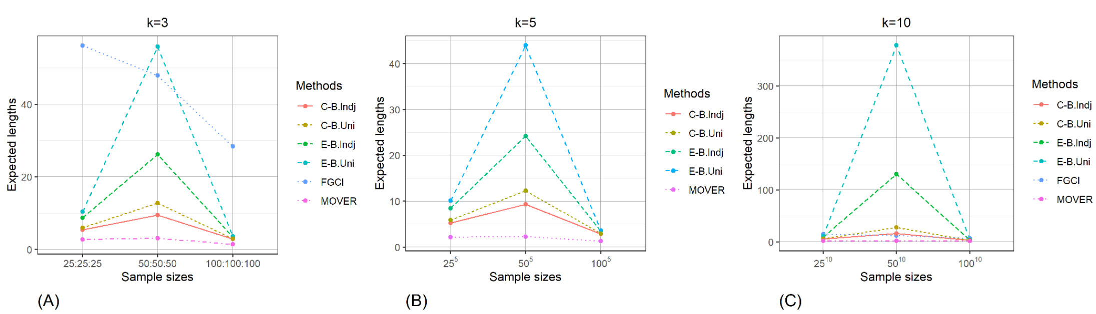

Figure 4. Comparison of the expected lengths of the proposed methods according to sample sizes for (A) $k = 3$ (B) $k = 5$ (C) $k = 10$.

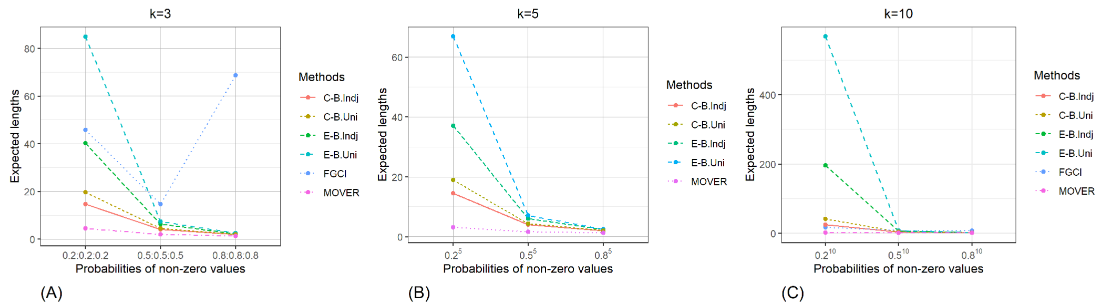

Figure 5. Comparison of the expected lengths of the proposed methods according to probabilities of non-zero values for (A) $k = 3$ (B) $k = 5$ (C) $k = 10$.

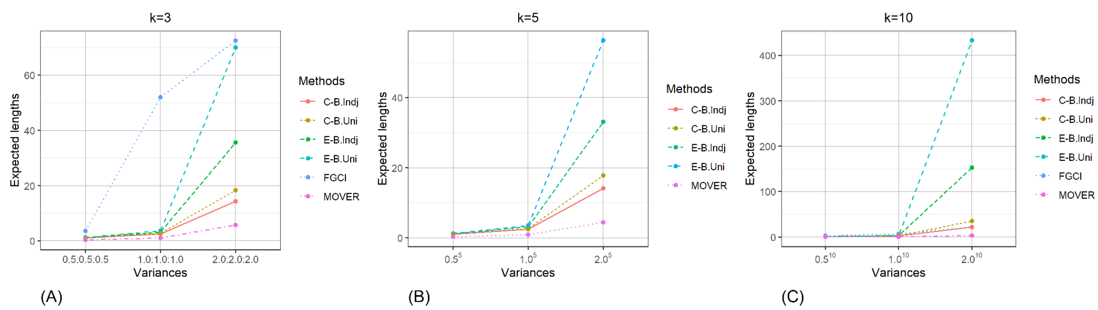

Figure 6. Comparison of the expected lengths of the proposed methods according to variances for (A) $k = 3$ (B) $k = 5$ (C) $k = 10$.

Application of the methods to real datasets

Datasets of daily rainfall from Chiang Klang, Tha Wang Pha, and Pua in Nan, Thailand, were obtained from the Upper Northern Region Irrigation Hydrology Center and used to demonstrate the applicability of the proposed methods for generating confidence intervals. The reason for using these datasets is because these areas are near the origin of the Nan River that flows into the Chao Phraya River. Moreover, throughout the year, the precipitation in Nan fluctuates between a precipitation deficit and heavy rainfall. The latter accompanied by thunderstorms occurs in the late summer period, and due to the southwest monsoon, the amount of rainfall increases from mid-May to early October with the highest rainfall frequently in August or September, which can cause flooding in some areas (Thai Meteorological Department, 2015).

In this study, the rainfall data from the three areas in Nan province in August 2018 and 2019 were selected. These data contain both true zero and non-zero values, as presented in Fig 7. Since the non-zero observations follow a right-skewed distribution, the minimum Akaike information criterion (AIC) and the lowest Bayesian information criterion (BIC) were used to test the distribution of these data. The AIC and BIC values in Tables 4 and 5, respectively, indicate that the non-zero observations from the three areas conform to lognormal distributions. Furthermore, the normal Q-Q plots via the log-transformation of non-zero observations shown in Fig 8 reveal that they follow normal distributions. By testing the non-zero observations together with the binomial distributions of the true zero observations indicate that the daily rainfall data from the three areas follow delta-lognormal distributions.

The summary statistics for the three rainfall datasets were $n_1 : n_2 : n_3 = 62 : 62 : 62$; $\hat{\delta}_1 : \hat{\delta}_2 : \hat{\delta}_3 = 0.7258 : 0.7903 : 0.7419$; $\hat{\mu}_1 : \hat{\mu}_2 : \hat{\mu}_3 = 2.1189 : 1.6448 : 1.8971$; $\hat{\sigma}_1^2 : \hat{\sigma}_2^2 : \hat{\sigma}_3^2 = 1.7857 : 3.4406 : 1.8346$; and $\tilde{\eta} = 3.2011$. Table 6 reports the 95% confidence intervals and credible intervals for the common CV of the rainfall datasets from three areas in Nan province, Thailand. The results reveal that the confidence intervals of all three methods could cover the parameter, which corresponds well with the simulation results. However, the expected length of FGCI was the shortest, thereby making it a good choice for estimating the common variance in the dispersion of precipitation from the three areas in Nan province, Thailand.

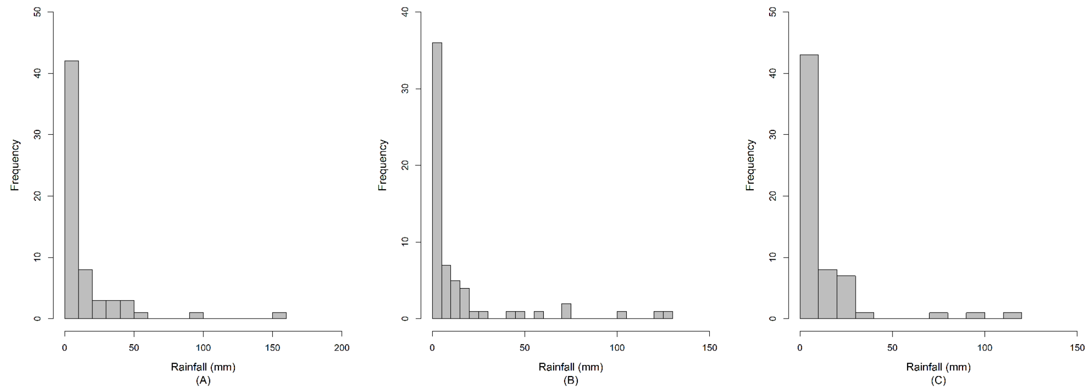

Figure 7. Histograms of the rainfall data from (A) Chiang Klang, (B) Tha Wang Pha, and (C) Pua in Nan, Thailand.

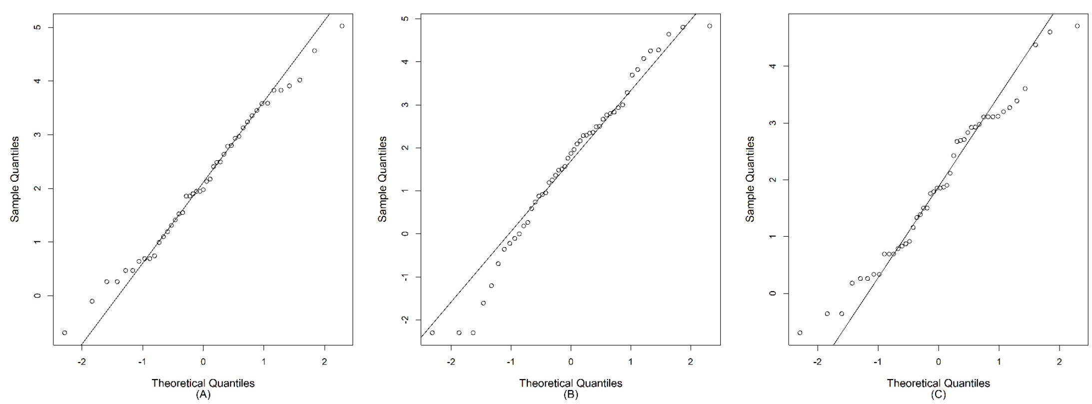

Figure 8. The normal Q-Q plots of the log-transformation of the positive rainfall data from (A) Chiang Klang, (B) Tha Wang Pha, and (C) Pua in Nan, Thailand.

CONCLUSION

Herein, we proposed methods to construct the confidence intervals for the common CV of delta-lognormal distribution, including FGCI, two Bayesian approaches constructed under the equal-tailed confidence intervals and credible intervals using the independent Jeffreys and uniform priors, and MOVER. The performances of the proposed methods were determined via their coverage probabilities together with their expected lengths under various circumstances. The results indicate that the equal-tailed Bayesian based on the independent Jeffreys prior outperformed the other methods in terms of the coverage probability. Moreover, the equal-tailed Bayesian based on the uniform prior can be used as an alternative. When considering the coverage probabilities together with the expected lengths, the Bayesian credible interval based on the independent Jeffreys prior should be chosen for cases with small variance, while MOVER is the best choice for cases with a high proportion of non-zero values together with large variance.

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Table 1. The coverage probabilities and expected lengths of 95% two-sided confidence intervals for the common CV of delta-lognormal distributions in case of $k = 3$.

n_i	δ_i	σ_i^2	Coverage probabilities (Expected lengths)					
			FGCI	E-B.Indj	E-B.Uni	C-B.Indj	C-B.Uni	MOVER
25	0.5	0.5	**0.9983**	**0.9689**	**0.9734**	**0.9518**	**0.9564**	0.5541
			(4.2363)	(1.5221)	(1.6132)	**(1.3548)**	(1.4144)	(0.4478)
		1.0	**0.9775**	**0.9613**	**0.9672**	0.9429	**0.9518**	0.7872
			(28.1984)	**(4.2206)**	(4.7049)	(3.2428)	(3.5018)	(1.4839)
		2.0	0.9027	**0.9528**	0.9600	0.9408	0.9485	0.9133
			(7.4009)	**(34.3846)**	(42.9115)	(18.1504)	(20.8831)	(8.0816)
	0.8	0.5	**0.9994**	**0.9658**	**0.9700**	**0.9575**	**0.9687**	0.7894
			(9.1319)	(0.8848)	(0.9205)	**(0.8193)**	(0.8486)	(0.3732)
		1.0	**0.9890**	**0.9563**	**0.9623**	**0.9526**	**0.9589**	0.9445
			(274.3619)	(2.0822)	(2.1950)	**(1.7915)**	(1.8716)	(1.1456)
		2.0	0.9204	**0.9518**	**0.9567**	0.9464	**0.9548**	0.9705
			(13.9706)	(9.4530)	(10.2449)	(6.8908)	**(7.3329)**	(4.7949)
50	0.2	0.5	**0.9940**	**0.9734**	**0.9721**	**0.9589**	**0.9580**	0.4001
			(5.3666)	(3.0387)	(3.3163)	**(2.5773)**	(2.7137)	(0.7416)
		1.0	**0.9668**	**0.9663**	**0.9700**	0.9436	0.9473	0.6903
			(61.6887)	**(10.2196)**	(12.7965)	(6.8382)	(7.7720)	(2.3985)
		2.0	0.8941	**0.9573**	**0.9674**	0.9403	0.9473	0.8591
			(10.2166)	**(205.9432)**	(470.3159)	(61.6888)	(89.7961)	(16.4537)
	0.5	0.5	**0.9991**	**0.9665**	**0.9689**	**0.9517**	**0.9533**	0.4400
			(2.3261)	(0.9184)	(0.9322)	**(0.8743)**	(0.8851)	(0.2156)
		1.0	**0.9977**	**0.9554**	**0.9608**	0.9435	0.9462	0.7862
			(27.7099)	**(2.0097)**	(2.0670)	(1.8046)	(1.8466)	(0.7810)
		2.0	0.9383	0.9498	**0.9545**	0.9385	0.9434	0.9348
			(24.2678)	(8.0467)	**(8.3999)**	(6.4058)	(6.6170)	(3.8741)
	0.8	0.5	**0.9997**	**0.9607**	**0.9665**	**0.9507**	**0.9593**	0.7357
			(1.6860)	(0.5521)	(0.5606)	**(0.5316)**	(0.5397)	(0.2003)
		1.0	**0.9980**	**0.9563**	**0.9584**	0.9458	**0.9513**	**0.9516**
			(5.1781)	(1.1695)	(1.1911)	(1.0905)	(1.1088)	**(0.7058)**
		2.0	0.9464	**0.9521**	**0.9554**	0.9483	**0.9515**	**0.9824**
			(293.3686)	(3.9878)	(4.0900)	(3.4771)	(3.5501)	**(2.7229)**
100	0.2	0.5	**0.9965**	**0.9691**	**0.9679**	**0.9508**	0.9470	0.2665
			(3.4026)	**(1.6493)**	(1.6684)	(1.5542)	(1.5659)	(0.3647)
		1.0	**0.9947**	**0.9613**	**0.9631**	0.9433	0.9420	0.6284
			(13.0351)	**(3.7143)**	(3.8315)	(3.2441)	(3.3177)	(1.0735)
		2.0	0.9326	**0.9547**	**0.9601**	0.9384	0.9410	0.8846
			(181.1185)	**(16.6849)**	(17.6932)	(12.4128)	(12.9191)	(6.1118)
	0.5	0.5	**0.9996**	**0.9604**	**0.9632**	0.9495	**0.9501**	0.3157
			(1.3697)	(0.6091)	(0.6121)	(0.5933)	**(0.5961)**	(0.1058)
		1.0	**0.9990**	**0.9553**	**0.9574**	0.9379	0.9415	0.7982
			(3.7435)	**(1.2257)**	(1.2378)	(1.1620)	(1.1725)	(0.4857)
		2.0	**0.9668**	**0.9545**	**0.9548**	0.9442	0.9475	**0.9549**
			(32.9162)	(4.0100)	(4.0691)	(3.6039)	(3.6477)	**(2.4343)**
	0.8	0.5	1.0000	**0.9562**	**0.9595**	0.9465	**0.9503**	0.7093
			(1.0189)	(0.3729)	(0.3754)	(0.3648)	**(0.3672)**	(0.1204)
		1.0	**0.9992**	**0.9537**	**0.9562**	0.9443	0.9465	**0.9560**
			(2.3875)	(0.7571)	(0.7631)	(0.7289)	(0.7344)	**(0.4796)**
		2.0	**0.9662**	0.9496	**0.9522**	0.9445	0.9479	**0.9872**
			(16.9898)	(2.3221)	(2.3448)	(2.1655)	(2.1851)	**(1.7740)**

Note: E-B.Indj and E-B.Uni represented the respective equal-tailed Bayesian intervals based on independent Jeffreys and uniform priors, and C-B.Indj and C-B.Uni represented the respective Bayesian credible intervals based on independent Jeffrey's and uniform priors.

Table 2. The coverage probabilities and expected lengths of 95% two-sided confidence intervals for the common CV of delta-lognormal distributions in case of $k = 5$.

n_i	δ_i	σ_i^2	Coverage probabilities (Expected lengths)					
			FGCI	E-B.Indj	E-B.Uni	C-B.Indj	C-B.Uni	MOVER
25	0.5	0.5	**0.9997**	**0.9710**	**0.9746**	**0.9531**	**0.9595**	0.4830
			(3.7038)	(1.5339)	(1.6273)	**(1.3637)**	(1.4246)	(0.3499)
		1.0	**0.9737**	**0.9600**	**0.9647**	0.9423	**0.9506**	0.7917
			(8.9088)	(4.2573)	(4.7602)	(3.2717)	**(3.5364)**	(1.2034)
		2.0	0.8603	**0.9541**	**0.9613**	0.9410	0.9483	0.9096
			(1933.3646)	**(32.2422)**	(40.4745)	(17.2785)	(19.8882)	(5.9389)
	0.8	0.5	1.0000	**0.9639**	**0.9713**	**0.9574**	**0.9671**	0.8460
			(3.4724)	(0.8808)	(0.9159)	**(0.8160)**	(0.8447)	(0.3396)
		1.0	**0.9836**	**0.9583**	**0.9637**	**0.9510**	**0.9614**	**0.9642**
			(10.7410)	(2.0830)	(2.1955)	(1.7916)	(1.8719)	**(1.0345)**
		2.0	0.8901	**0.9517**	**0.9576**	0.9493	**0.9560**	**0.9807**
			(3.67E+04)	(9.7354)	(10.5885)	(7.0602)	(7.5199)	**(4.1883)**
50	0.2	0.5	**0.9987**	**0.9743**	**0.9730**	**0.9556**	**0.9546**	0.2693
			(5.1024)	(3.0276)	(3.2924)	**(2.5680)**	(2.7012)	(0.5583)
		1.0	**0.9647**	**0.9655**	**0.9690**	0.9441	0.9467	0.6413
			(16.6022)	**(10.0192)**	(12.3389)	(6.7304)	(7.6160)	(1.6963)
		2.0	0.8562	**0.9539**	**0.9636**	0.9407	0.9498	0.8377
			(3.08E+05)	**(187.9496)**	(362.9737)	(60.4077)	(85.9882)	(10.2873)
	0.5	0.5	1.0000	**0.9661**	**0.9688**	**0.9519**	**0.9550**	0.3610
			(2.4601)	(0.9163)	(0.9303)	**(0.8721)**	(0.8834)	(0.1694)
		1.0	**0.9990**	**0.9582**	**0.9620**	0.9446	0.9489	0.8114
			(6.0419)	**(2.0201)**	(2.0770)	(1.8123)	(1.8540)	(0.6894)
		2.0	0.9276	**0.9529**	**0.9570**	0.9440	0.9463	0.9459
			(1419.0445)	**(7.9109)**	(8.2496)	(6.2988)	(6.5140)	(3.4257)
	0.8	0.5	1.0000	**0.9650**	**0.9703**	**0.9546**	**0.9618**	0.8014
			(1.8753)	(0.5542)	(0.5624)	(0.5337)	**(0.5412)**	(0.1868)
		1.0	**0.9993**	**0.9541**	**0.9547**	0.9456	**0.9501**	**0.9760**
			(4.2360)	(1.1716)	(1.1949)	(1.0922)	(1.1119)	**(0.6724)**
		2.0	0.9321	**0.9552**	**0.9576**	**0.9518**	**0.9555**	**0.9932**
			(45.8789)	(4.0252)	(4.1253)	(3.5054)	(3.5809)	**(2.5923)**
100	0.2	0.5	**0.9995**	**0.9667**	**0.9640**	0.9484	0.9459	0.1510
			(3.6099)	(1.6424)	**(1.6607)**	(1.5490)	(1.5595)	(0.2957)
		1.0	**0.9980**	**0.9626**	**0.9638**	0.9438	0.9443	0.6174
			(8.7078)	**(3.7265)**	(3.8501)	(3.2539)	(3.3293)	(0.8938)
		2.0	0.9178	**0.9578**	**0.9612**	0.9435	0.9464	0.8842
			(1302.6130)	**(16.4958)**	(17.4678)	(12.2904)	(12.7883)	(5.0720)
	0.5	0.5	1.0000	**0.9618**	**0.9621**	0.9494	0.9494	0.2473
			(1.5439)	**(0.6091)**	(0.6120)	(0.5933)	(0.5958)	(0.0838)
		1.0	1.0000	**0.9541**	**0.9577**	0.9425	0.9459	0.8244
			(3.9566)	**(1.2291)**	(1.2421)	(1.1653)	(1.1763)	(0.4456)
		2.0	**0.9672**	**0.9541**	**0.9558**	0.9444	0.9464	**0.9717**
			(15.8155)	(4.0068)	(4.0614)	(3.6001)	(3.6428)	**(2.2992)**
	0.8	0.5	1.0000	**0.9607**	**0.9647**	0.9483	**0.9549**	0.7699
			(1.1485)	(0.3738)	(0.3762)	(0.3656)	**(0.3679)**	(0.1127)
		1.0	1.0000	0.9499	**0.9530**	0.9430	0.9461	**0.9857**
			(2.6290)	(0.7549)	(0.7615)	(0.7270)	(0.7329)	**(0.4680)**
		2.0	**0.9598**	**0.9516**	**0.9523**	0.9466	0.9489	**0.9969**
			(6.1687)	(2.3383)	(2.3604)	(2.1803)	(2.1988)	**(1.7304)**

Table 3. The coverage probabilities and expected lengths of 95% two-sided confidence intervals for the common CV of delta-lognormal distributions in case of $k = 10$.

n_i	δ_i	σ_i^2	Coverage probabilities (Expected lengths)					
			FGCI	E-B.Indj	E-B.Uni	C-B.Indj	C-B.Uni	MOVER
25	0.5	0.5	**0.9999**	**0.9682**	**0.9716**	**0.9500**	**0.9550**	0.3282
			(3.7907)	(1.5242)	(1.6165)	**(1.3565)**	(1.4164)	(0.2387)
		1.0	**0.9721**	**0.9568**	**0.9646**	0.9396	0.9486	0.7670
			(7.9902)	**(4.2284)**	(4.7050)	(3.2447)	(3.5022)	(0.9681)
		2.0	0.8170	**0.9544**	**0.9617**	0.9399	0.9486	0.9124
			(27.2845)	**(35.0401)**	(46.9413)	(18.2674)	(21.6147)	(4.5799)
	0.8	0.5	1.0000	**0.9658**	**0.9731**	**0.9566**	**0.9677**	0.9021
			(3.4758)	(0.8807)	(0.9150)	**(0.8155)**	(0.8441)	(0.3032)
		1.0	**0.9796**	**0.9538**	**0.9594**	0.9494	**0.9584**	**0.9804**
			(7.0148)	(2.0894)	(2.2045)	(1.7966)	(1.8782)	**(0.9335)**
		2.0	0.8306	**0.9552**	**0.9601**	0.9488	**0.9557**	**0.9900**
			(37.3845)	(9.6166)	(10.4834)	(6.9970)	(7.4504)	**(3.6911)**
50	0.2	0.5	**0.9979**	**0.9723**	**0.9724**	**0.9541**	**0.9518**	0.1051
			(5.3392)	(3.0554)	(3.3326)	**(2.5815)**	(2.7157)	(0.3897)
		1.0	**0.9650**	**0.9636**	**0.9685**	0.9442	0.9463	0.5255
			(11.0842)	**(10.4725)**	(12.9382)	(6.9200)	(7.8607)	(1.1454)
		2.0	0.8124	**0.9575**	**0.9658**	0.9442	**0.9522**	0.8009
			(53.4270)	(1142.6720)	(3369.2176)	(122.0329)	**(222.4721)**	(6.6526)
	0.5	0.5	1.0000	**0.9672**	**0.9702**	**0.9516**	**0.9553**	0.2112
			(2.6159)	(0.9159)	(0.9302)	**(0.8720)**	(0.8828)	(0.1163)
		1.0	**0.9970**	**0.9596**	**0.9621**	0.9458	**0.9507**	0.8171
			(6.1760)	(2.0213)	(2.0769)	(1.8134)	**(1.8535)**	(0.6006)
		2.0	0.8956	**0.9540**	**0.9584**	0.9415	0.9459	**0.9652**
			(15.6125)	(8.0203)	(8.3825)	(6.3810)	(6.5924)	**(3.0276)**
	0.8	0.5	**0.9596**	**0.9557**	0.9487	0.9433	0.8688	0.9300
			(2.0073)	**(0.5524)**	(0.5606)	(0.5322)	(0.5395)	(0.1726)
		1.0	**0.9975**	**0.9554**	**0.9598**	0.9451	**0.9516**	**0.9910**
			(4.2894)	(1.1691)	(1.1913)	(1.0899)	(1.1089)	**(0.6371)**
		2.0	0.8936	**0.9554**	**0.9579**	**0.9513**	**0.9566**	**0.9992**
			(7.8557)	(4.0458)	(4.1465)	(3.5224)	(3.5975)	**(2.4638)**
100	0.2	0.5	**0.9996**	**0.9680**	**0.9669**	**0.9550**	0.9487	0.0384
			(3.8651)	(1.6467)	(1.6645)	**(1.5521)**	(1.5622)	(0.2242)
		1.0	**0.9942**	**0.9614**	**0.9623**	0.9459	0.9445	0.5322
			(8.9324)	**(3.7215)**	(3.8417)	(3.2516)	(3.3245)	(0.6967)
		2.0	0.8966	**0.9539**	**0.9579**	0.9389	0.9412	0.8751
			(22.0831)	**(16.7659)**	(17.8464)	(12.4326)	(12.9750)	(4.1159)
	0.5	0.5	1.0000	**0.9623**	**0.9624**	**0.9512**	**0.9519**	0.1266
			(1.6621)	(0.6085)	(0.6115)	**(0.5927)**	(0.5954)	(0.0594)
		1.0	1.0000	**0.9561**	**0.9582**	0.9433	0.9462	0.8441
			(4.1570)	**(1.2303)**	(1.2424)	(1.1663)	(1.1766)	(0.4028)
		2.0	**0.9565**	0.9497	**0.9523**	0.9404	0.9409	**0.9899**
			(8.2323)	(4.0172)	(4.0756)	(3.6098)	(3.6542)	**(2.1795)**
	0.8	0.5	1.0000	**0.9590**	**0.9631**	**0.9501**	**0.9547**	0.8216
			(1.2451)	(0.3745)	(0.3770)	**(0.3663)**	(0.3686)	(0.1055)
		1.0	1.0000	**0.9551**	**0.9568**	0.9476	0.9483	**0.9975**
			(2.7509)	(0.7554)	(0.7609)	(0.7273)	(0.7324)	**(0.4582)**
		2.0	**0.9518**	0.9497	**0.9526**	0.9469	0.9480	**0.9999**
			(4.0077)	(2.3383)	(2.3610)	(2.1800)	(2.1998)	**(1.6955)**

Table 4. The AIC values of the non-zero observations from Chiang Klang, Tha Wang Pha, and Pua in Nan, Thailand.

Areas	Distributions					
	Normal	Lognormal	Cauchy	Exponential	Gamma	Weibull
Chiang Klang	430.7372	**347.4835**	387.5394	356.1853	355.0152	353.0018
Tha Wang Pha	477.9087	**363.7785**	415.4823	386.6203	366.9576	363.8529
Pua	425.2069	**335.9760**	379.7269	346.6206	344.5560	342.1551

Table 5. The BIC values of the non-zero observations from Chiang Klang, Tha Wang Pha, and Pua in Nan, Thailand.

Areas	Distributions					
	Normal	Lognormal	Cauchy	Exponential	Gamma	Weibull
Chiang Klang	434.3506	**351.0968**	391.1527	357.9920	358.6286	356.6151
Tha Wang Pha	481.6923	**367.5621**	419.2659	388.5121	370.7412	367.6365
Pua	428.8642	**339.6333**	383.3842	348.4493	348.2133	345.8124

Table 6. The 95% confidence intervals and credible intervals for the common CV of rainfall datasets from Chiang Klang, Tha Wang Pha, and Pua in Nan, Thailand.

Methods	Lower	Upper	Lengths
FGCI	2.0363	4.4528	2.4165
E-B.Indj	1.8975	4.7920	2.8945
E-B.U	1.9039	5.0631	3.1592
C-B.Indj	1.7583	4.3644	2.6061
C-B.U	1.7540	4.4703	2.7163
MOVER	2.3642	5.1399	2.7757